THE EXISTENCE OF THE STRUCTURAL ANISOTROPY OF THE JAGIELLONIAN FIELD OF THE GALAXIES

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Abstract. The total galaxy set of the Jagiellonian catalogue ($6° \times 6°$, $N = 15650$ galaxies) has undergone two-dimensional clusterization procedure. The position angle of the major axis and the ratio of axes of the effective ellipse of inertion are calculated for every cluster. The histograms 'position angle – number of galaxies' are investigated for the different samples of clusters. The histograms at the certain radii of clusterization ($R = 2'$–$3'$) and galaxy population levels ($6 \le N_{min} \le 15$) demonstrate the pronounced and statistically significant anisotropy with the smooth trend along the position angle F and preferable orientation in the range of $F = 105°$–$120°$.

When we take into consideration the systematical catalogue errors, the problem of reality of this effect has been analysed. The question about possible space scale of anisotropy is discussed. Its lower limit is evaluated by magnitude 500 Mps. One cannot explain the result obtained by influence of systematical errors of catalogue, as in that case they would have to be enormous.

1. Introduction

A search and investigation of the anisotropy in the observed part of the Universe are important at least with the two points of view: cosmological – by the construction of evolutional models of the Universe – and cosmogonical – by the choice of a real scenario of genesis of galaxies and their clusters. An adoption of hypothesis of the Universe isotropy unambiguously leads to the Friedmann world model, which occupies a dominant position in modern cosmological works. Alternative models could be adopted if only a real Universe as a whole, has or had in the past a deviation from isotropy. Now this idealized statement of question digresses slightly into the background in connection with the idea of an inflationary Universe. The basic theoretical predictions of the existence of the structural anisotropy appeared last in the modern distribution of the visible matter which have subcosmological and cosmological scale. Similarly, in the works by Chibisov and Shtanov (1989, 1990) an anisotropy of the modern structure of the galaxies distribution was predicted as a consequence of chaotic inflationary expansion of the Universe on the early stage. An expected space-scale of such an anisotropy is limitted only from below by the magnitude of a few tens Mpc. In accordance with the authors a structural anisotropy arises owing to the development of one fluctuation on the background of another, which have essentially larger scale. The authors consider that the discovery of the structural anisotropy could be served the observational test of the chaotic inflation scenario. Of course, such an investigation is also important regardless of any cosmological scenario. On the other hand, the anisotropy manifestations on limited space-scales could serve as an important information source in a cosmogonical sense. The theoretical investigations show (see, for example, Oort, 1958; Ozernoy, 1974;

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Icke, 1973; Doroshkevich, 1973; Peebles, 1983; Bhattacharjee, 1989) that the character of galaxies orientational alignment within their clusters or superclusters could serve as a key in the choice of the real variant of the theory of galaxies and their systems formation.

The main directions of modern investigations of an anisotropy are defined by three objects (i) background radiations, (ii) field of galaxy velocities, (iii) structure of the distribution of the visible matter.

The contemporary quest of large-scale quadrupole anisotropy of the microwave relict radiation meanwhile leads to negative results (see, for example, Lubin and Vilella, 1986; Fixen *et al.*, 1980; Lukash and Novikov, 1987; Strukov *et al.*, 1988). The attempts to discover global cosmological magnetic field from polarization of the background radiations (for example, see Bernandis, 1989) are interesting.

The indicated large-scale anisotropy of Hubble expansion (effect of the Rubin *et al.*, 1973) refuted a long idea about rather high level of isotropy of the galaxy velocity field. This result was so unexpected that it has excited lively reaction of leading specialists since these years. The observed magnitudes of galaxy velocities deviations from the Hubble expansion field are explained by gravitational influence of the great attractor. By efforts of many groups of observers (for example, see Aaronson *et al.*, 1986; Dressler *et al.*, 1987) its residence is localized at a distance of 45 Mpc ($H = 100$ km s^{-1} Mpc^{-1}) in Hydra–Centaurus direction. The generally accepted cosmological interpretation of this phenomenon is absent; yet theoretical investigations (for example, see Peebles, 1987; Vittorio *et al.*, 1988; Kaiser, 1988; Steblins and Turner, 1989; Hoffman and Zurek, 1988) are contradictory.

The existence of sufficiently complicated structure in distribution of the visible matter of the Universe allows to set a question about possible orientational alignment of elements of this structure on any space-scales.

