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CLUSTERING OF LOW SURFACE BRIGHTNESS DWARF GALAXIES. I. GENERAL PROPERTIES

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ABSTRACT. *A two-dimensional cluster analysis was executed on a homogeneous sample of the Catalogue of low surface brightness (LSB) dwarf galaxies (Karachentseva, Sharina, 1988) covering the whole sky.*

The total set of dwarfs at certain radii of clustering is divided into clusters corresponding to real clusters and groups of galaxies.

Gas-rich (dSm, dIr, dIm) and gas-poor (dE, dSph) dwarfs cluster in a wholly different manner, which confirms the known segregation in dwarf galaxies distribution. The peculiarities of clustering have been noted for every galaxy type.

The cluster analysis was carried out for the catalogue divided into sub-samples on the basis of the different properties of the galaxies: mean surface brightness, gradient of brightness, etc. Here differences in the clustering properties are statistically significant, but are not as expressive as for other types.

The cluster analysis was made separately for the dwarfs located in dense regions (Virgo, Fornax) and a background (groups of galaxies of different population). The surrounding density determines both the morphological structure and the character of dwarfs clustering.

Having "cut off" the dense fluctuation we find that in the region with a low density of galaxies the spheroidal, magellanic and irregular dwarfs cluster in the same way.

Выполнен двумерный кластерный анализ однородной выборки Каталога карликовых галактик низкой поверхностной яркости (Караченцева, Шарина, 1988), охватывающего все небо. При определенных радиусах кластеризации вся *совокупность карликов распадается на карлики, соответствующие реальным скоплениям и группам галактик.*

Скучивание карликов богатых газом (dSm, dIm, dIr) и бедных (dE, dSph) происходит совершенно различным образом, что подтверждает известную сегрегацию в распределении карликовых галактик. Отмечены особенности скучивания для каждого типа. Проведен кластерный анализ каталога, разбитого на подвыборки по различным признакам галактик: средняя поверхностная яркость, градиент яркости и др. Здесь различия в скучивании статистически значимы, но не столь выразительны, как для разных типов.

Анализ скучивания проведен также отдельно для карликов, расположенных в плотных (Virgo, Fornax) областях и в рассеянном фоне, состоящем из групп галактик разной населенности. Плотность окружения определяет как морфологический состав, так и характер скучивания карликов. "Срезав" плотную флуктуацию, мы получаем, что в области с низкой плотностью галактик сфероидальные, магеллановы и иррегулярные карлики скучиваются одинаковым образом.

I. INTRODUCTION

For the last few years LSB dwarfs have been generally recognized as testing particles for checking the theories of galaxy formation and formation of the large-scale structure. There are problems in this context which are being actively discussed: does the space distribution of dwarfs repeat the distribution of normal galaxies and what determines the observed characteristics - initial conditions of their formation or the influence of their surroundings?

The measurements of redshifts at 21 cm for large samples of dwarfs and normal galaxies allow us to compare their mutual distributions by different methods (see, for example, Eder et al., 1989; Thuan et al., 1991, and references therein). According to these authors there is no large difference between the clustering properties of dwarfs, LSB and bright galaxies.

A comparison of the optical and radio characteristics of over 300 gas-rich dwarfs (spiral, magellanic, irregular) in clusters, groups and background was made by Karachentseva (1990). It was shown that luminosities, linear diameters, colours, and HI-line widths do not change on the average for a given type, depending on the surrounding density. An increase of the "mass-luminosity" ratio has been noted only for magellanic dwarfs when passing from the more to less dense regions.

Our goal was to solve the question about the influence of the surroundings on the morphology and peculiarities of distribution of LSB dwarfs applying the method of cluster analysis to a sufficiently full and homogeneous sample of the Catalogue of

LSB dwarf galaxies (Karachentseva and Sharina, 1988). The two-dimensional variant was chosen so as to take into consideration all types of dwarfs, including the ellipticals and spheroidals.

The general character of LSB dwarf distribution over the sky allows us to place them in the volume of the Local Supercluster (Karachentseva, Sharina, 1988).

A short description of the method used is given in Sec. II. The results of the cluster analysis which has been applied to the whole sample, to the Virgo and Fornax clusters, and to the less dense region consisting of different population groups, are presented in Sec. III. The discussion of results and conclusions are summarized in Sec. IV.

II. ORIGINAL MATERIAL AND TECHNIQUE OF ANALYSIS

Let us briefly describe the Catalogue. The objects in the Catalogue (covering the whole sky) were chosen by low surface brightness, with a weak or lacking gradient of brightness. Where it was possible the luminosities were determined, and the value $M = -16$ was chosen as the upper limit. The catalogue thus contains mainly LSB dwarfs with the confusion effects of the far LSB normal galaxies kept to a minimum.

Morphologically the objects of the Catalogue were classified as elliptical (dE), spheroidal (dSph), spiral (dSm), irregular magellanic (dIm) and irregular without magellanic signs (dIr). Note also, such signs as the mean surface brightness 1 (SB ≈ 24.5 m/□"); 2 (24.6 - 25.5 m/□"); 3 (weaker than 25.6 m/□"), the presence of a weak (W) or absent (0) surface brightness gradient, description of signs of a nucleous: star-like "nucleous" (N*), diffuse one (ND), nucleous is absent (NO), and others are involved into the coded description of the object. A detailed description is given in the work of Karachentseva and Sharina (1988). We emphasize that the classification of dwarfs by different signs was used for all objects of the Catalogue uniformly. It was not possible to determine all characteristics with confidence for all dwarfs, so the overall numbers do not coincide with the total number of objects of the Catalogue in the tables of data.

We have attempted to elucidate how the observed clustering in the distribution of dwarfs over the sky reflects their morphological peculiarities. Among the different mathematical models of solving this task we have chosen the method of clustering galaxies according to the principle of geometrical search for the nearest neighbour, when the nearest neighbours are declared to be such galaxies for which the distance *d* does not exceed the pre-assigned radius of clustering *r*. As a consequence of this process the set of galaxies is divided into the subsets of ones isolated from other clusters with different intrinsic galaxy populations (Gregul et al., 1991).

The choice of the nearest neighbour strategy was due primarily to the importance placed on the concept of contiguity for any pair of galaxies. One of the confirmations of a successful choice for the clustering model can be seen in the fact that the centers of the clusters picked out by the model coincided with the centers of groups of dwarfs in the places where they were really located. We note that a typical property of this kind of methods is the contrast between the extraordinary mathematical simplicity of the algorithm and the complication of calculations.

In this task the calculation of distance on the sphere between every pair of galaxies was made in angular units by the well-known formula:

$$\cos d = \sin b_i \sin b_j + \cos b_i \cos b_j \cos (l_i - l_j),$$

where l, b - the galactic coordinates, d - the angular distance between the galaxies. The radius of clustering was varied within the optimal limits in the analysis of the "subcatalogues" of galaxies sorted out by the morphological features. After this the calculation of intrinsic population of all clusters was made for a given chosen r and the corresponding summary histograms were plotted. The method of calculation of different characteristics inside the cluster will be described in the next work.

III. RESULTS

III.1. The whole catalogue

Firstly all dwarfs of the Catalogue (except the Local group members) were considered without their division according to the types and other features.

For a broad variety of radii of clustering (0.5; 1; 2; 2.5; 3; 3.5; 4; 4.5; 6; 10°) the corresponding distributions were constructed. The results are shown in Fig. 1.

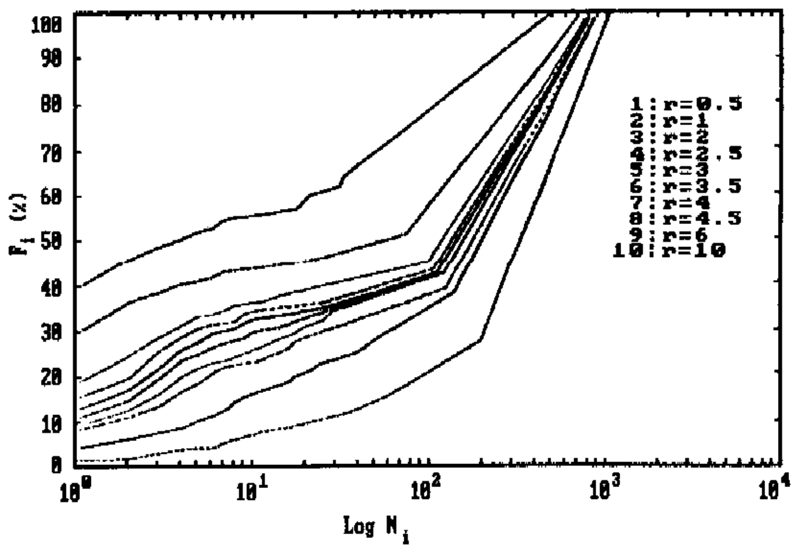


Fig. 1. Clustering of all LSB dwarfs of the Catalogue for a set of clustering radii. The abscissa is the logarithm of the number of dwarfs in a cluster ($\log N_i$), the ordinate is the integral fraction of galaxies entering clusters with a population $N_g < N_i$, F_i (%). The upper curve corresponds to the minimum clustering radius. The clustering radii are expressed in angular degrees. The designations in all the following figures are the same as in Fig. 1

Here the abscissa axis is the logarithm of the number of galaxies in the cluster, log N_i - (cluster population) and the ordinate - the integral function $F_i(\%)$ - the fraction of galaxies in clusters with the $N_g < N_i$.

