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CONSTRAINTS ON THE INTERACTION STRENGTH IN THE MODEL OF INTERACTING DYNAMICAL DARK ENERGY WITH LINEAR AND NON-LINEAR INTERACTING TERMS

In this work, the observational constraints on the coupling parameter of the interaction between dynamical dark energy and cold dark matter were obtained using cosmic microwave background data, baryon acoustic oscillations, and type Ia supernova data. The dark energy in considered models is dynamic, and the evolution of its equation of state parameter depends on dark coupling and internal properties of the dark energy itself. Such a model is believed to be more physically consistent than models of interacting dark energy considered in previous works. Constraints were made for three types of interaction. The first two are the types often considered in other works on interacting dark energy and are linearly dependent on the energy densities of dark components. The third type has a non-linear dependence on these densities and is studied for the first time. Observational constraints on the Hubble constant H_0 for the first two models strongly disagree with so-called local measurements of H_0. The third model aligns more closely with local measurements than the ΛCDM model. Also, for the first two types of interaction models, only the existence of small upper bounds of the interaction parameter was found, as for the last non-linear model, the existence of non-zero interaction was found at greater than 1σ significance level.

Keywords: *interacting dark energy, dark matter, cosmological perturbations.*

1. INTRODUCTION

Interacting dark energy (IDE) is an extension of a cosmological model whose aim is to explain the accelerated expansion of the Universe [25, 26]. In this model, there is some form of new interaction between dark energy (DE), which causes this acceleration, and another component, the dark matter (DM), in addition to the four known fundamental interactions [4, 31]. The existence of these dark components follows from their gravitational impact on visible matter and radiation, as they do not interact via the other three fundamental forces. As a result, the presence of such DE-DM interaction can be concluded if it makes a significant impact through gravitational interaction, leaving an imprint in the cosmic microwave background and other astrophysical data. This fact will be a possible indication that DE and DM have a quantum-field nature. In the most well-studied IDE model, the DE equation of state (EoS)

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parameter does not vary in time, and the DE-DM interaction is proportional to the energy densities of DE, DM, or the sum of both and is generated by the expansion rate of the Universe [5, 8, 10, 12, 13]. The constraints on parameters of such models using data on cosmic microwave background, baryon acoustic oscillations, and type Ia supernova give the non-zero energy transfer between DE and DM with a low confidence level or interaction is absent at all. [11, 24]. Furthermore, the constraints on the DE-DM interaction parameter proportional to the density of DM or the densities' sum of dark components were imposed only for the phantom DE model [6, 9]. For the quintessence model, such analysis is impossible due to non-adiabatic instabilities of cosmological perturbations in the radiation-dominated epoch of the Universe for these IDE models [30]. However, observational constraints for such models are possible when the DE EoS parameter varies in time, and its evolution can be tuned in such a way that non-adiabatic instabilities will not arise. Hence, in this work, the Markov Chain Monte-Carlo constraints on parameters of dynamical quintessence IDE with these interaction forms were done for the first time. The model of quintessence IDE EoS parameter evolution proposed in works [20, 22] and used here is more physically consistent than the well-known linear model for EoS evolution $w(a) = w_0 + w_1(1-a)$ [19]. The second part of this work presents the analysis of another type of DE-DM interaction (also for the first time), which does not depend on the expansion rate of the Universe and has the form of a Coulomb-type interaction function (e.g., the energy-momentum exchange rate between dark components is proportional to the product of densities of this components). Such interaction form is physically well justified as it does not vanish when the Universe does not expand, and its form is also often found among other interactions in nature.

Chapter 2 of this work provides a brief introduction to the analyzed models of dynamical IDE. In Chapter 3, the observational data and method of statistical constraints used are described. In Chapter 4, the impact of DE-DM interaction on the formation of the high-scale structure of the Universe and the results of parameters' observational constraints of considered models are shown and discussed.

