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SPECTRAL PROBLEM FOR THE JONES MATRIX IN REMOTE SCATTERING

The paper addresses the study of anisotropy in remote scattering basing on the spectral problem. The spectral problem is formulated as the determination of eigenpolarizations and eigenvalues for the Jones matrix, which describe the optical anisotropy of the medium. Jones matrices of media with complex anisotropy (media characterized by several types of anisotropy) are considered in terms of a homogeneous (differential) approach. The essence of this approach is that the anisotropy of the class of media under consideration does not depend on the thickness of this medium. An analysis of the Jones matrices for arbitrary homogeneous media (media characterized by all four main types of optical anisotropy: linear, circular, phase and amplitude anisotropy) and media characterized by two types of anisotropy as a special case has been peformed. The main tool for such an analysis was the inhomogeneity parameter of the medium, which allows characterizing the latter as a medium with orthogonal or non-orthogonal eigenpolarizations. The study reveals the peculiarities of complex anisotropy types (elliptical birefringence and Hermitian dichroism, improper dichroism, non-Hermitian dichroism, and degenerate anisotropy) based on the inhomogeneity parameter. A geometric interpretation of eigenpolarizations using the inhomogeneity parameter is demonstrated. The conditions for the anisotropy parameters under which the above-mentioned complex types of anisotropy are realized in the studied classes of medium were calculated. The research was motivated by the fundamental results of van de Hulst and Hovenier formed the basis of the analysis of the Jones and Mueller matrices inner structure. The results obtained contribute to a deeper understanding of polarization phenomena in electromagnetic scattering and provide a basis for future research in polarization diagnostics and remote sensing.

Keywords: *Mueller matrix, Jones matrix, phase linear and circular anisotropy, amplitude linear and circular anisotropy, spectral problems.*

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INTRODUCTION

Natural scattering scenes include, but are not limited to, various types of vegetation [4, 16], water surfaces [3, 18], buildings [26, 27], snow [24, 25, 28], clouds, fog, and aerosols [5, 6, 10, 17], which can be considered as natural polarization converters and/or depolarizers. Reflection, transmission, and scattering of electromagnetic radiation by natural scattering scenes create polarization patterns determined by the nature of these scenes. Therefore, measuring the polarization patterns is an important concept of modern remote sensing. Thus, polarization represents a new quality in understanding the properties of scattering scenes.

In many cases, Mueller polarimetry is a powerful modality to characterize completely the anisotropy and depolarization properties of the studied scattering scenes. This assumes, on the one hand, the measurement of the Mueller matrices in the given geometry of the experiment and the wavelength of the electromagnetic radiation. On the other hand, the measured Mueller matrices are further analyzed using a wide range of decomposition methods that exist today to get anisotropy and depolarization characterization of studied scenes.

Anisotropic properties are described by non-depolarizing or pure Mueller matrices. The latter have a one-to-one correspondence with the Jones ones or, as they are called in [12], scattering matrices. Based on a number of so-called equivalence theorems [13, 15, 20], the Jones matrices allow one to obtain the values of the anisotropy parameters (i.e., the values and azimuths of the linear and the values of the circular phase and amplitude anisotropy) characterizing the scattering scene under study.

The goal of this paper is to analyze the Jones matrix, which contains information on the anisotropy of the scattering scene, based on the solution of the spectral problem [1, 23]. The spectral problem is to determine the eigenpolarizations and eigenvalues for the Jones matrix. Despite the fact that the eigenpolarizations and eigenvalues carry very extensive and important information about the anisotropic properties of the object under study, in our opinion, insufficient attention is paid to the solution of the spectral problem in modern polarimetric bibliography. This study was largely motivated by seminal results obtained by van de Hulst and Hovenier et al. in their analysis of the structure of the Jones matrix and the pure Mueller matrix [2, 8, 9, 11, 12].