Analysis of the distribution of clusters over the sky shows that they closely outline the known groups and clusters. For all *r* a smooth rise from isolated galaxies to groups is seen and after the break corresponding to the Fornax cluster the sharp turn to the Virgo cluster is observed. The fraction of isolated dwarfs changes from about 40% for $r = 0.^{\circ}5$ to 2% for $r = 10^{\circ}$. It is seen that for *r* from 2° to $4.^{\circ}5$ the clustering of dwarfs is stable. These values have therefore been chosen to construct suitable graphs for two subsamples of dwarfs: dIr+dIm+dSm (N = 540) and dSph + dE (N = 995), Figs.2 and 3. Their comparison shows an impressive distinction in the character of clustering reflecting the well-known fact of morphological segregation of dwarfs. Gas-rich dwarfs cluster more or less smoothly from isolated to clusters, meanwhile elliptical and spheroidal dwarfs are located mainly in the Virgo and Fornax galaxy clusters. The fraction of isolated dwarfs for a given *r*, and the whole shape of curves differ significantly.

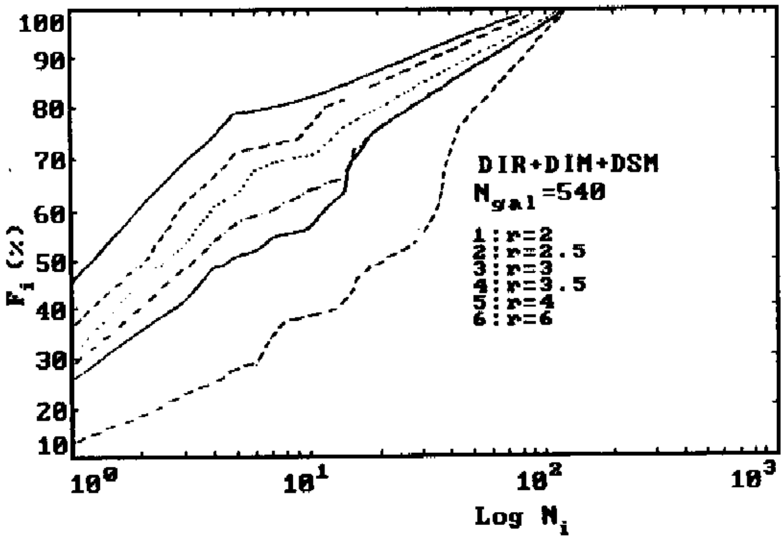


Fig.2. Clustering of dIr+dIm+dSm dwarfs (the whole Catalogue).

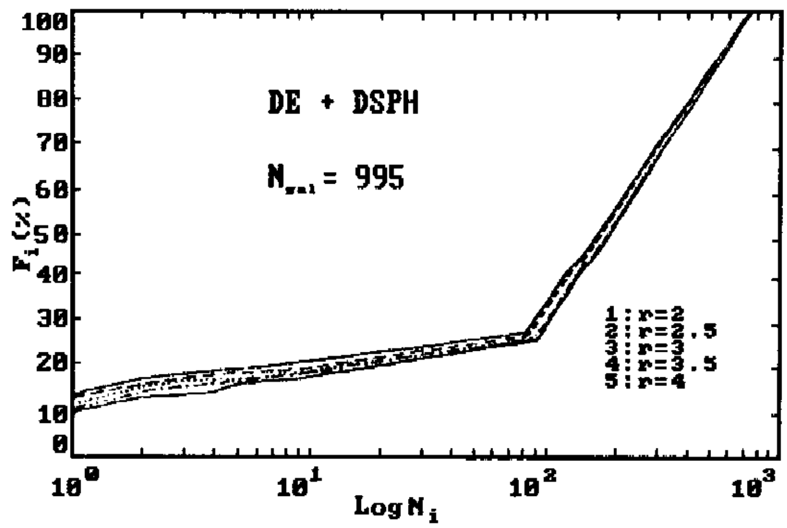


Fig.3. Clustering of dE+dSph dwarfs (the whole Catalogue).

a) Types

The Catalogue has been divided into the subsamples according to the dwarfs types: dSm (N=101), dE (N=539), dSph (N=400), dIr (N=244) and dIm (N=139). Classes dSph and dE are likely to have small admixture of types dE, dIr and dSph, respectively, due to the difficulties of morphological classification.

For each type a clustering procedure has been carried out at $r = 0.5;\ 1;\ 1.5;\ 2;\ 3^{\circ}$ and the graphs similar to Figs.1-3 have been constructed. It is seen from Figs. 4a-e that each dwarf type clusters in its own way; this is valid for all r.

The Kolmogorov-Smirnov test shows that the H-hypothesis is not acceptable only for the dIm-dSm and dIm-dIr pairs for the smallest $r = 0\overset{\circ}{.}5$ and 1°. In all remaining cases the distribution functions of dwarfs of different morphological types are statistically different at a 99 per cent confidence level.

In a compact form the results of dwarfs clustering (the F_1 function) by type and isolation degree depending on r are presented in Table 1. Here the clusters are grouped according to their population as " isolated" ($N_i = 1$), "pairs" ($N_i = 2$), "groups" ($N_i = 2 - 20$) and "clusters" ($N_i \geqslant 21$).

The results are clear enough and do not need detailed comments. It can be seen that dSph, dE and dSm, dIr, dIm dwarfs cluster in different ways. Even without this known fact, however, it is possible to distinguish more subtle features of the different types of dwarfs clustering. *Spiral* dwarfs are practically single or form "pairs". *Irregular* dwarfs are mainly single, but also cluster into "pairs", "groups", and at $r = 3^{\circ}$ in more populated "clusters" also. *Magellanic* dwarfs demonstrate a comparatively stronger tendency to clustering with dSm and dIr: thus about 30%

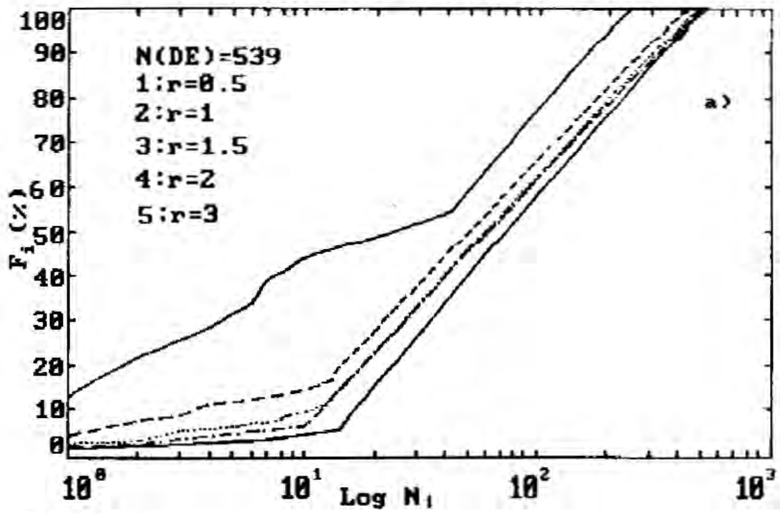


Fig.4a.

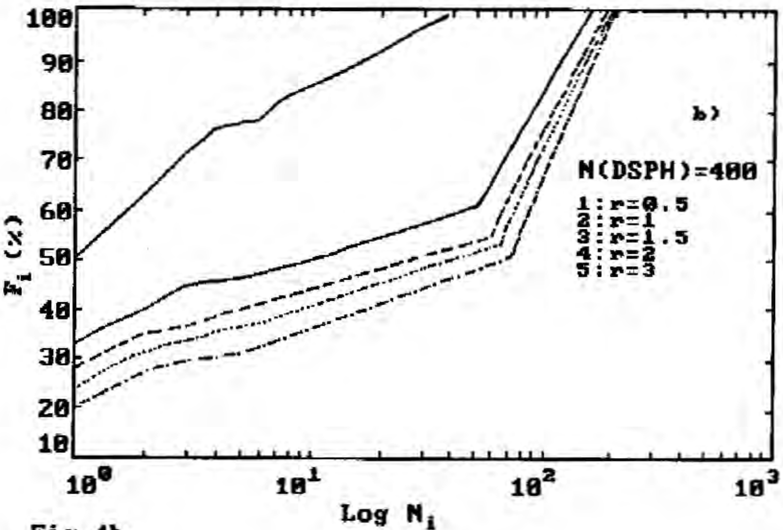


Fig.4b.