2. MODELS OF DYNAMICAL IDE

The description of each component of the Universe is done in the perfect fluid approximation with the following stress-energy tensor:

$$T_i^k = (\rho + p)u_i u^k + p\delta_i^k .$$

The Universe is considered to be homogeneous and isotropic, which is described by the Friedman-Lemaître-Robertson-Walker (FLRW) metric with zero spatial curvature in relation to which the small perturbations of metric (perturbations are given in synchronous gauge):

$$ds^2 = a^2(\eta)[-d\eta^2 + (\delta_{\alpha\beta} + h_{\alpha\beta})dx^\alpha dx^\beta],$$

where a denotes a scale factor, η is conformal time, and $h_{\alpha\beta}$ is the perturbation of metric tensor. For each component, the general-covariant equation of stress-energy tensor conservation is true except for DE and DM, which, as a result of non-gravitational interaction between them, take the following form:

$$T^k_{(de)i;k} = J_{(de)i} , \tag{1}$$

$$T^k_{(c)i;k} = J_{(c)i} . \tag{2}$$

Here, ";" denotes the general-covariant derivative and J_i is the 4-vector of energy-momentum exchange between DE and DM, or, in other words, it describes the DE-DM interaction. The demand of conservation of energy and momentum of total DE and DM fluid implies that $J_{(c)i} = -J_{(de)i} = J_i$.

To solve the system of equations (1)-(2) along with Einstein's gravitational field equations, the 4-vector J_i must be given as a function of variables, which describes the state of DE and DM. In most works on IDE, this interaction is taken in the form, which in FLRW Universe is proportional to Hubble parameter H and some function of dark components' densities $\overline{\rho}_{de}$, $\overline{\rho}_c$. In the cases considered in this work, $\overline{J}_0$ is taken in the following forms [5, 8]:

$$\overline{J}_0 = 3\beta a H \overline{\rho}_c , \tag{3}$$

$$\overline{J}_0 = 3\beta a H (\overline{\rho}_{de} + \overline{\rho}_c) . \tag{4}$$

Here, β is the interaction parameter, and when it goes to zero, the DE-DM interaction disappears. When the consideration of these interaction forms is extended to the small linear cosmological perturbations in the relation to FLRW Universe, then, as was mentioned above, the problem of instabilities of these perturbations in the radiation-dominated epoch occurs [30]. To avoid this problem, the DE EoS parameter must be allowed to evolve with the Universe's expansion. In this study, the model of IDE is considered, which in the evolution of this EoS parameter is given by the DE-DM interaction parameter and DE adiabatic sound speed. Consequently equations (1) and (2) with the additional equation for DE EoS parameter evolution in the FLRW Universe take the following form:

$$\dot{\bar{\rho}}_{de} + 3aH(1+w)\bar{\rho}_{de} = -\bar{J}_0\,, \qquad (5)$$

$$\dot{\bar{\rho}}_{c} + 3aH\rho_{c} = \bar{J}_0\,, \qquad (6)$$

$$\dot{w} = 3aH(1+w)(w-c_a^2) + \frac{\bar{J}_0}{\bar{\rho}_{de}}(w-c_a^2)\,. \qquad (7)$$

Here, dot over quantity is the derivative on conformal time η, w is the DE EoS parameter, and $c_a^2 = \dot{\bar{p}}_{de}/\dot{\bar{\rho}}_{de}$ is the square of DE adiabatic sound speed ($\bar{p}_{de}$ is the DE pressure). The solutions of these equations were obtained in works [20, 22]. To extend our models to the case of small perturbations relative to the background Universe, we first need to specify the general-covariant form of DE-DM interactions. In this study, we use the form proposed in [18, 21]:

$$J_i = \beta\rho_c u^k_{;k} u_i^{(c)}\,, \qquad (8)$$

$$J_i = \beta(\rho_{de}+\rho_c) u^k_{;k} u_i^{(c)}\,, \qquad (9)$$

where $u_i^{(c)}$ is a four-vector of DM velocity and u^k is a velocity four-vector of all components' center of mass.