JONES MATRIX METHOD

There are four main anisotropic mechanisms characterizing a homogeneous medium for a given wavelength: linear and circular phase and amplitude anisotropy [24]. In the case of phase anisotropy two orthogonal linearly or circularly polarized eigenpolarizations propagate with different phase velocities. In the case of amplitude anisotropy, two orthogonal linearly or circularly polarized eigenpolarizations are absorbed differently, propagating through the appropriate medium. To quantify these four anisotropic mechanisms, one uses the following parameters: $\delta=(2\pi/\lambda)(n_0-n_e)z=\delta_0 z$ and α — the value and azimuth of the linear phase anisotropy, respectively; $\phi=(\pi/\lambda)(n_l-n_r)z=\phi_0 z$ — the value of the circular phase anisotropy; $\xi=(2\pi/\lambda)(k_o-k_e)z=\xi_0 z$ and θ — the value and azimuth of the linear amplitude anisotropy, respectively; $r=(2\pi/\lambda)(k_l-k_r)z=r_0 z$ — the value of the circular amplitude anisotropy; n_0, n_e, n_l, n_r and k_0, k_e, k_l, k_r — linear and circular refractive indices and absorption coefficients of the phase and amplitude anisotropy, respectively; $\delta_0, \phi_0, \xi_0, r_0$ — values of relative phase shift and absorptions per unit length in the light propagation direction.

The essence of the Jones matrix formalism is that the polarization properties of an infinitely thin layer of anisotropic medium can be represented by the differential Jones matrix $\mathbf{N}$ [25]. In the case of a homogeneous medium, such a matrix does not depend on the coordinate z in the direction of light propagation. Jones matrix for an arbitrary homogeneous anisotropic medium (i. e., the case when the medium is simultaneously characterized by all four mechanisms of anisotropy) is as follows:

$$\mathbf{N}=\begin{bmatrix} \frac{1}{2}\left(-i\delta_0-\xi_0+i\delta_0\cos 2\alpha+\xi_0\cos 2\theta\right) & \frac{1}{2}\left(-ir_0+2\phi_0+i\delta_0\sin 2\alpha+\xi_0\sin 2\theta\right) \\ \frac{1}{2}\left(-2\phi_0+i\left(r_0+\delta_0\sin 2\alpha\right)+\xi_0\sin 2\theta\right) & -\frac{1}{2}i\left(\delta_0-i\xi_0+\delta_0\cos 2\alpha-i\xi_0\cos 2\theta\right) \end{bmatrix}. \quad (1)$$

To calculate the Jones matrix for a medium Eq. (1) of thickness d, we use the Jones vector transfer equation:

$$\frac{d\mathrm{E}}{dz} = \mathrm{NE}\,. \tag{2}$$

Solving this equation with initial conditions $E_1(0) = E_{01}$, $E_2(0) = E_{02}$, one can get a system of linear equations of the form:

$$\begin{cases} E_1 = T_{11}E_{01} + T_{12}E_{02}, \\ E_2 = T_{21}E_{01} + T_{22}E_{02}. \end{cases} \tag{3}$$

Then elements of the differential Jones matrix can be found using the equation [18]:

$$T_{11} = \left.\frac{E_1}{E_{01}}\right|_{E_{02}=0}; \quad T_{12} = \left.\frac{E_1}{E_{02}}\right|_{E_{01}=0}; \quad T_{21} = \left.\frac{E_2}{E_{01}}\right|_{E_{02}=0}; \quad T_{22} = \left.\frac{E_2}{E_{02}}\right|_{E_{01}=0}. \tag{4}$$

Applying this method to the differential matrix Eq.(1), we obtain the following form of the integral Jones matrix describing an arbitrary homogeneous anisotropic medium:

$$\mathbf{T} = K\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}, \tag{5}$$

where

$$K = \exp(-z(i\delta_0 + \xi_0 + A)/2)/2A\,,$$

$$T_{11} = A + A\exp(zA) + [-1 + \exp(zA)]\left(i\delta_0\cos 2\alpha + \xi_0\cos 2\theta\right),$$

$$T_{12} = [-1 + \exp(zA)] \times \left(-ir_0 + 2\phi_0 + i\delta_0\cos 2\alpha + \xi_0\cos 2\theta\right),$$

$$T_{21} = [-1 + \exp(zA)] \times \left(ir_0 - 2\phi_0 + i\delta_0\cos 2\alpha + \xi_0\cos 2\theta\right),$$

$$T_{22} = A + A\exp(zA) - [-1 + \exp(zA)]\left(i\delta_0\cos 2\alpha + \xi_0\cos 2\theta\right),$$

$$A = \sqrt{-\delta_0^2 + \xi_0^2 + (r_0 + 2i\phi_0)^2 + 2i\delta_0\xi_0\cos 2(\alpha - \theta)}\,.$$

EIGENANALYSIS OF THE MEDIUM POLARIZATION PROPERTIES

To analyze the anisotropy of the medium described by the Jones matrix $\mathbf{T}$, the polarimetry spectral problem is solved [18,19]. The first step is to calculate eigenpolarizations $\mathbf{E}_{1,2}$ and eigenvalues $V_{e1,2}$ describing the eigenpolarizations propagation peculiarities in the medium.