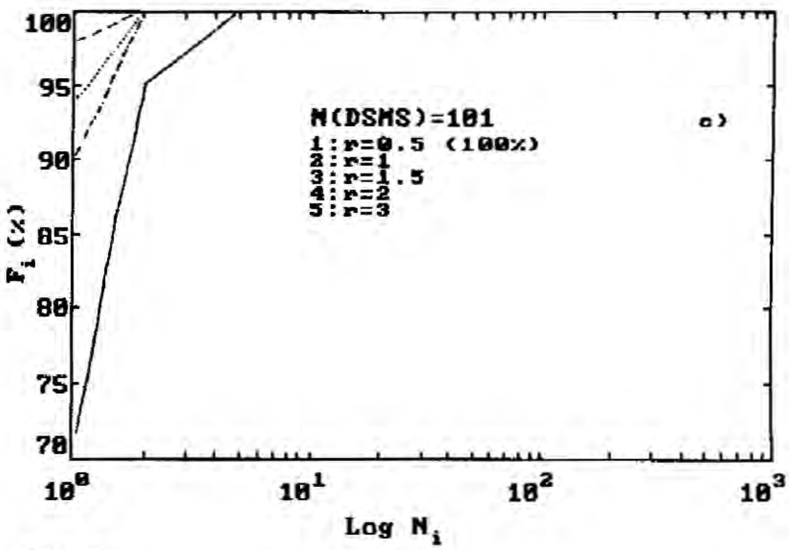


Fig.4c.

of magellanic dwarfs enter into rich "groups" for $r=2$ and 3°. *Spheroidal* dwarfs at the smallest $r = 0.5^\circ$ prefer to form "pairs" and "groups" and beginning from $r=1^\circ$ to 3° about 60% of them enter into rich "clusters". *Elliptical* dwarfs demonstrate the strongest clustering (more than 50% for $r = 0^\circ.5$ and near 90% for $r = 3^\circ$). Here the influence of the Virgo cluster causes the fraction of elliptical dwarfs among the dwarf galaxies to be dominating.

b) Surface brightness

Consider the way the dwarfs of different surface brightness classes cluster. In the Catalogue they contain: (1) N = 139, (2) N=590, (3) N=792. These subsamples correspond quite closely to luminosity classes IV-V, V, and VI according to van den Bergh (1966).

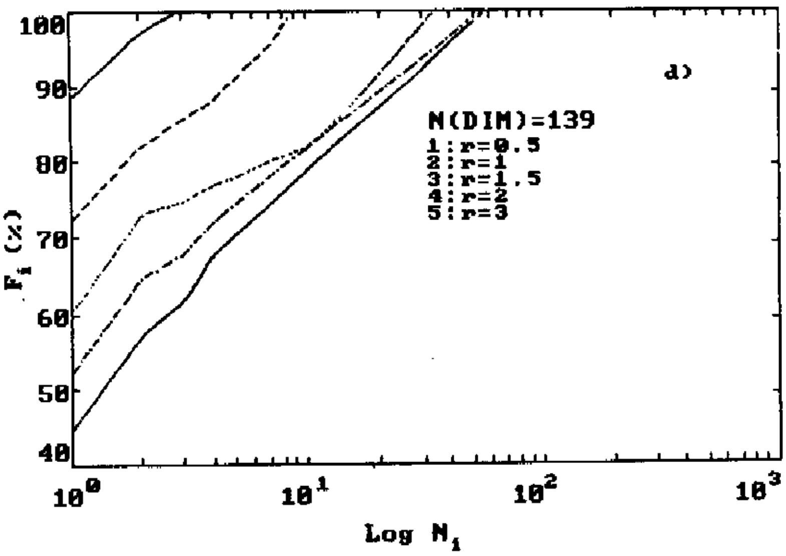


Fig.4d.

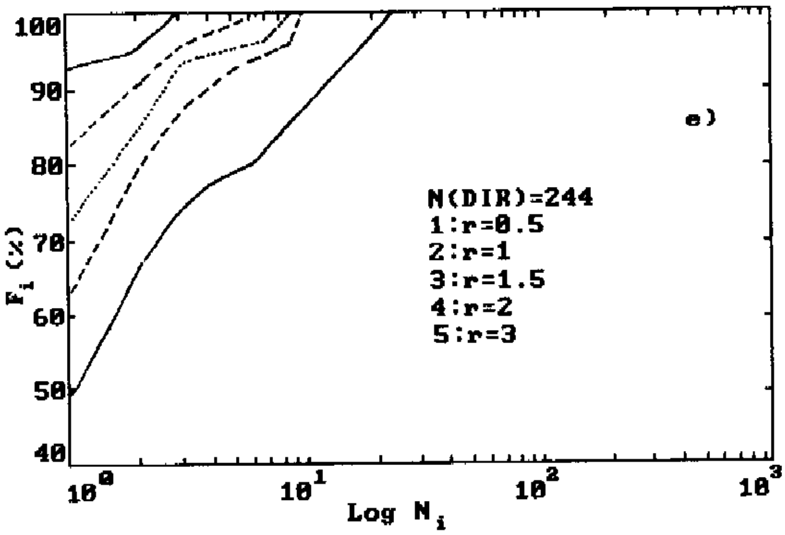


Fig.4. Clustering of dwarfs of the Catalogue divided into subsamples by types: a) dE, b) dSph, c) dSm, d) dIm, e) dIr (the whole Catalogue).

For r the same as in the preceding case the clusters have been constructed and corresponding cumulative functions are drawn (Figs. 5a-c). The Kolmogorov-Smirnov test confirms the significant difference of all distributions taken in pairs for each r.

The brightest dwarfs (1) cluster more rapidly in comparison with (2) and (3) and they show a significantly greater part of singles for small r. This is attributable to the fact that the surface number density of dwarfs of class (1) is less than those of (2) and (3).

The lowest surface brightness dwarfs (3) cluster more smoothly than (2) and the fraction of single galaxies of class (3) is greater than of (2) despite their greater surface number density. Overall one can conclude that with higher mean surface brightness a more rapid clustering of dwarfs takes place with increasing r. Without detailing we give in Table 2 the data for clusters with $N_1 = 1$ and $N_1 \geqslant 21$ for the three classes of surface brightness.

Table 1

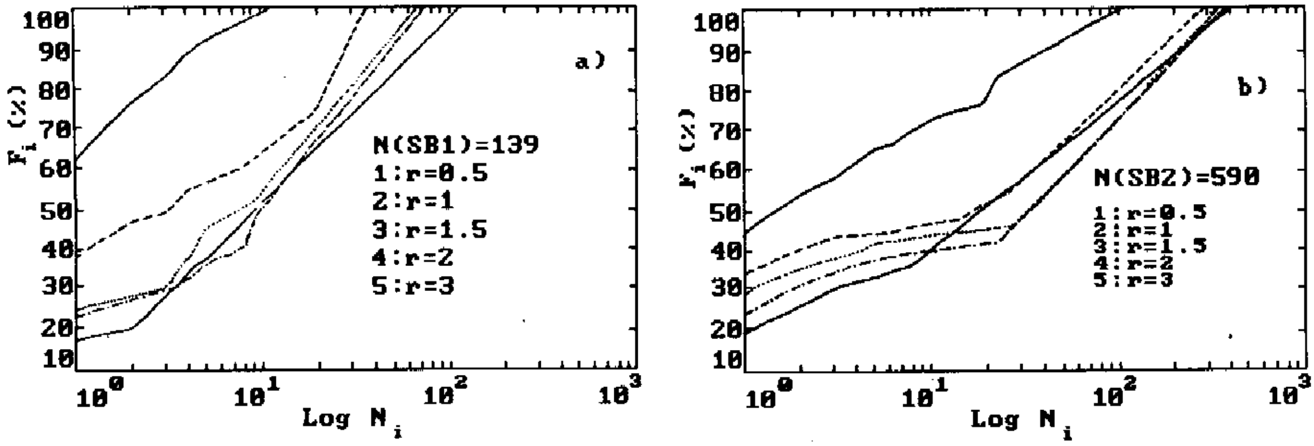
Clustering of the dwarfs of different types depending on the radius of clustering r (the whole sky)

Type/r	0°.5	1°.0	1°.5	2°.0	3°.0
		$N_i = 1$			
dSm	100%	98.0	94.0	90.1	71.3
dIr	92.6	82.4	72.5	63.8	48.6
dIm	88.6	72.5	60.6	52.3	44.6
dSph/dSpI	50.0	33.2	28.0	23.75	19.75
dE/dSph	14.1	5.2	3.3	2.4	2.2
		$N_i = 2$			
dSm	0	2.0	6.0	9.9	23.8
dIr	2.5	8.2	12.3	16.4	18.0
dIm	8.3	19.3	12.4	12.4	12.4
dSph/dSpI	13.0	7.0	7.0	7.5	7.5
dE/dSph	8.2	3.1	0.4	0.4	0
		$N_i = 3 - 20$			
dSm	0	0	0	0	4.9
dIr	4.9	9.4	15.2	20.5	22.9
dIm	3.1	18.2	9.4	7.3	13.5
dSph/dSpI	27.0	4.75	4.75	5.75	5.0
dE/dSph	24.3	11.2	11.0	3.6	3.9
		$N_i \geq 21$			
dSm	0	0	0	0	0
dIr	0	0	0	0	10.3
dIm	0	0	17.6	28.0	29.5
dSph/dSpI	10.0	60.25	60.25	63.0	67.75
dE/dSph	53.4	80.5	89.0	93.6	93.9