In addition to the two forms of DE-DM interactions (8) and (9), this study considers another form of J_i, which is not generated by the expansion rate of the Universe. In other words, it is not proportional to the Hubble parameter H in the FLRW Universe as the previous two types. Also, such interaction is proportional to the product of DE and DM densities. So its general-covariant form is as follows:

$$J_i = 3\beta H_0 \frac{\rho_{de}\rho_c}{\rho_{de}+\rho_c} u_i^{(c)}\,. \qquad (10)$$

The presence of the Hubble constant H_0 in this interaction form is only necessary to normalize the interaction parameter β. The interaction (10) is motivated by those interactions that are frequently encountered in various fields of physics, such as the Coulomb electrostatic interaction, the Newtonian gravitational interaction, etc. Such interaction is being studied for the first time.

The resulting equations for the evolution of cosmological perturbations for DE and DM with interaction (8) in synchronous gauge comoving to DM are as follows:

$$\dot{\delta}_{de} = -3aH(c_s^2 - w)\delta_{de} - (1+w)\frac{\dot{h}}{2} - (1+w)[k^2 + 9a^2H^2(c_s^2 - c_a^2)]\frac{\theta_{de}}{k^2} - \beta\frac{\bar{\rho}_c}{\bar{\rho}_{de}}\left[3aH(\delta_c - \delta_{de}) + \frac{\dot{h}}{2} + \theta + 9a^2H^2(c_s^2 - c_a^2)\frac{\theta_{de}}{k^2}\right],$$

$$\dot{\theta}_{de} = -aH(1-3c_s^2)\theta_{de} + \frac{c_s^2k^2}{1+w}\delta_{de} + 3aH\frac{\beta}{1+w}\frac{\bar{\rho}_c}{\bar{\rho}_{de}}(1+c_s^2)\theta_{de}\,, \qquad (11)$$

$$= -\frac{\dot{h}}{2} + \beta\left(\frac{\dot{h}}{2} + \theta\right),$$

where $\theta_N = i\left(\sum_\alpha k_\alpha v^\alpha\right)$, c_s^2 is a comoving effective DE sound speed, which in this work is taken as $c_s^2 = 1$ and

$$\theta = \frac{\sum_N(\bar{\rho}_N + \bar{p}_N)\theta_N}{\sum_N(\bar{\rho}_N + \bar{p}_N)}\,,$$

where N is an index of Universe's each component.

For the interaction (9), we have such equations:

$$\dot{\delta}_{de} = -3aH(c_s^2 - w)\delta_{de} - (1+w)\frac{\dot{h}}{2} - (1+w)[k^2 + 9a^2H^2(c_s^2 - c_a^2)]\frac{\theta_{de}}{k^2} - \beta\frac{\bar{\rho}_{de}+\bar{\rho}_c}{\bar{\rho}_{de}} \times \left[3aH\left(\frac{\bar{\rho}_{de}}{\bar{\rho}_{de}+\bar{\rho}_c}\delta_{de} + \frac{\bar{\rho}_c}{\bar{\rho}_{de}+\bar{\rho}_c}\delta_c - \delta_{de}\right) + \frac{\dot{h}}{2} + \theta + 9a^2H^2(c_s^2 - c_a^2)\frac{\theta_{de}}{k^2}\right],$$

$$\dot{\theta}_{de} = -aH(1-3c_s^2)\theta_{de} + \frac{c_s^2k^2}{1+w}\delta_{de} + 3aH\frac{\beta}{1+w}\frac{\bar{\rho}_{de}+\bar{\rho}_c}{\bar{\rho}_{de}}(1+c_s^2)\theta_{de}\,, \qquad (12)$$

$$\dot{\delta}_c = -\frac{\dot{h}}{2} + \beta \frac{\overline{\rho}_{de} + \overline{\rho}_c}{\overline{\rho}_c} \left[3aH \left(\frac{\overline{\rho}_{de}}{\overline{\rho}_{de} + \overline{\rho}_c} \delta_{de} + \right.\right.$$
$$\left.\left. + \frac{\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c} \delta_c - \delta_c \right) + \frac{\dot{h}}{2} + \theta \right].$$