The second step is the calculation of the inhomogeneity value η [16]:

$$\eta = \left|\mathbf{E}_1^{\dagger}\mathbf{E}_2\right|, \quad 0 \le \eta \le 1\,, \tag{6}$$

$$\eta^2 = \frac{\mathrm{tr}\left(\mathbf{T}^{\dagger}\mathbf{T}\right) - \frac{1}{2}\left|\mathrm{tr}\mathbf{T}\right| - \frac{1}{2}\left|\left(\mathrm{tr}\mathbf{T}\right)^2 - 4\det\mathbf{T}\right|}{\mathrm{tr}\left(\mathbf{T}^{\dagger}\mathbf{T}\right) - \frac{1}{2}\left|\mathrm{tr}\mathbf{T}\right| + \frac{1}{2}\left|\left(\mathrm{tr}\mathbf{T}\right)^2 - 4\det\mathbf{T}\right|}\,, \tag{7}$$

where † stands for Hermitian conjugation.

The case η = 0 corresponds to the orthogonal $\mathbf{E}_1$ and $\mathbf{E}_2$; η = 1 is the case of $\mathbf{E}_1$ and $\mathbf{E}_2$ coincidence (i. e., degenerate anisotropy). Main concern is to show for what types of anisotropy the eigenpolarizations $\mathbf{E}_{1,2}$ are orthogonal with η = 0 (birefringence and Hermitian dichroism [1,7,23]), non-orthogonal with 0 < η < 1 (non-Hermitian and improper dichroisms [19, 21, 22]) and coincident with η = 1 (degenerate anisotropy [19, 21, 22]).

For the medium with an elliptical phase anisotropy (a combination of linear and circular phase anisotropy) and the medium with an elliptical amplitude anisotropy (a combination of linear and circular amplitude anisotropy), from Eq. (1), we get:

$$\mathbf{T}^{EP} = \frac{\exp\left(-\frac{iz\delta_0}{2}\right)}{A_1} \times \begin{bmatrix} A_1\,\mathrm{c}_{zA_1/2} + i\delta_0 c_{2\alpha} s_{zA_1/2} & \mathrm{s}_{zA_1/2}(i\delta_0\,\mathrm{s}_{2\alpha} + 2\phi_0) \\ \mathrm{s}_{zA_1/2}(i\delta_0\,\mathrm{s}_{2\alpha} - 2\phi_0) & A_1\,\mathrm{c}_{zA_1/2} - i\delta_0\,\mathrm{c}_{2\alpha}\mathrm{s}_{zA_1/2} \end{bmatrix},$$

$$\mathbf{T}^{EA} = \frac{\exp(-z\xi_0/2)}{A_2} \times \begin{bmatrix} A_2\cosh(zA_2/2) + \xi_0\cos(2\theta)\sinh(zA_2/2) & \sinh(zA_2/2)(\xi_0\sin(2\theta) - ir_0) \\ \sinh(zA_2/2)(\xi_0\sin(2\theta) + ir_0) & A\cosh(zA_2/2) - \xi_0\cos(2\theta)\sinh(zA_2/2) \end{bmatrix}, \tag{8}$$

where

$$A_1 = (\delta_0{}^2 + 4\phi_0{}^2)^{1/2}, \; A_2 = (\xi_0{}^2 + r_0{}^2)^{1/2},$$

$$\mathrm{c}_x \equiv \cos x, \; \mathrm{s}_x \equiv \sin x\,.$$

Substituting the elements of the Jones matrices Eq. (8) into Eq. (7) we can obtain η = 0, since these classes of media are unitary and Hermitian and,