Table 2

Clustering of dwarfs of different classes of the surface brightness depending on r (the whole sky)

SB \ r	0°.5	1°.0	1°.5	2°.0	3°.0
		$N_i = 1$			
1	61.9%	38.1	24.5	22.3	16.5
2	43.4	33.0	28.6	23.2	18.5
3	50.1	38.9	34.0	29.5	21.8
		$N_i \geqslant 21$			
1	0	25.8	48.1	52.3	80.6
2	20.6	50.3	59.3	62.8	64.4
3	16.1	44.5	51.1	52.9	55.7



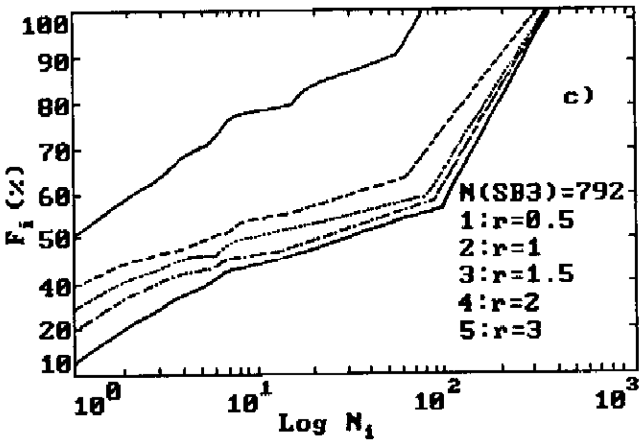


Fig.5. Clustering of dwarfs of the Catalogue divided into subsamples by the classes of surface brightness: a) brightest b) intermediate brightness, c) lowest (the whole Catalogue).

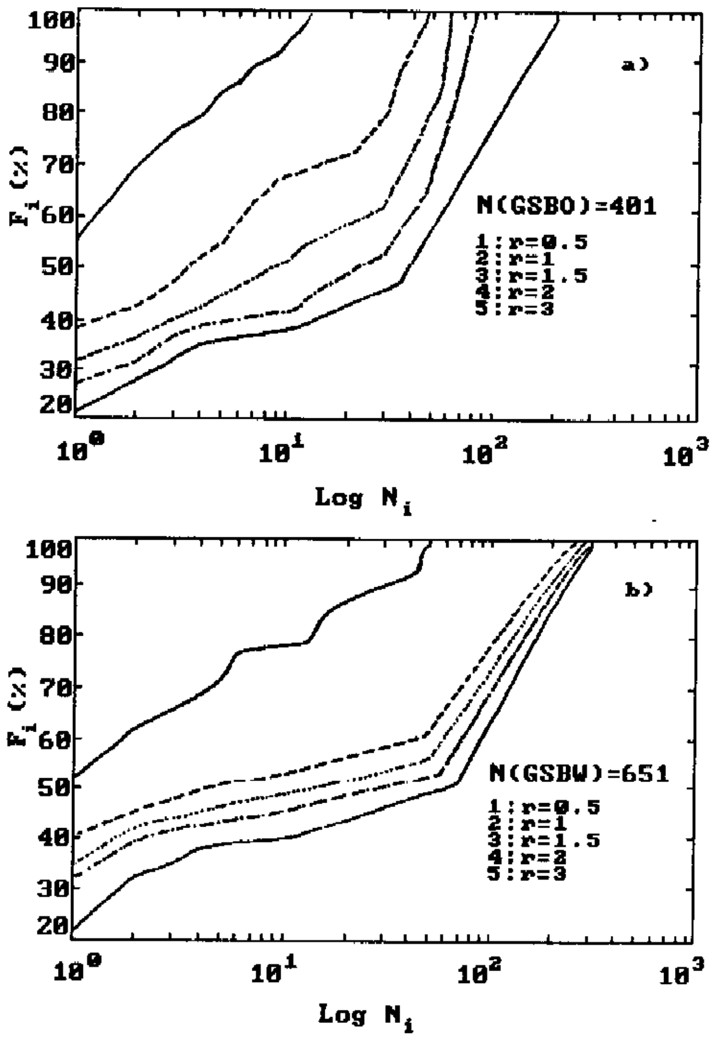


Fig.6. Clustering of dwarfs of the Catalogue divided into subsamples by sign of surface brightness gradient: a) gradient is absent, b) gradient is weak (the whole Catalogue).

c) Gradient of surface brightness

This feature was determined visually. The Catalogue's objects have been divided into two subclasses: dwarfs having a weak surface brightness gradient, GSBW (N=651), and dwarfs without an appreciable one, GSB0 (N=401). The cumulative functions F (log N_1) are given in Figs. 6 a-b. Here for all *r* the differences are significant at a confidence level *p* higher than 99%.

A general conclusion notes that although the surface number density of dwarfs W is 1.5 times higher than of dwarfs 0, for minimum $r = 0.5^{\circ}$ the fractions of isolated galaxies for these two subclasses are practically equal (52 and 55%, respectively). But they have different slopes of clustering: for all *r* all cluster population dwarfs without brightness gradient cluster more rapidly than dwarfs W.

d) Sign of "nucleus"

Dividing the Catalogue's objects by this feature, the subsamples contain differences in the number of galaxies: the subsample with star-like nucleus N* (32), with diffuse nucleus ND (110), with uncertain sign of nucleus N? (297), and without nucleus NO (635). In several cases of (N*) we are not sure if it is a real star-like nucleus or a star projected near the centre. It must be a subject of special investigation. The small number of objects in the first two subsamples is caused by the principle of selection of dwarfs in our catalogue, i.e. with the designation of low and extremely low surface brightness dwarfs. It is clear that scarce dwarfs N* and ND are either mainly isolated or form poorly populated clusters. It is more interesting to compare the dwarfs NO and N? More than half of NO dwarfs are single and that fraction is equal to 20% even at 3° (similar to dSph, see Table 1). This feature distinguishes them from N? dwarfs, the latter clustering more rapidly and constitutes a smaller fraction of single dwarfs at all *r*. Note also that at $r = 0.5^{\circ}$ it distinguishes from the rest.

Thus, this method of cluster analysis, applied to all Catalogue objects, shows the clearly defined dense regions of dwarf clustering (the Virgo and the Fornax clusters) and loose background containing more or less populated groups. We obtained the more prominent clustering differences of all investigated characteristics with morphological types of dwarfs. Having a sufficient collection of graphs on dwarfs clustering by different features, it is possible to try and find some correlations between them. From comparison we found a similarity of clustering for dSph with W, W with NO, dSph with NO, and dSph with 3, i.e. from the whole set of characteristics the class of spheroidal dwarf galaxies without nucleus, with low brightness gradient, and luminosity class V-VI according to van den Bergh is distinguished from the rest. This type of dwarfs seems the most homogeneous. The other types have no such distinct correlations with other features indicated in the Catalogue.

III. 2. The Virgo cluster

a) Types

For the analysis of clustering the following set of clustering radii: r = 0.2, 0.3, 0.35, 0.5, 1, 2, 3° has been chosen. 493 dE from 593 dwarfs of the Catalogue, 197 dSph from 400, 31 dIr from 244, 55 dIm from 139 have been entered into the Virgo cluster. Spiral dwarfs in the Virgo cluster are practically absent.

The character of clustering of gas-rich and gas-poor dwarfs is completely different, clearly apparent from the graphics presented in Figs. 7a-d.