And for the interaction (10):

$$\dot{\delta}_{de} = -3aH(c_s^2 - w)\delta_{de} - (1+w)\frac{\dot{h}}{2} -$$
$$-(1+w)[k^2 + 9a^2H^2(c_s^2 - c_a^2)]\frac{\theta_{de}}{k^2} -$$
$$-3\beta aH_0 \frac{\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c} \left[\delta_c + 3aH(c_s^2 - c_a^2)\frac{\theta_{de}}{k^2} - \right.$$
$$\left. - \frac{\overline{\rho}_{de}}{\overline{\rho}_{de} + \overline{\rho}_c}\delta_{de} - \frac{\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c}\delta_c \right],$$
$$\dot{\theta}_{de} = -aH(1 - 3c_s^2)\theta_{de} + \frac{c_s^2 k^2}{1+w}\delta_{de} + \quad (13)$$
$$+ \frac{3aH_0\beta}{1+w}\frac{\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c}(1 + c_s^2)\theta_{de},$$
$$\dot{\delta}_c = -\frac{\dot{h}}{2} + 3\beta aH_0 \frac{\overline{\rho}_{de}}{\overline{\rho}_{de} + \overline{\rho}_c} \times$$
$$\times \left(\delta_{de} - \frac{\overline{\rho}_{de}}{\overline{\rho}_{de} + \overline{\rho}_c}\delta_{de} - \frac{\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c}\delta_c \right).$$

To make numerical integration of this system of equations, the initial conditions for the background system (5)—(7) and for the perturbed system (11)—(13) must be set up. The background initial conditions are given for the present epoch at $a_0 = 1$, and the perturbation initial conditions are given for the early epoch of electromagnetic radiation dominance.

Initial conditions for perturbation equations are taken as their solutions for the radiation-dominated epoch when the perturbations have not yet entered the Hubble horizon. These solutions satisfy the following condition for the arbitrary two components x and y

$$S_{x,y} = aH\left(\frac{\delta_x}{\dot{\overline{\rho}}_x / \overline{\rho}_x} - \frac{\delta_y}{\dot{\overline{\rho}}_y / \overline{\rho}_y} \right) = 0,$$

and as a result, the fluids are adiabatic. When DE does not interact with DM, the small deviations from adiabatic perturbations are damped, and as a result, these perturbations stay stable until they enter the Hubble horizon. But when DE-DM interaction of form (8) or (9) is present, and DE is quintessential, this adiabatic mode could become unstable if DE EoS parameter w is close to –1 [30]. To avoid this problem, the stability analysis of adiabatic solutions of perturbation equations (11)—(13) was made. From this, the ranges of values for the interaction parameter and DE adiabatic sound speed c_a^2, for which adiabatic mode is stable, were derived in [18, 21] for each of the interactions (8), (9). It should be noted that, for the interaction of type (10), the adiabatic mode, based on the early epoch analysis, is always stable. Hence, for all three types of DE-DM interaction, the standard adiabatic initial conditions can be used without interaction even if they differ by a small value from the true initial conditions with non-zero interaction because, as was mentioned above, small deviations in the true initial conditions disappear.

3. OBSERVATIONAL DATA AND STATISTICAL METHOD

To impose the constraints on parameters of IDE models (8) (it will be called Model I), (9) (Model II), and (10) (Model III), the following observational data were used:

Cosmic Microwave Background (CMB) anisotropies: the dataset consisting of high-l TT, EE, TE power spectra and low-l TT, EE power spectra of Planck collaboration (2018 data release) [1]; this dataset is complemented by additional CMB weak gravitational lensing data of the same collaboration (2018 data release) [2];

Baryon Acoustic Oscillations (BAO): the 6dF Galaxy Survey [7] consisting of one data point at effective redshift z_{eff} = 0.106, SDSS DR7 Main Galaxy Sample [28] of data point at z_{eff} = 0.15, and SDSS-III Baryon Oscillation Spectroscopic Survey, DR12 [3] consisting of three data points at z_{eff} = 0.38, 0.51, 0.61;

Type Ia supernova (SN Ia): Pantheon dataset consisting of data on 1048 type Ia supernovae [29].