First of all, the question is about searches for anisotropy at galaxy orientations. This problem was investigated by many authors, beginning from F. Brown's pioneer work in 1938, and a number of essential results were obtained (see, for example, Gregory *et al.*, 1981; Binggelli, 1982; Dekel, 1985; Kaprandis and Sullivan, 1983; MacGillivray and Dodd, 1985; Lambas *et al.*, 1988; Mandzhos *et al.*, 1987). Besides the investigation of orientations of galaxies themselves, the question about the existence of the orientational alignment of other structural units of any level in the cosmic hierarchy of visible matter is actual. Both the morphologically pronounced cosmic systems (for example, clusters of the galaxies) and apparent conglomerates of the matter, formally selected according to the strict mathematical algorithm (for example, percolational clusters of galaxies on the celestial sphere) can serve as these units. In any case an algorithm of the selection of structural units must be submitted to one necessary condition: namely, it must be isotropic (i.e., it must not introduce the anisotropy) in those cases when the structure itself is obviously isotropic.

Unlike the investigations of galaxy orientations which drew rather much attention, the number of works on orientation of other structural units is very small. In particular, the question of existence on different space-scales of the structural anisotropy in galaxy distribution was not yet sufficiently investigated. It is to the point here to emphasize once again that the structural anisotropy in that context is orientational alignment of the identical structural units. It goes without saying that a certain scale is not necessary to be *a priori* set up, but may transpire in the process of investigation.

To the works in this direction we refer the investigation of orientations of galaxy clusters of Zwicky's catalogue (Mandzhos, 1976a), the article on orientation of the Abell clusters (Binggelli, 1982) and investigation of structural anisotropy by methods of the percolationary cluster-analysis: three-dimensional analysis of the galaxies distribution of CGCG catalogue (Chincarini *et al.*, 1988) and two-dimensional analysis of the Jagiellonian field (Mandzhos and Telnjuk-Adamchuk, 1976b, 1979).

The present investigation is a development of these works (Mandzhos and Telnjuk-Adamchuk, 1976b, 1979). The subject of this investigation by the cluster-analysis method is the Jagiellonian field of galaxies from the point of view of the existence of the structural anisotropy.

2. Jagiellonian Catalogue

The *Jagiellonian Catalogue* (Rudnicki *et al.*, 1973) contains 15650 galaxies, which are distributed on the sky area of $6° \times 6°$ with the center coordinates $\alpha = 11^h 19^m$ and $\delta = 35°53'$ (2000.0). The limiting apparent magnitude of the catalogue (in red rays) is equal to $19^m\!.5$, the galaxies are marked on the atlas 1127×1127 mm in size. One can roughly estimate the size of the cone, which this catalogue 'cuts off' in the three-dimensional space. For this aim we used the contemporary Hubble diagrams (Tammann, 1984) and carried out estimation in the frames of Friedmann cosmological model ($q_0 = \frac{1}{2}, H = 75$ km s^{-1} Mpc^{-1}). For these cosmological parameters the distance (photometric) to the cone base and cone volume are $R_{\rm Jag} \cong 3000$ Mpc and $V_{\rm Jag} \cong 10^8$ Mpc, respectively. For the comparison we note that the radius and volume of the Local Supercluster of galaxies is evaluated by magnitudes of $R_{\rm LS} \cong 30$ Mpc, $V_{\rm LS} \cong 10^5$ Mpc, respectively. Thus the Jagiellonian cone has cosmological space scale and its volume contains thousands of systems like the Local supercluster. It is necessary to emphasize that the galaxy sets became considerably incomplete while approaching to the cone base due to the threshold effect, and it is necessary to take into account this circumstance in interpretating the results.

3. The Method

As we noted, the two-dimensional percolational cluster of galaxies was chosen as an elementary structural unit in this work. Although the procedure of selection these clusters is repeatedly described in literature (see, for example, Mandzhos and Telnjuk-Adamchuk, 1979; Einasto *et al.*, 1984) we draw a brief attention to this question. The Cartesian coordinates of galaxies were measured on the map. The origin of the coordinates was placed to the vertex of the lower right angle of the Jagiellonian field. The axes x and y are oriented along the increasing of the right ascension α and the declination

δ, respectively (the coordinates are expressed in the mm of the Jagiellonian map). It is necessary to emphasize that we have in mind a two-dimensional cluster-analysis of the galaxy distribution in the sky plane of the Jagiellonian field. Although, on one hand, this is an obligatory question-decision, because we have no third galaxy coordinates; but, on the other hand,, the specific properties of the observational data of galaxies are such that a two-dimensional approach is the only possible one for the present. We will discuss this problem in detail below.