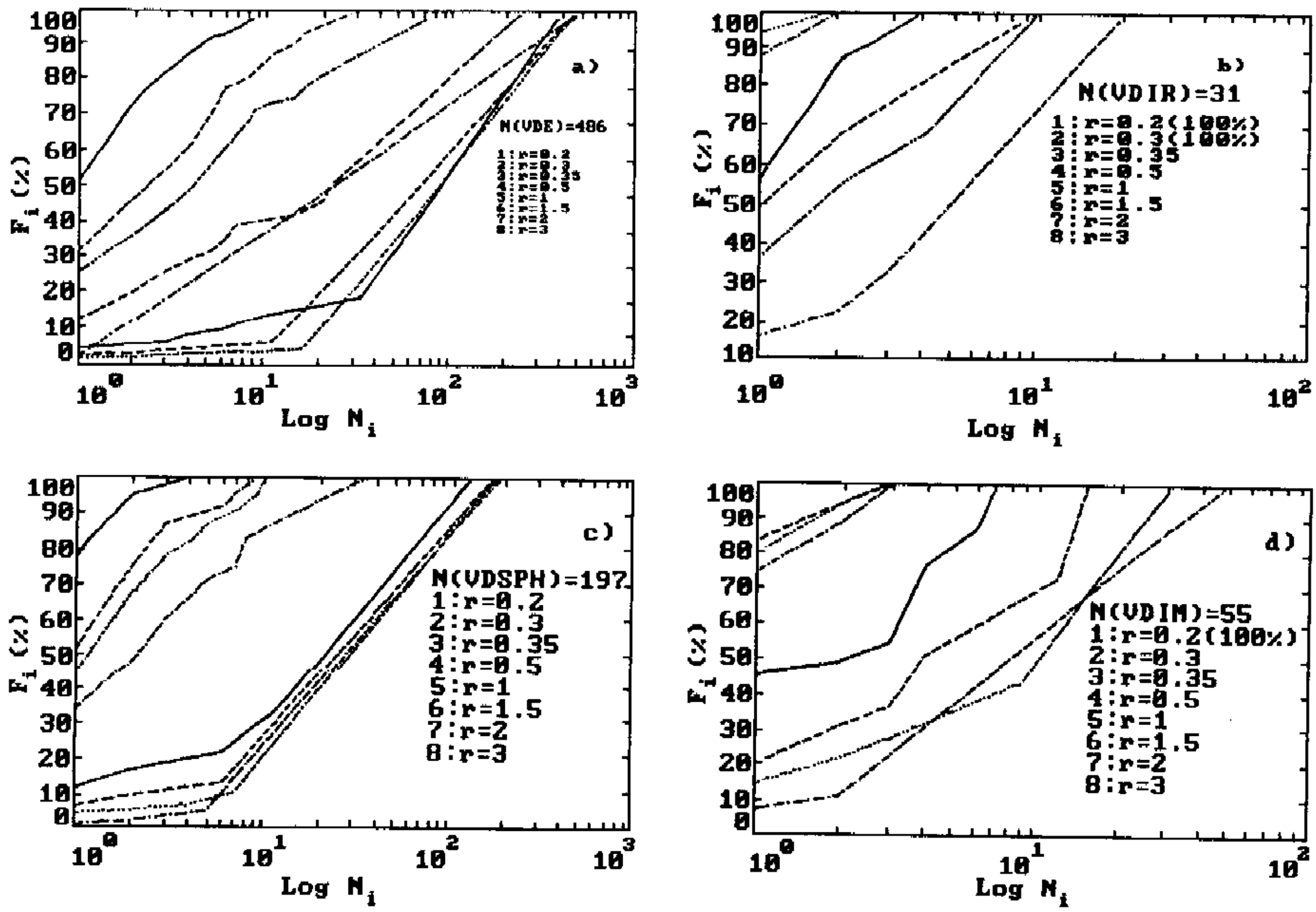


Fig.7. Clustering of dwarfs of the Catalogue divided into subsamples by types: a) dE, b) dSph, c) dIm, d) dIr (the Virgo cluster).

For the irregular galaxies, as compared to the magellanic ones, the fraction of isolated dwarfs is two times larger at all r and especially starting from r = 1.5°; the clusters are very small and the clustering process goes rapidly.

The spheroidal dwarfs in the Virgo are more diffused compared with dE and are collected into less populated clusters.

In a compact form the results are presented in Table 3.

b) Surface brightnesses

The distribution of the set of dwarfs over the classes of mean surface brightness in the Virgo cluster is as follows: N(1)=111, N(2)=370, and N(3)=330. The results are presented in Table 4 and in Figs. 8 a–c.

Table 3

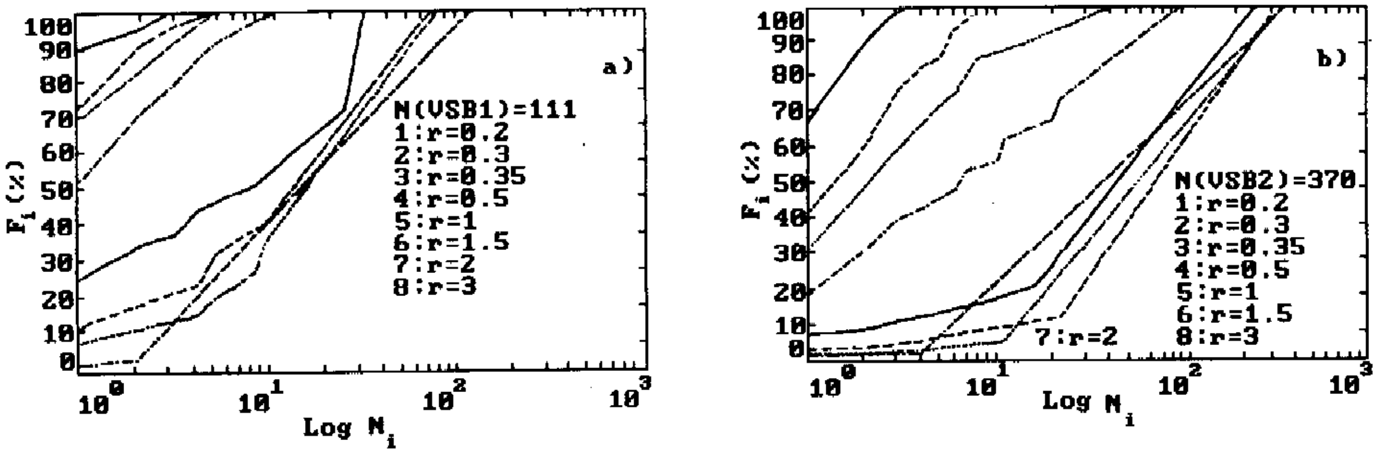
The clustering of dwarfs of different types depending on r (the Virgo cluster)

Type \ r	$0\overset{\circ}{.}2$	$0\overset{\circ}{.}3$	$0\overset{\circ}{.}35$	$0\overset{\circ}{.}5$	$1\overset{\circ}{.}0$	$1\overset{\circ}{.}5$	$2\overset{\circ}{.}0$	$3\overset{\circ}{.}0$
			$N_i = 1$					
dIr	100%	100	93.5	87.1	54.8	48.4	35.4	16.1
dIm	100	83.6	80.0	74.5	45.5	20.0	14.5	7.3
dSph	77.3	51.3	44.7	33.5	11.7	6.6	4.5	1.5
dE	51.2	31.2	25.1	11.5	3.3	1.8	0.6	0.6
			$N_i = 2$					
dIr	0%	0	6.5	12.9	32.2	19.4	19.4	6.5
dIm	0	10.9	14.5	14.5	3.6	10.9	7.3	3.6
dSph	17.3	23.4	22.3	14.2	5.0	3.0	2.0	1.0
dE	22.6	16.0	11.5	8.6	1.6	0.4	0	0
			$N_i = 3 \div 20$					
dIr	0%	0	0	0	13.0	32.2	45.2	9.7
dIm	0	5.5	5.5	11.0	50.9	69.1	21.8	0
dSph	5.0	25.3	33.0	22.8	16.7	3.0	3.6	2.5
dE	26.2	46.4	41.8	26.6	8.1	3.6	3.3	0
			$N_i \geq 21$					
dIr	0%	0	0	0	0	0	0	67.7
dIm	0	0	0	0	0	0	56.4	89.1
dSph	0	0	0	17.3	66.6	87.4	89.9	95.0
dE	0	6.4	21.6	53.3	87.0	94.2	96.0	99.4

The highest surface brightness dwarfs in Virgo (class **1**) are more isolated as compared with (2) and (3) (from the smallest r to $r = 1\overset{\circ}{.}5$), and their groupings constitute an insignificant fraction.

Unlike class 1, over 50-60 % of dwarfs of classes 2 and 3 group into poor and medium clusters at $r = 0\overset{\circ}{.}3 - 0\overset{\circ}{.}5$. At these r the clustering degree for the faintest dwarfs is highest, as different from the data over the entire catalogue.

In other words, in a dense surrounding the minimum mutual distances are noted for the weakest surface brightness dwarfs. For larger r the fraction of dwarfs of class 1 grows in medium and rich clusters. The Kolmogorov-Smirnov test gives a significant difference at a 0.99 confidence level between all the classes (1), (2), (3) at all clustering radii exept (2) - (3) at r = $0\overset{\circ}{.}2$.



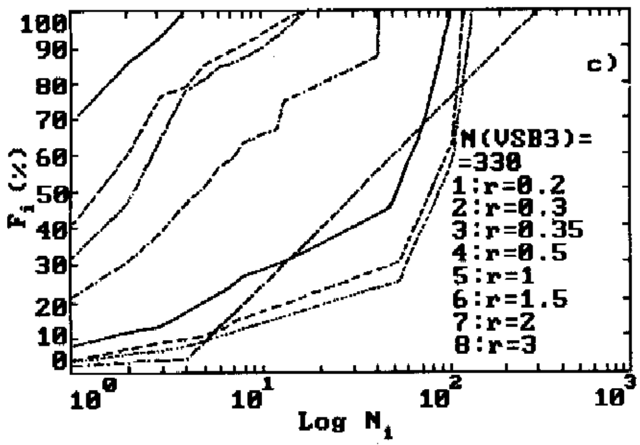


Fig.8. Clustering of dwarfs of the Catalogue divided into subsamples by classes of surface brightness: a) brightest, b) mean brightness, c) lowest (the Virgo cluster).

c) Gradient of surface brightness

The number of dwarfs without a visible gradient of surface brightness (GSBO) in Virgo totals 214 and those with a weakly visible one (GSBW) total 302. The main results of clustering are presented in Table 5 and Figs.9 a-e.