To confront Models I, II, and III with these observational data, the corresponding observable quantities should be calculated. For this purpose, the code IDECAMB [17] was modified. This code is the modification of the program package CAMB [15] and is specially designed for considering IDE models. In this program, the Parameterized Post Friedman (PPF) method adapted for IDE models was used

[16] to calculate the evolution of cosmological perturbations. To be suitable for Models I and II, it must consider the local Hubble parameter perturbations, described by the perturbing part of $u^k_{;k}$ in expressions (8) and (9). It was done by modifying the expressions (3.14) and (3.15) given in the work [17]:

$$\Delta Q = C_1\delta_{de} + C_2\delta_c + Q\left(\frac{kV}{3aH} + \frac{\zeta'}{aH} - \xi\right), \quad (14)$$

$$f_k = Q(\theta_c - \theta),$$

where $\overline{J}_0 = -aQ$ and ζ' is given by expression (4.9) in [17]. For Model I, $Q = -3\beta H\overline{\rho}_c$, $C_1 = 0$, $C_2 = Q$, and for Model II, $Q = -3\beta H(\overline{\rho}_{de} + \overline{\rho}_c)$,

$$C_1 = \frac{\overline{\rho}_{de}}{\overline{\rho}_{de} + \overline{\rho}_c}Q,\ C_2 = \frac{\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c}Q.$$

For Model III, the expression (14) takes the following form:

$$\Delta Q = C_1\delta_{de} + C_2\delta_c,$$

where

$$Q = -3\beta H_0\frac{\overline{\rho}_{de}\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c},$$

$$C_1 = \frac{\overline{\rho}_c}{\overline{\rho}_{de} + \overline{\rho}_c}Q,\ C_2 = \frac{\overline{\rho}_{de}}{\overline{\rho}_{de} + \overline{\rho}_c}Q.$$

Constraints on interaction parameters and other parameters of IDE models were obtained using the Markov Chain Monte-Carlo method realized in the CosmoMC program package [14] modified for this purpose. There, 12 Monte-Carlo chains were run for each of the studied IDE models with a convergence condition (using the Gelman-Rubin parameter) of $R - 1 < 0.01$. The priors for independent parameters, which describe the pressure of DE, w_0 and c_a^2, were taken in the quintessence range of values, and for interaction parameter β — in the positive range of values (the case when energy flows from DE to DM) for Model I and Model II. Also, the additional priors for these models were derived from the conditions of the positivity of energy density of dark components [18, 20] and conditions of stability of early cosmological perturbations [18, 21]. For Model III, priors for c_a^2 were taken in the phantom range, for w_0 — in quintessence and phantom ranges, and β is bounded by negative lower value and positive upper value. For Models I and II, the H_0 — parametrization was used, and for Model III, the $100\theta_{MC}$ — parametrization was used. Besides, 12 Monte-Carlo chains were run for the ΛCDM model with the same observational data to compare its constraints with the results for IDE models. Priors for the ΛCDM and all three interaction models are given in Table 1.

4. RESULTS

At first, the dependence of the scale structure of the Universe on DE-DM interaction coupling was studied for Model I, Model II, and Model III. In Fig. 1 and Fig. 2, the modification of the matter power spectrum at redshift $z = 0$ by the value of interaction parameter β is shown for all three models. For Models I and II, the modifications are similar with the suppression of structure formation on small scales and with some larger inhomogeneities on very high

Table 1. **Priors of independent parameters for each IDE model**

Parameter	ΛCDM	Model I	Model II	Model III
$\Omega_b h^2$	[0.005, 0.1]	[0.005, 0.1]	[0.005, 0.1]	[0.005, 0.1]
$\Omega_c h^2$	[0.001, 0.99]	[0.001, 0.99]	[0.001, 0.99]	[0.001, 0.99]
$100\theta_{MC}$	[0.5, 10]	—	—	[0.5, 10]
H_0	—	[40, 100]	[40, 100]	—
τ	[0.01, 0.8]	[0.01, 0.8]	[0.01, 0.8]	[0.01, 0.8]
$\log(10^{10}A_s)$	[1.61, 3.91]	[1.61, 3.91]	[1.61, 3.91]	[1.61, 3.91]
n_s	[0.8, 1.2]	[0.8, 1.2]	[0.8, 1.2]	[0.8, 1.2]
w_0	—	[–1, –0.333]	[–1, –0.333]	[–3, –0.333]
	—	[–1, 0]	[–0.533890, 0]	[–3, –1]
β	—	[0, 0.08]	[0, 0.5]	[–1.5, 1.5]