The clusterization procedure consists in dividing the complete set of galaxies of the Jagiellonian field into subsets-clusters. As percolating cluster we call such set of galaxies in which the distance for every one of its members to the nearest neighbour does not exceed some before-hand assigned radius of clusterization R. In the present two-dimensional case, the radius of clusterization is measured in angular units. As a result of such a procedure the field of galaxies is represented by the collection of clusters with different levels of galaxy population (singles, couples, triplets, etc.). After the clusterization of the galaxy field finished for every cluster (except the isolated galaxies) we calculated the parameters of the effective flat inertial ellipse: the center of cluster, the ratio E of the values of the inertial momenta relative to the main axis, the position angle F of the major axes orientation: i.e.,

$$F = \begin{cases} \pi - \varphi\,, & \text{for} \quad \varphi \in [0, \pi/2]\,, \\ -\varphi\,, & \text{for} \quad \varphi \in [0, -\pi/2]\,, \\ \pi/2\,, & \text{for} \quad I_{yy} > I_{xx}, I_{xy} = 0\,, \\ \pi\,, & \text{for} \quad I_{yy} < I_{xx}, I_{xy} = 0\,, \end{cases} \qquad \varphi = \operatorname{arctg}\left(\frac{\Lambda - I_{xx}}{I_{xy}}\right), \tag{1}$$

$$E = 2\Lambda(I_{xx} + I_{yy} - ((I_{xx} - I_{yy})^2 + 4I_{xy}^2)^{1/2})\,; \tag{2}$$

where

$$I_{xx} = \sum_{i=1}^{N} y_i^2 - Ny_c^2\,, \qquad I_{yy} = \sum_{i=1}^{N} x_i^2 - Nx_c^2\,, \qquad I_{xy} = \sum_{i=1}^{N} y_i x_i + Ny_c x_c\,,$$

the meanings of inertial momenta about the main axis;

$$x_c = \sum_{i=1}^{N} x_i \Big/ N\,, \qquad y_c = \sum_{i=1}^{N} y_i \Big/ N$$

the coordinates of center;

$$\Lambda = \tfrac{1}{2}(I_{xx} + I_{yy} + ((I_{xx} - I_{yy})^2 + 4I_{xy}^2)^{1/2}\,;$$

N is the number of the galaxies in the cluster.

To receive a general information about structural anisotropy, it is necessary to have the data on galaxy distribution in the three-dimensional space. In fact, we have at our disposal a projection of this distribution on to the sky plane. For this, the structural formations, which are placed at different distances, can overlay each other – that is a distorted factor. How one can judge by this flat picture about the existence of structural anisotropy in the investigated region? Due to the fact that the algorithm of selection structural units is isotropical by definition, only isotropical structure will be displayed on the plane for the isotropical three-dimensional structure. On the other hand, if the real anisotropy exists in the three-dimensional space, in the plane it may be smoothed up to null because of overlay effect. In other words, to confine the structural anisotropy at the plane is a sufficient, but not necessary, condition of the existence of the structural anisotropy in three-dimensional space. The question concerns the statistically-significant effects.

It should be supposed, that the knowledge of red shifts of all galaxies of the Jagiellonian field would permit to realize the three-dimensional variant of investigation for structural anisotropy. Indeed this is wrong. Our own peculiar galaxy velocities (noise at background of the Hubble expansion) are $\delta V = 200–500 \text{ km s}^{-1}$. For the red shifts from the 0 to the 1 the errors of distance will be equal to 3 Mpc and greater because of the peculiar galaxy velocities. Such errors are comparable with the dimensions of the clusters of galaxies. To be sure, we note that the dimensions of the selected Jagiellonian clusters are as a rule smaller than those dimensions. It points out that, in the three-dimensional analysis such errors would lead to a full disintegration of orientational alignment (if it exists) and an introduction of fictitious anisotropy. As an illustration of such situation we note the work by Chincarini *et al.* (1988) on the three-dimensional percolation clusterization of the galaxies in the space volume of the order of that of the Local Supercluster. Its authors noted that just such a peculiar velocity effect and, in particular, the effect of virial velocities in clusters and groups, leads to false stretching along the line-of-sight. Consequently, the three-dimensional analyis of the orientational structural alignment in galaxy distribution (for definition of the distances to galaxies by the red shifts) is not correct. However, the knowledge of red shifts would be very useful as far as they could serve though rough but rather important test of a true (three-dimensional) neighbourhood in every case of the close galaxies pair at the two-dimensional analysis of the structural anisotropy.