It is clear that for all r and for all population levels the GSBO dwarfs cluster more strongly than GSBW. This difference is significant at a 0.99 confidence level by the Kolmogorov-Smirnov test.

Comparison of these results with the data of Table 4 indicates that the character of clustering for dwarfs of class 1 and GSBW is similar.

d)The sign of nucleus

The number of dwarfs with a star-like nucleus (N*) in Virgo totals 15, and with a diffuse nucleus (ND) totals 20 they are mainly singles or form clusters with N_i = 2-3. Compare clustering of the dwarfs without the nucleus sign (NO), (they total 291) and those with a nucleus being implied (N?) (they total 212). The results are presented in Table 6.

Table 4

Clustering of dwarfs of different classes of surface brightness depending on r (the Virgo cluster)

SB \ r	0°.2	0°.3	0°.35	0°.5	1°.0	1°.5	2°.0	3°.0
			N_1 = 1					
SB1	89.2%	73.9	70.3	53.2	25.2	12.6	8.1	1.8
SB2	66.5	40.8	30.3	18.1	7.0	2.7	1.4	1.1
SB3	69.1	40.6	31.8	21.2	7.9	3.9	3.6	2.7
			N_1 = 2					
SB1	5.4%	16.2	12.6	18.0	9.0	0	3.6	1.8
SB2	21.6	18.4	17.8	11.8	1.6	0.5	0.5	0
SB3	16.3	19.4	15.2	9.7	3.6	3.0	1.8	0
			N_1 = 3 ÷ 20					
SB1	5.4%	9.9	17.1	28.8	17.1	27.9	23.4	0
SB2	11.9	40.8	40.1	37.5	18.2	3.6	4.1	1.1
SB3	14.6	40.0	53.0	43.8	19.1	7.2	3.7	1.2
			N_1 ≥ 21					
SB1	0	0	0	0	48.7	59.5	64.8	96.4
SB2	0	0	11.8	32.6	73.2	93.2	94.0	97.8
SB3	0	0	0	25.3	69.4	85.9	90.9	96.1

Table 5

Clustering of dwarfs without brightness gradient and with weak gradient depending on r (the Virgo cluster)

GSB \ r	0°.2	0°.3	0°.35	0°.5	1°.0	1°.5	2°	3°
			N_1=1					
GSB0	80.8%	52.3	44.9	33.2	13.1	6.5	4.7	2.8
GSBW	94.4	92.4	91.0	75.2	28.8	16.0	9.9	4.6
			N_1 = 2 ÷ 20					
GSB0	19.2%	46.7	55.1	66.8	34.5	15.0	7.5	0
GSBW	5.6	7.0	9.0	24.8	30.8	24.8	19.5	9.6
			N_1 ≥ 21					
GSB0	0 %	0	0	0	52.4	78.5	87.8	97.2
GSBW	0	0	0	0	40.4	59.2	70.6	85.8

Fig.9. Clustering of dwarfs of the Catalogue divided into subsamples by sign of surface brightness gradient: a) gradient is absent, b) gradient is weak (the Virgo cluster).

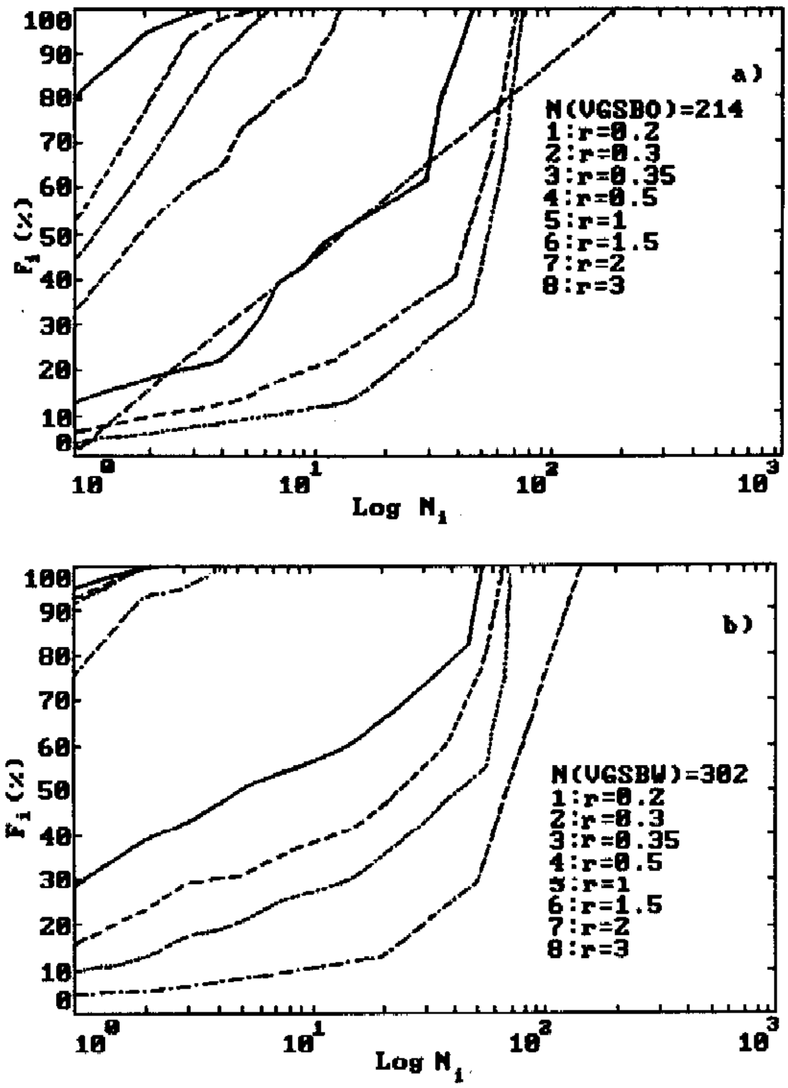


Table 6

Clustering of dwarfs without nucleus and with implied nucleus as dependent on r (the Virgo cluster)

r / Nucleus	0°.2	0°.3	0°.35	0°.5	1°.0	1°.5	2°	3°
			N_i =1					
NO	69.4%	44.7	37.8	25.4	7.9	4.1	2.4	2.1
N?	74.1	49.0	8.7	21.2	11.3	4.7	3.3	1.4
			N_i = 2-20					
NO	30.6%	55.3	2.2	63.8	20.5	10.7	5.9	2.4
N?	25.9	51.0	1.3	62.7	9.0	6.6	1.9	2.3
			$N_i \geq 21$					
NO	0 %	0	0	10.8	71.6	85.2	91.7	95.5
N?	0	0	0	16.1	79.7	88.7	94.8	96.3

It is seen that the character of clustering for NO and N? dwarfs is nearly similar. However the comparison by the Kolmogorov-Smirnov test gives a significant difference in the clustering degree of these classes of dwarfs.

Dwarfs without the nucleus demonstrate at all r a tendency to clustering into small clusters. N? dwarfs are the most isolated at small r, and at large r their fraction in the most populated clusters is higher than for NO dwarfs.

III. 3. The Fornax cluster

The Fornax cluster differs from the Virgo in both the total number of dwarfs (111), and distribution over the types: dSm, dIm are absent, N(dIr)=17, N(dE)=15, N(dSph)=74. The results of recent works devoted to the search for dwarf galaxies in Fornax (Caldwell and Bothun, 1987; Ferguson, 1989) have not been taken into account in our analysis. We therefore consider these results as preliminary.

The clustering has been executed for r = 0.3, 0.5, 1.0, 1.5, 2.0 and 3°0 for different types, different classes of surface brightness, brightness gradient, and sign of nucleus.

Note only some results. Elliptical and irregular dwarfs in Fornax are clustered rapidly, and the fraction of single irregular dwarfs is larger than that of dE for all r. Spheroidal dwarfs in Virgo demonstrate the larger fractioning in clustering than in Fornax, which is likely to be caused by the presence of a few subsystems of normal galaxies in Virgo. One can see the same when comparing Fornax dwarfs and Virgo dwarfs of class 3 and class W. In other words the larger fractioning in clustering of the faintest spheroidal dwarfs in Virgo reflects the presence of the subsystems of bright galaxies as different from the more compact Fornax cluster.

III. 4. A galaxy background

a) Types

The set of dwarfs of the Catalogue avoiding the Virgo and Fornax clusters we have called a background.

The procedure of clustering for the background has been executed at r = 0.5, 1, 1.5, 2, 2.5, 3, 4, 5° for the subsamples of types: N(dSm)=89, N(dIr)=187, N(dIm)=136, N(dSph)=129, N(dE)=25. The results are presented in Figs. 10(a-e).

First of all note that the dwarfs in the background are either singles and their fraction smoothly decreases from dSm to dSph for all r, or they enter into thinly populated groups. Only 3.7% of irregular dwarfs are clustered into the clusters with the $N_i > 21$ for r = 0.5 and 1°. The Kolmogorov-Smirnov test demonstrates that a mutual run of clustering for dIr, dIm, dSph types of dwarfs at all r is practically indiscernible.