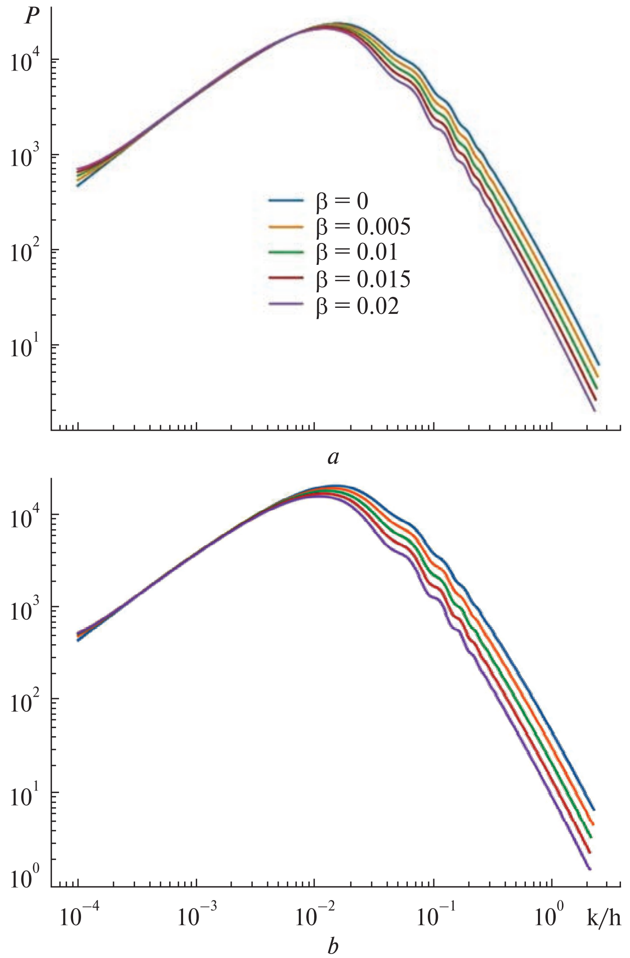

Figure 1. Dependence of the matter power spectrum at redshift $z = 0$ on the interaction parameter β for Model I (panel *a*) and for Model II (panel *b*). The independent model parameters that were used are as follows: $\Omega_b h^2 = 0.0226$, $\Omega_c h^2 = 0.112$, $H_0 = 68.2$, $\Omega_K = 0$, $A_s = 2.1\times10^{-9}$, $n_s = 0.96$, $\tau = 0.09$, $c_s^2 = 1$, $w_0 = -0.9$, $c_a^2 = -0.5$