4. The Results

To choose the magnitude of the clusterization radius R we proceeded from that it is necessary for the statistical analysis to obtain a sufficiently large number of clusters with the population in every one not lower than 6–10 galaxies. We note that for too small R the overwhelming majority of the selected clusters are isolated galaxies, pairs, triplets. And at excessively large R all the galaxies of the field merge into the one large cluster.

We determined that the optimum range of the radius of clusterization is 6–10 mm on the scale of the catalogue map ($1'.9–3'$). We note that, in order to avoid the edge effects and possible systematic observational errors (the authors of the catalogue pointed out these facts), we considered not all Jagiellonian field, but the circle with the $R = 563$ mm, which was cut out from the field.

Several words about algorithm and program of the calculations. The task of the cluster-analysis of the type, which we have chosen, has certain difficulties in the sense of the quick-act and necessary size of memory for sufficiently large array of the primordial data. The optimization carried out by the algorithm founded on the ideas of the book by Dijkstra (1976) and use of the possibilities of the recursion methods permitted to solve effectively these two problems.

After finishing the clusterization procedure we formed a number of samples of galaxy clusters for different values of the parameters of clusterization radius R, minimum permitted number of galaxies in the cluster N, maximum permitted coefficient of the main ellipse axes ratio. For every sample the histograms 'position angle – number of cluster' were investigated. The most typical histograms are presented in Figure 1(a–e).

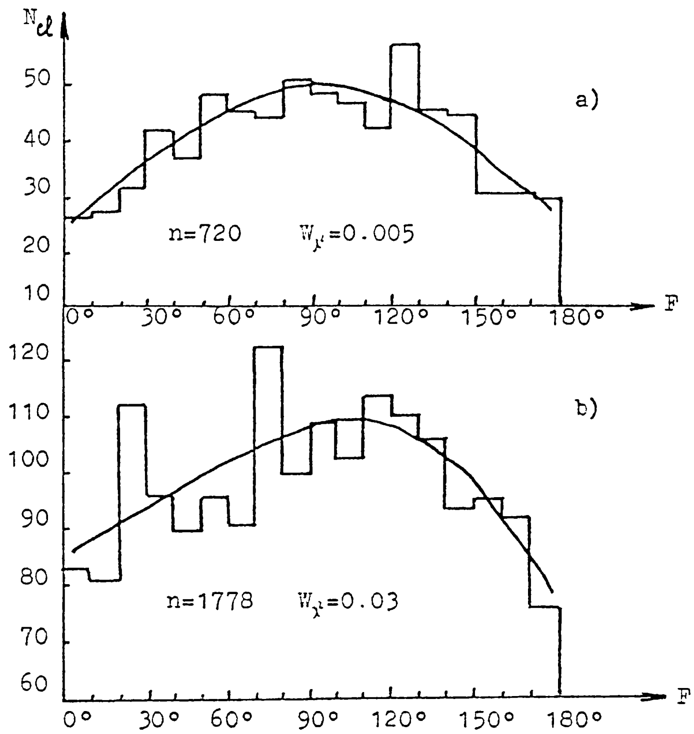

Fig. 1a–e. The histograms 'position angle-number of clusters' for different sets of the clusters of glaxies of the Jagiellonian field: (a) radius of the clusterization $R = 8$ mm, number of the galaxies in the cluster $N \geq 6$, (b) $R = 6$ mm, $N \geq 3$, (c) $R = 8$ mm, $N \geq 8$, ratio of the axis of the inertial ellipse $E \leq 0.25$, (d) $R = 8$ mm, $N \geq 6$, $E \leq 0.5$, (e) $R = 7$ mm, $N \geq 8$, $E \leq 0.25$.

The significance of deviation from isotropy was estimated by the κ^2-criterion of the Pearson – i.e., the possibility W, that the given histogram presents a random fluctuation from the isotropic distribution – has been estimated. Furthermore, for each histogram the regression curve was drawn on basis of least-squares method with the estimation of significance of the expansion coefficients by the Fisher criterion.