Only dE and dSm dwarfs demonstrate a significant difference in clustering as compared with the rest of the types. Here we see a principal difference between the clustering of dwarfs in the dense Virgo region and in the region with low density of galaxies.

b) Surface brightness

In the background the number of dwarfs of class 1 is N = 28, N (2) = 209 and N (3) = 362. The results of clustering are presented in Figs.11(a-c). Brighter galaxies are practically all singles. As compared with the Virgo cluster the dwarfs of class 2 demonstrate a similar run of curves of clustering, and an approximately identical fraction of isolated dwarfs at displacement of r (5° in the background corresponds to $0^\circ.3$ in Virgo). The faintest dwarfs are practically isolated up to $r = 2^\circ.5$, and after that they are clustered into clusters of a small and medium population. The difference between the character of clustering of dwarfs of 2 and 3 by the Kolmogorov-Smirnov test is reliable only for $r = 2^\circ.5$, 4°, and 5°. These values of r are of the same order as the characteristic sizes of nearby groups, i.e. one may speak about a weak segregation

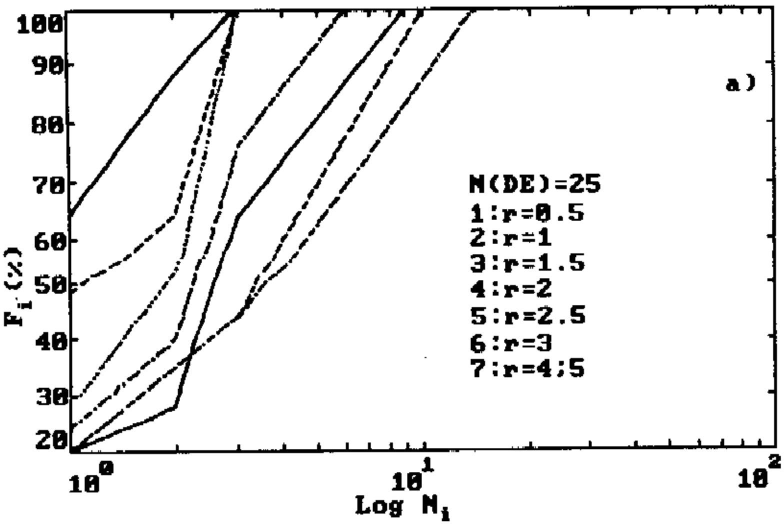


Fig.10a.

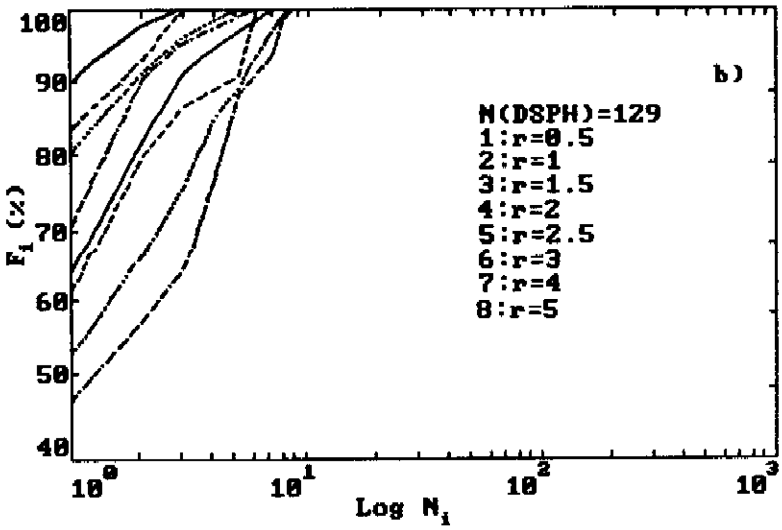


Fig.10.b.

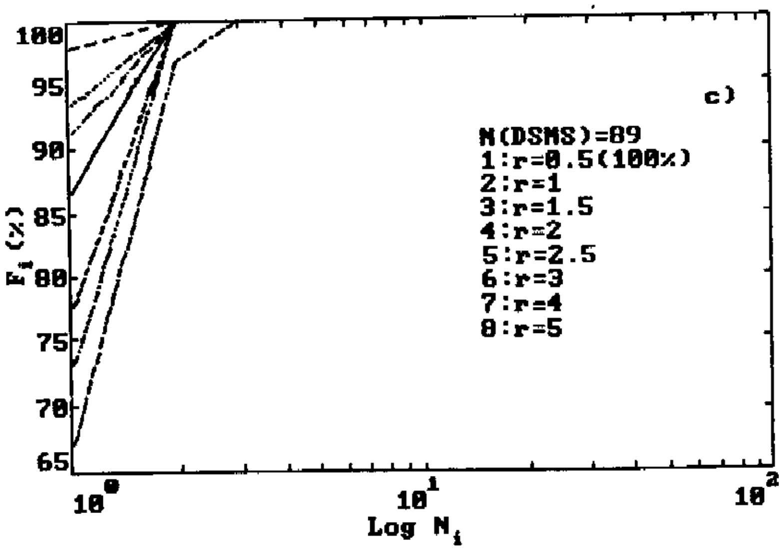


Fig.10.c.

of dwarfs according to the surface brightness in groups of galaxies.

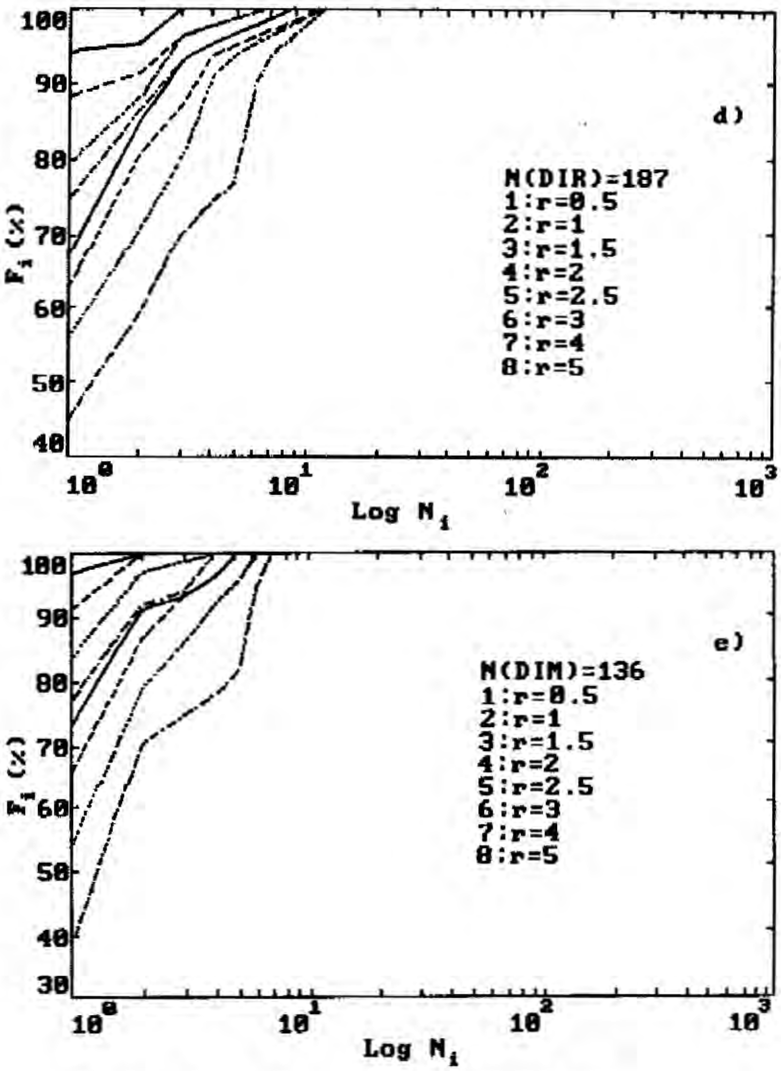


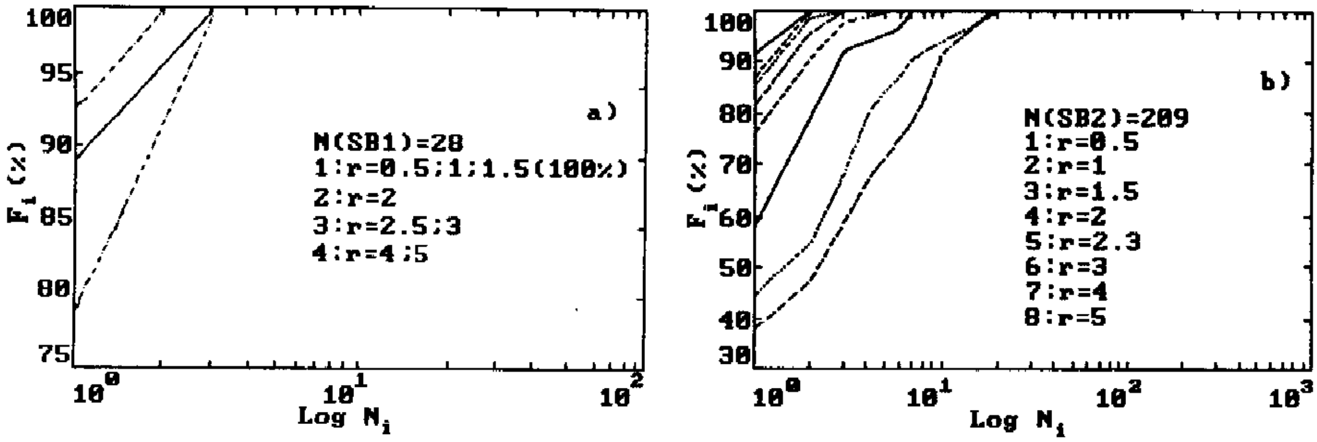
Fig. 10. Clustering of dwarfs of the Catalogue divided into subsamples by types: a) dE, b) dSph, c) dSm, d) dIm, e) dIr (a background).