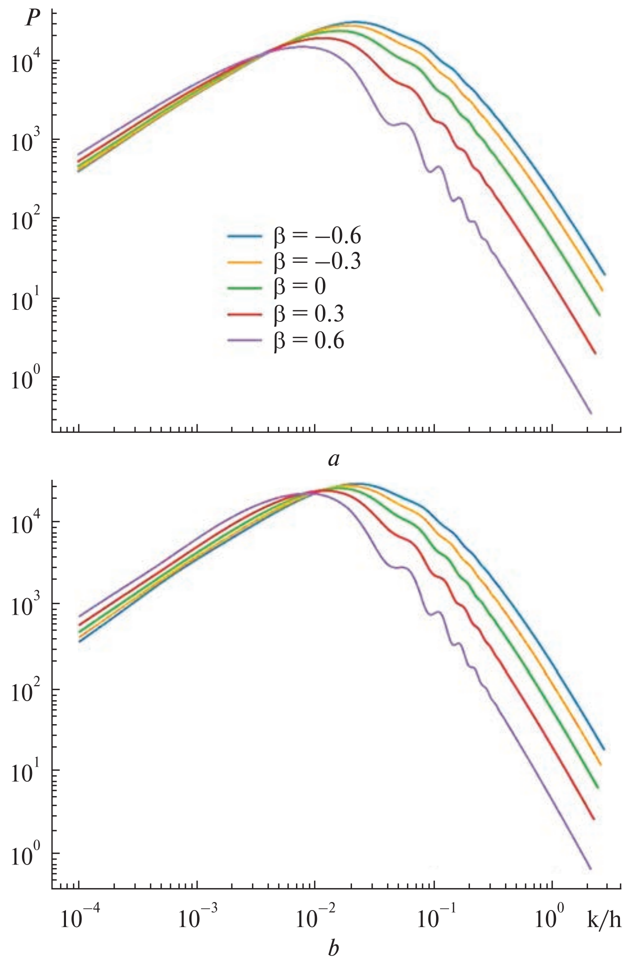

Figure 2. Dependence of the matter power spectrum at redshift $z = 0$ on the interaction parameter β for Model III with $c_a^2 = -0.5$ (panel *a*) and $c_a^2 = -1.2$ (panel *b*). The independent model parameters used are the same as those in Fig. 1

scales compared to non-interacting cases (in these figures, for Models I and II, the interaction parameter β is bounded to positive values only, the same as in priors in MCMC simulations). For Model III, the distribution of matter in the Universe is more inhomogeneous on high scales and sufficiently more homogeneous on small scales when β is positive. When we have negatively-valued β (it corresponds to the case when energy flows from DM to DE), the impact of DE-DM interaction is exactly opposite — on high scales, the matter is distributed slightly less homogeneous, while on small scales, the matter structure growth is larger.

The observational constraints on parameters of Model I, Model II, and Model III obtained from MCMC simulation at 68 % CL are given in Table 2.

As we can see for the quintessence IDE of Model I and Model II, due to the presence of DE-DM interaction, the relative part of the DE component is much lower and DM much higher compared to the ΛCDM model. As a result, the Hubble constant H_0 is much lower than the value obtained in work [27].

Table 2. **Constraints on model parameters at 68 % CL**

Parameter	ΛCDM	Model I	Model II	Model III
$\Omega_b h^2$	0.02242 ± 0.00014	0.02282 ± 0.00015	0.02282 ± 0.00015	0.02239 ± 0.00014
$\Omega_c h^2$	0.11932 ± 0.00092	0.1142 ± 0.0010	0.1142 ± 0.0011	$0.151^{+0.027}_{-0.017}$
τ	0.0573 ± 0.0074	0.083 ± 0.010	0.083 ± 0.010	0.0539 ± 0.0074
w_0	—	$-0.99424^{+0.00086}_{-0.0057}$	< -0.994	$-0.83^{+0.17}_{-0.29}$
c_a^2	—	$-0.24553^{+0.00075}_{-0.0046}$	$-0.24557^{+0.00076}_{-0.0046}$	$-1.130^{+0.083}_{-0.074}$
β	—	$<9.43 \cdot 10^{-5}$	$<9.71 \cdot 10^{-5}$	$0.27^{+0.23}_{-0.15}$
$\log(10^{10} A_s)$	3.049 ± 0.014	3.092 ± 0.020	3.092 ± 0.020	3.042 ± 0.014
n_s	0.9664 ± 0.0037	0.9805 ± 0.0041	0.9804 ± 0.0041	0.9658 ± 0.0040
H_0	67.66 ± 0.42	56.51 ± 0.25	56.51 ± 0.25	68.37 ± 0.83
Ω_{de}	0.6889 ± 0.0056	0.5690 ± 0.0066	0.5688 ± 0.0066	$0.627^{+0.041}_{-0.055}$
Ω_m	0.3111 ± 0.0056	0.4310 ± 0.0066	0.4312 ± 0.0066	$0.373^{+0.055}_{-0.041}$
σ_8	0.8110 ± 0.0060	0.6843 ± 0.0067	0.6843 ± 0.0067	$0.756^{+0.036}_{-0.055}$
S_8	0.826 ± 0.011	0.820 ± 0.011	0.820 ± 0.011	$0.839^{+0.012}_{-0.011}$

So, such models only worsen the so-called Hubble tension, which is one of the major problems in modern cosmology. Also, for both of these models, only the upper positive bounds on interaction parameter β were obtained. The constraints on the DE EoS parameter at the present time w_0 and EoS parameter evolution, which are mainly determined by DE's squared adiabatic sound speed c_a^2, strongly prefer the dynamical nature of the quintessence DE. In general, the constraints on β using CMB, BAO, and SN Ia data described in Chapter III do not allow us to determine whether the DE-DM interaction of Model I and Model II exists.