Both a trigonometrical basis of the Fourier expansion and polynomial basis were adopted as a primordial basis of regression functions that led to practically similar

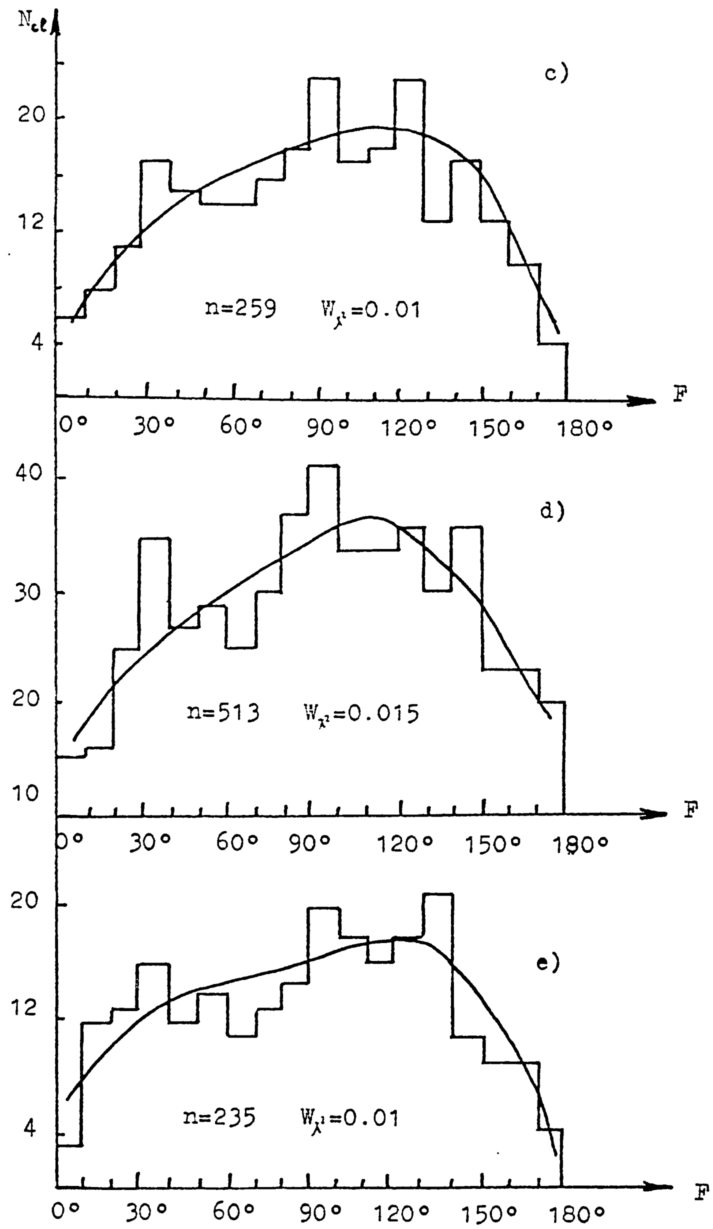

Fig. 1c–e.

results. In most of the cases the present histograms demonstrate the statistically-significant and strongly pronounced anisotropy of orientations of clusters with the preferential direction of major axes of effective inertial ellipses in the range of the position angles $F = 105°–120°$ and obvious minimum in direction of the sky meridian.

If we consider this effect to be real, then first of all a question is bound to arise about its space interpretation: does this effect concern the whole cone of Jagiellonian field, or – only a small part of it? The first checks, which can be done at once consists in dividing the field into the subzones. We picked out 5 subzones in the form of the circle with the $r = 281$ mm; four being inscribed each quadrants of the field, and the fifth circle had the centre in the centre of Jagiellonian field. The histograms for these subzones are presented in Figure 2(a–b) for samples which are characterized by the same parameters $R = 8$ mm, $N \geq 8$, $E \leq 0.5$. The figures demonstrate that the clusters in each subzones display the same tendency to preferential orientation in the range of position angles (105°–120°). The effects of anisotropy have no sufficiently reliable statistical significance because of small number of clusters in these samples; however, at amalgamation of subzones in the pairs (Figure 2(b)) the effect, of course, is conserved but the statistical