c) Gradient of the surface brightness

The number of GSBO dwarfs in the background is equal to 147, and N(GSBW) = 275. Figs. 12 (a-b) demonstrate that from the least radii to $r = 3^{\circ}$ the fraction of singles in both classes exceeds 50%. For larger r the GSBO dwarfs cluster steeper than GSBW dwarfs. The difference is significant at a 99% confidence level.

d) Sign of nucleus

The number of dwarfs in the background with sign N* is equal to 15, N(ND) = 89, N(N?)= 54 and N(NO) = 268. The data for ND, N? and NO are listed in Table 7. From the data of Table 7 it is seen that the character of clustering is different for all subsets of dwarfs. Dwarfs without the nucleus cluster most tightly. The weakest clustering is seen for dwarfs with a diffuse nucleus.



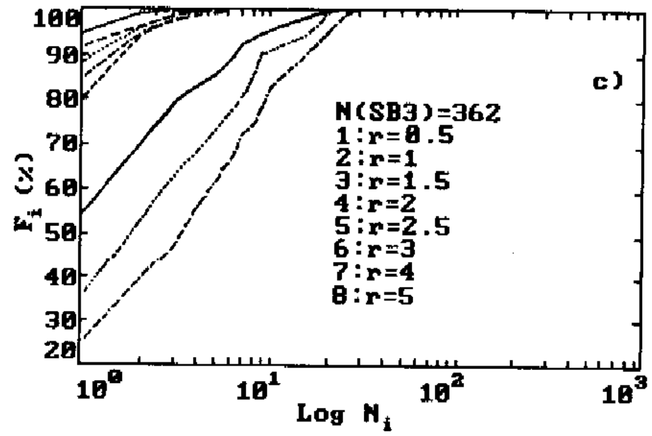


Fig. 11. Clustering of dwarfs of the Catalogue divided into subsamples by the classes of surface brightness: a) brightest, b) mean brightness: c) lowest (a background).

d) Sign of nucleus

The number of dwarfs in the background with sign N* is equal to 15, N(ND) = 89, N(N?)= 54 and N(NO) = 268. The data for ND, N? and NO are listed in Table 7. From the data of Table 7 it is seen that the character of clustering is different for all subsets of dwarfs. Dwarfs without the nucleus cluster most tightly. The weakest clustering is seen for dwarfs with a diffuse nucleus.

Table 7

Clustering of the dwarfs without nucleus and with implied and diffuse nuclei depending on r (background)

r \ Type	0°.5	1°.0	1°.5	2°.0	2°.5	3°.0	4°.0	5°.0
			N_i = 1					
ND	97.8%	95.5	91.0	88.8	83.1	76.4	73.0	62.9
N?	92.6	81.5	77.8	72.2	68.5	66.7	59.2	57.4
NO	90.7	79.1	66.8	63.0	53.7	47.4	33.9	27.2
			N_i > 1					
ND	0	0	0	0	0	3.4	9.0	14.6
N?	0	0	0	13.0	13.0	22.2	29.7	31.5
NO	4.8%	10.5	17.5	24.3	29.1	35.4	47.5	58.6

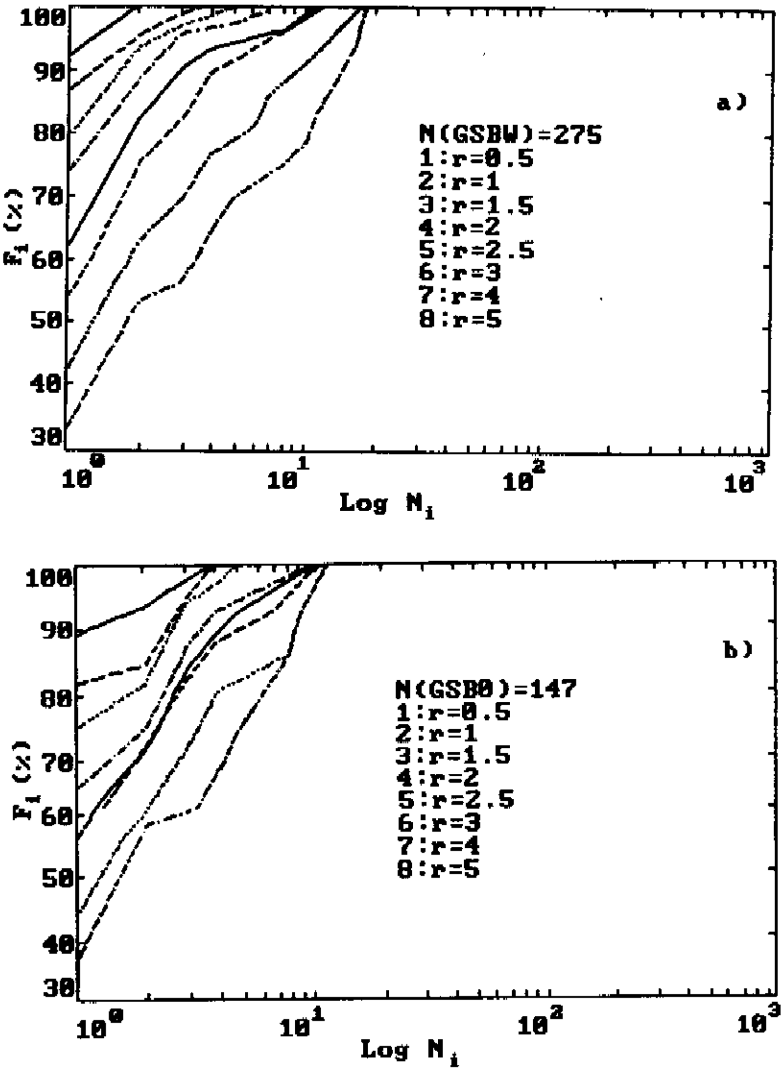


Fig.12. Clustering of dwarfs of the Catalogue divided into subsamples by sign of surface brightness gradient: a) gradient is absent, b) gradient is weak (a background).

IV. DISCUSSION OF RESULTS AND CONCLUSIONS

Having considered numerous versions of the two-dimensional cluster analysis applied to LSB dwarfs examined in the volume of the Local Supercluster, we note the following results:

1. Considering dwarfs as "indiscernible particles" we have found that they are clustered into two clusters: with a high population coinciding with the Virgo and Fornax clusters, and in a number of clusters with a smaller population coinciding with the known groups of galaxies.

2. Dividing the whole set of dwarfs according to their different features, we obtain significant differences for the different types (dE, dSph, dIr, dIm, dSm) of dwarfs. Other signs are not so pronounced and give a smoothed notion about the clustering degree.

3. In the dense region of the Virgo cluster the dE and dSph dwarfs dominate accounting for 3/4's of the total number of dwarfs in the cluster. Spiral dwarf galaxies are practically absent in the Virgo cluster. The character of clustering for

all types is reliably different moreover. Elliptical and spheroidal dwarfs cluster more strongly. The clustering of dwarfs is also evidence of the existence of several galaxy subsystems in Virgo.

4. In the region with a low galaxy density an abundance of morphological types of dwarfs is observed, differing from that of the Virgo region. Elliptical dwarfs are extremely rare and together with dSph dwarfs make up only 1/4 of the total number of LSB dwarfs of the "background". The character of clustering for dIm, dIr and dSph dwarfs is the same.

5. It is known that there are no principal differences between spheroidal dwarfs located in the Local group, group M 81, the Virgo cluster or background (Karachentseva et al., 1987; Richter et al., 1989). One can say the same about the properties of gas-rich dwarfs located in Virgo, groups and background (Karachentseva, 1990). However intrinsic differences between the dwarfs of different morphological types are essential and defined mainly by the ratio of their stellar and gas components . Thus the high density of the surrounding of dwarfs defines both their "morphological abundance" and the character of clustering. Having excluded the dense fluctuation (galaxy clusters) we see both gas-poor (dSph) and gas-rich (dIm, dIr) dwarfs are mixed well enough and clustered indiscernibly.

6. At last we note that for an investigation of clustering of LSB dwarfs it is necessary to distinguish their "dominant" feature - the morphological type and "recessive" ones - the surface brightness, gradient of brightness and etc., and to stipulate clearly in which region by density this investigation is under way.

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