The constraints for Model III on interaction parameter β give the existence of its non-zero positive value on $> 1\sigma$ significance level. Also, constraints on the EoS parameter prefer that DE has the quintessential nature in epochs closer to modern time and behaves as the phantom in the early epochs of the Universe. It means that DE energy density $\overline{\rho}_{de}$ begins to increase from some constant value after the Universe's expansion starts and, after approaching some maximum, follows the gradual decrease of DE density till the present epoch at $a = 1$. Such a model (but in a non-interacting case) was studied in the work [23]. In this model, there is a higher proportion of DM and a lower of DE components compared to the ΛCDM model as in the previous two IDE models. The Hubble constant H_0 in the constraints of Model III is slightly higher than in the ΛCDM model. This difference suggests that Model III might help resolve the Hubble tension, particularly if future measurements from the next generation of BAO and SN Ia data are used to constrain its parameters.

5. CONCLUSIONS

In this work, we studied the cosmological models of interacting dynamical dark energy. These models have a non-gravitational interaction between dynamical dark energy and dark matter and are described by three different functions. The first two functions, well known in the literature, are proportional to the Hubble parameter, one of which is also proportional to the dark matter energy density (Model I) and the other to the sum of the energy densities of both dark components (Model II). The third one does not depend on the expansion rate of the Universe and is proportional to the product of energy densities of interacting components (Model III). Such interaction is studied for the first time and is expected to be more physically realistic in comparison to the previous two types of interaction and other types, which are proportional to the Hubble parameter. By imposing Markov Chain Monte-Carlo constraints on the parameters of these three models using the CMB, BAO, and SN Ia data, it was found that Models I and II significantly disagree in their estimates of the Hubble constant H_0 compared to the so-called local measurement of H_0. Also, it was determined only the upper bounds of the interaction parameter for these models. In contrast, Model III provides better agreement in estimating H_0 with local measurements than the ΛCDM model using the same observational data. Also, the constraints give the non-zero positive value of the interaction parameter (which corresponds to the energy flow from dark energy to dark matter) at $>1\sigma$ significance level for Model III. The use of next-generation data on BAO and SN Ia, along with current CMB data, is expected to impose tighter constraints on the interaction in the dark sector.

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ОБМЕЖЕННЯ НА СИЛУ ВЗАЄМОДІЇ У МОДЕЛІ ВЗАЄМОДІЮЧОЇ ДИНАМІЧНОЇ ТЕМНОЇ ЕНЕРГІЇ З ЛІНІЙНИМИ ТА НЕЛІНІЙНИМИ ЧЛЕНАМИ

У цій роботі спостережувані обмеження на параметр взаємодії між динамічною темною енергією та холодною темною матерією були отримані з використанням даних по реліктовому випромінюванню, баріонних акустичних осциляцій і наднової типу Ia. Темна енергія в розглянутих моделях є динамічною, і еволюція її параметра рівняння стану залежить від взаємодії між прихованими компонентами та внутрішніх властивостей самої темної енергії. Вважається, що така модель є більш фізично послідовною, ніж моделі взаємодіючої темної енергії, розглянутих в попередніх роботах. Обмеження були зроблені для трьох типів взаємодії. Перші дві є типами взаємодії, які часто розглядаються в інших роботах по взаємодіючій темній енергії та лінійно залежать від густини енергії прихованих компонентів. Третій тип має нелінійну залежність від цих густин і досліджується вперше. Спостережувані обмеження на сталу Хаббла H_0 для перших двох моделей є в сильному протиріччі з так званими локальними вимірюваннями H_0. А третя модель краще узгоджується з локальними вимірюваннями, ніж ΛCDM-модель. Також для перших двох типів моделей взаємодії було знайдено лише існування малих верхніх меж на параметр взаємодії, а для останньої нелінійної моделі існування ненульової взаємодії було встановлено на рівні значущості, що перевищує 1σ.

Ключові слова: взаємодіюча темна енергія, темна матерія, космологічні збурення.