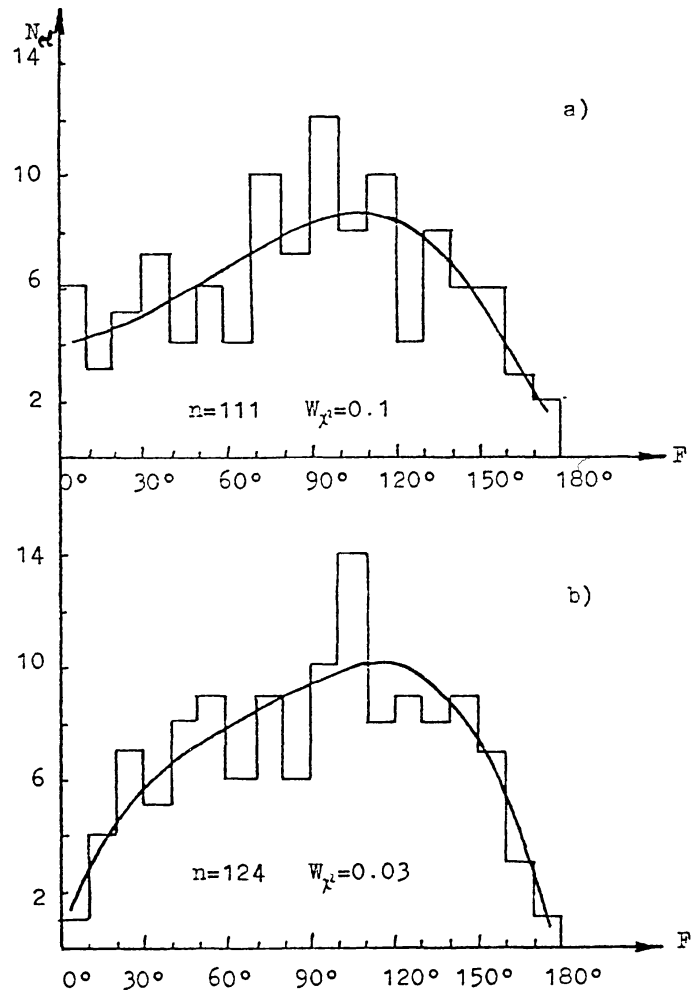

Fig. 2a–b. The histograms 'position angle-number of the clusters' for the sets of the clusters of the galaxies ($R = 8$ mm, $N \geq 8$, $E \leq 0.5$) for the two subzones of the Jagiellonian field: (a) central circle with $r = 281$ mm, (b) the subzones which are consisted from the third and fourth quadrants of the Jagiellonian field.

significance arrives to the sufficient magnitude. This allows to draw a conclusion that the effect of anisotropy is inherent in all Jagiellonian field.

Thus, the histograms of clusters demonstrate a strongly pronounced and statistically significant anisotropy. The effect takes place both for the Jagiellonian field, and for its parts. The question about statistical interpretation of the results arises.

It may be supposed that all the field consists of separate zones ('orientational domains'). Inside every such zone the more strict orientational order is present. The histogram of all the field is represented as result of the independent statistical summing up of orientational structures of all domains. In this case the orientation of an arbitrary-chosen cluster must be as a rule similar to orientation of its near neighbours, which are the members of the same domain. In reality, some of these orientational domains are clearly distinguished in the field. One of them is demonstrated in Figure 3 (R = 8 mm ($\approx 2\rlap{.}'5$), $N \geq 7$). These domains are interesting in themselves; however, does this picture define the typical tendency for the entire field? To clear up this question we calculated the mean values of modulus of the position angle differences of the of arbitrary chosen

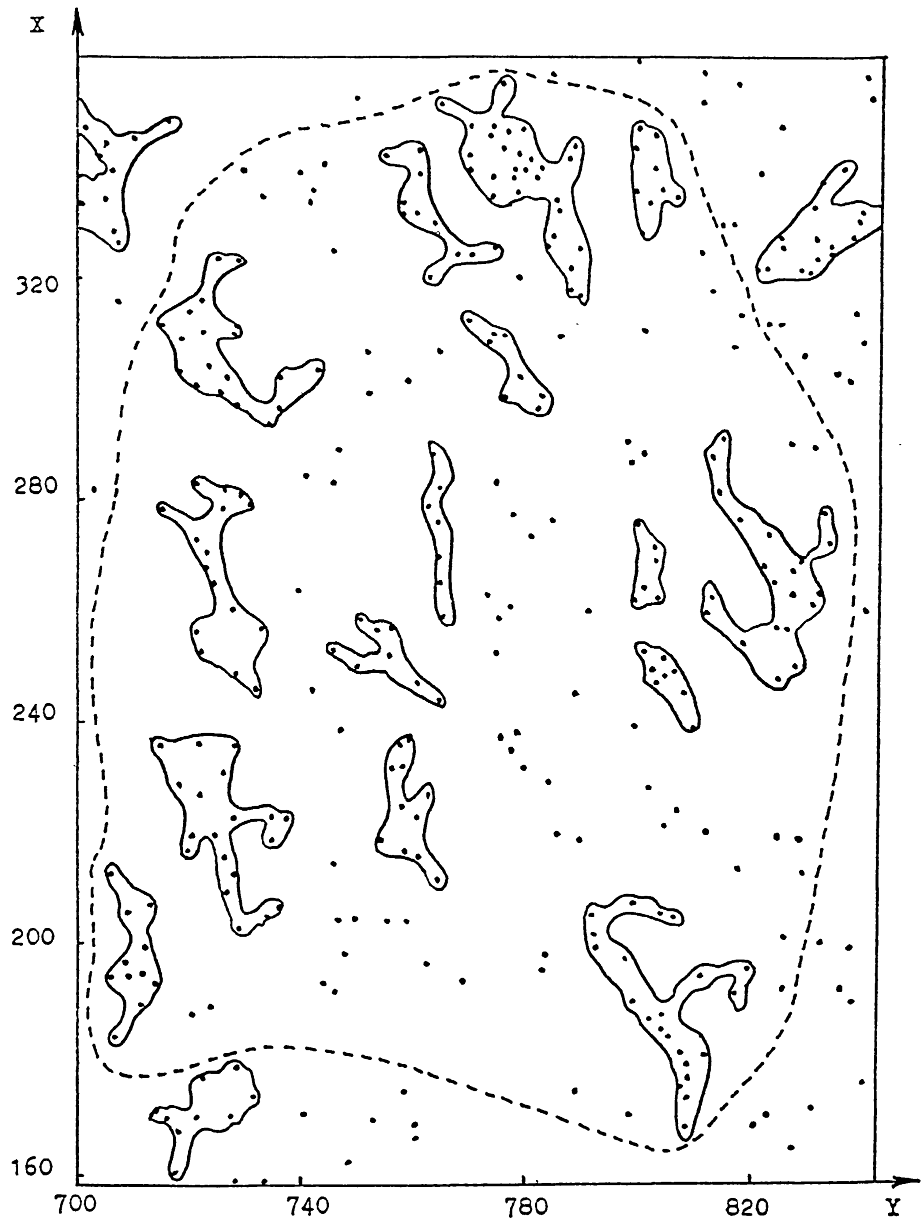

Fig. 3. The part of the Jagiellonian field with the evolved clusters for the R = 8 mm, $N \geq 7$ (orientational domain).

cluster and its neighbour ($i = 1, 2, \ldots, 10$). The results of this calculation are shown in Table I.

TABLE I

The differences of the position angles of the picking out clusters and its neighbours for the Jagiellonian field. N_i-number of neighbours, $\overline{|F - F_i|}$ – moduls of the differences of the position angle, σ_i – r.m.s. deviation.

N_i	$\overline{\lvert F - F_i \rvert}$	σ_i	N_i	$\overline{\lvert F - F_i \rvert}$	σ_i
1	42.1	25.8	6	43.8	25.9
2	42.7	27.0	7	43.2	26.3
3	43.2	25.2	8	43.7	26.2
4	43.8	26.8	9	43.7	25.7
5	46.5	27.2	10	44.8	25.6

For the isotropic case the magnitude $\overline{|F - F_i|}$ must be equal to 45°. In fact these values are slightly differ from the one of 45° (as compared with δ_i). Thus one can conclude that near neighbours-clusters have no tendency to be similarly oriented as to respect to each other. This demonstrates the absence of the common orientational-domain structure of the Jagiellonian field.

5. Discussion

The first question which arises in connection with the above-mentioned result: is this anisotropy a real effect or consequence of the certain systematical errors? First of all we note that the initial data of the present task are the coordinates of galaxies. Apparently, the homogeneous deformation of the galaxy coordinates of the field along one axis is the most simple and effective way of 'generation' of the fictitious anisotropy. Suppose that it is the axis x: $\hat{x} = x/(1 + \delta)$, where $\delta \ll 1$ – the small parameter of deformation. Which δ must be taken to ensure quantitatively the above anisotropy effect? To answer this question we searched for such a 'reverse' deformation of the Jagiellonian field which practically removes anisotropy in cluster orientations. In detail this procedure was as follows. As far as in most of cases the histograms of clusters showed anisotropy along the direction of the axis x, all x-coordinates of galaxies of the Jagiellonian field were changed according to formula $\hat{x} = x_i/(1 + \delta)$. Furthermore, the above-described algorithm of the investigation was carried out from beginning to end: selection of clusters and determination of the main parameters of the inertial ellipse, the formation of different samples and drawing up of the histograms. Such a procedure was repeated for a set of parameters δ. By such a procedure we determined such a value of this parameter at which anisotropy of histograms practically removed.

We noted that, for different samples of clusters, these values of δ may somewhat differ. The results of the calculations are represented in Figure 4(a–d). As it is clear from these Figures, the isotropy is eliminated at $\delta = 0.5$ (for other samples δ is different but of the same order of magnitude). Such strong deformations at compilation of the Jagiellonian

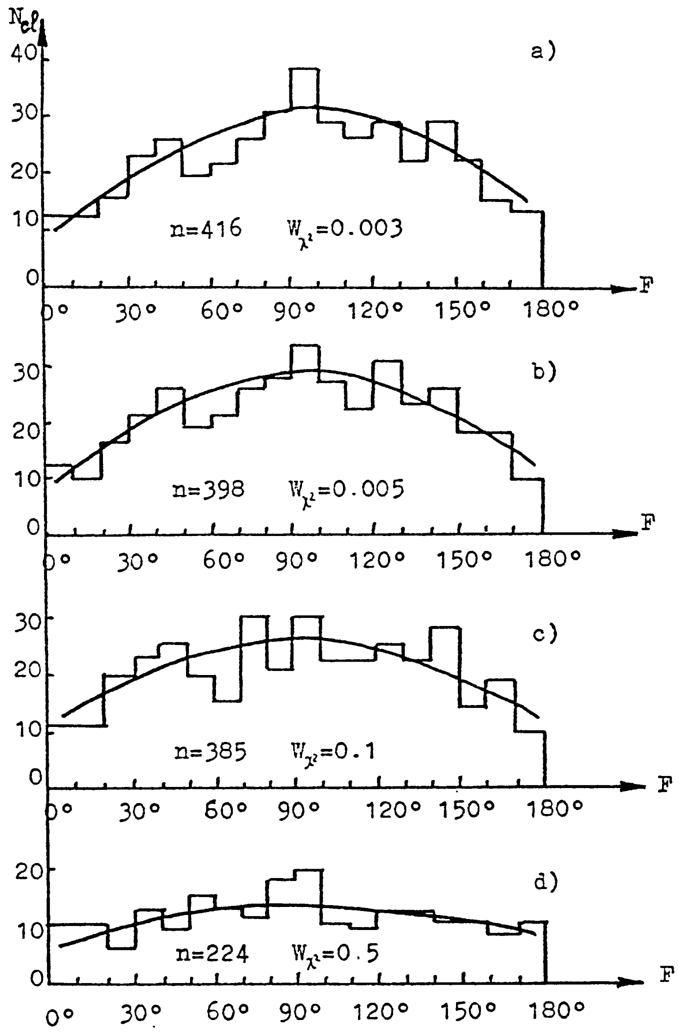

Fig. 4a–d. The histograms 'position angle-number of clusters' for the sets of the clusters with the parameters: $R = 8$ mm, $N \geq 8$, $E \leq 0.75$. The compressing of the Jagiellonian field along the axes X is characterized by the coefficients P: (a) $P = 1.01$, (b) $P = 1.05$, (c) $P = 1.1$, (d) $P = 1.5$.

map and its processing are excluded. In other words, it is difficult to explain the discovered anisotropy by the systematic errors in observational data.

6. Conclusions

Until the question about reality of the effect is finally solved it is not expedient to make a serious attempt at such an interpretation. Instead, we restrict ourselves to two remarks. First, the observed anisotropy in the case of its reality can be the result of orientational order of certain structural details of the galaxy large-scale distribution, in particular, filaments (see, for example, Haynes and Giovanelli, 1986; Bhavsar and Ling, 1988). It is necessary to take into consideration that we analysed the two-dimensional picture – i.e., the projection of the large thickness on the plane. Secondly, if one supposes that galaxies, which are 'responsible' for structural anisotropy occur roughly in the middle of the catalogue depth, then a spatial scale of the anisotropy is equal, at least, to 500 Mpc. Consequently, the effect has the sub-cosmological or cosmological character.

In conclusion, we note that the primlary task in connection with the received results is to answer the question about the reality of the discovered effect carrying out high-precise measurements of the galaxy coordinates of the catalogue and repeating of the investigations on the basic of these new data.

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