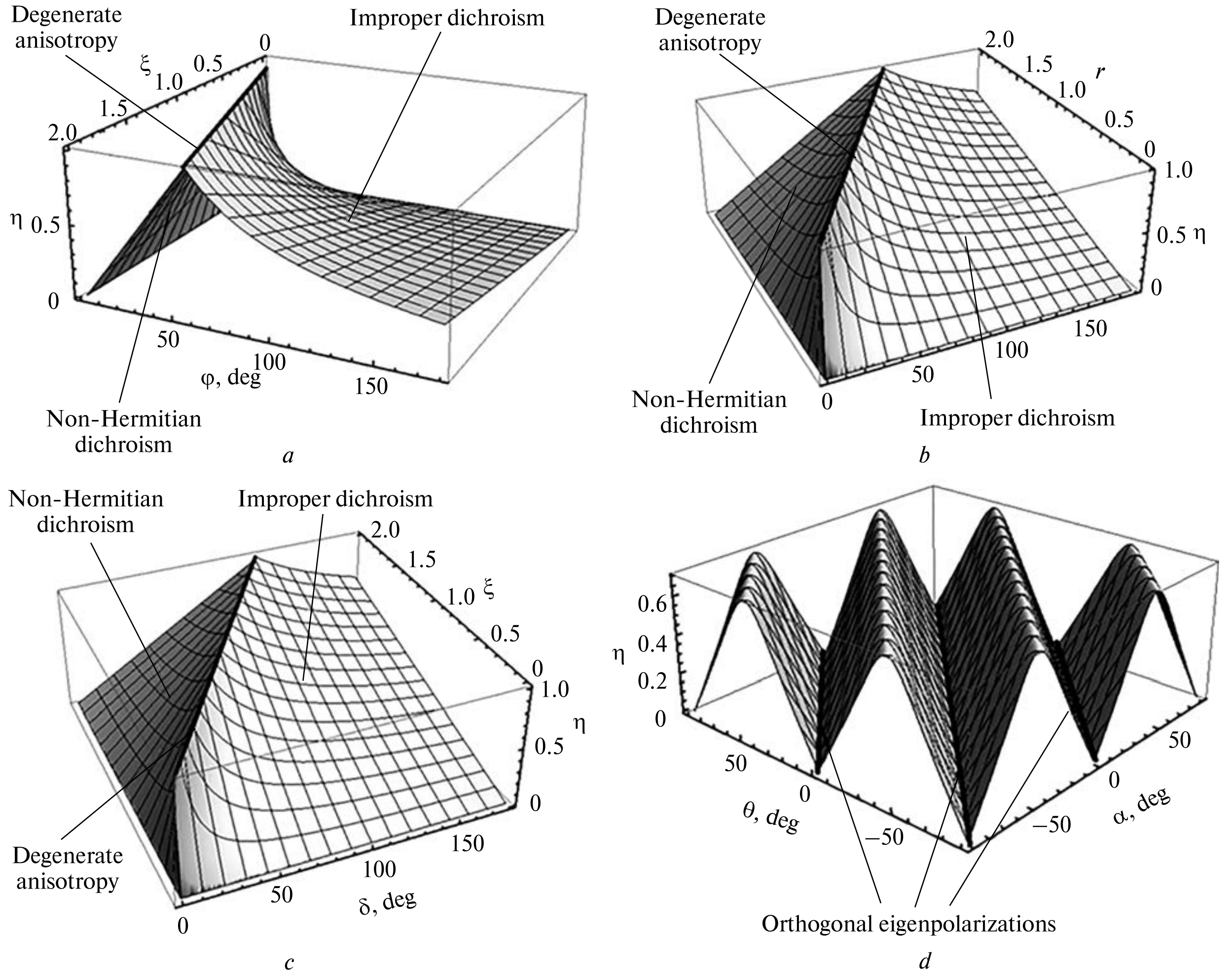

Figure 1. Dependence of inhomogeneity h on the anisotropy parameters: *a* — ξ and φ; *b* — δ and r; *c* — δ and ξ; *d* — α and θ

therefore, are described by orthogonal eigenpolarizations.

The same value of inhomogeneity, i. e., $\eta = 0$, can be obtained for the class of media with a combination of circular phase and amplitude anisotropy described by the following Jones matrix:

$$\mathbf{T}^{CACP} = \\ = \begin{bmatrix} \cosh(z(r_0 + 2i\phi_0)/2) & -i\sinh(z(r_0 + 2i\phi_0)/2) \\ i\sinh(z(r_0 + 2i\phi_0)/2) & \cosh(z(r_0 + 2i\phi_0)/2) \end{bmatrix}. \quad (9)$$

The polarization properties of other media characterized by two anisotropy types are more complex. Consider these cases.

Medium with linear amplitude and circular phase anisotropy. The Jones matrix for this medium is as follows:

$$\mathbf{T}^{LACP} = \frac{\exp(-z\xi_0/2)}{A_3} \times \\ \times \begin{bmatrix} A_3\cosh(zA_3/2) + \xi_0\cos(2\theta)\sinh(zA_3/2) & (\xi_0\sin(2\theta) + 2\phi_0)\sinh(zA_3/2) \\ (\xi_0\sin(2\theta) - 2\phi_0)\sinh(zA_3/2) & A_3\cosh(zA/2) - \xi_0\cos(2\theta)\sinh(zA_3/2) \end{bmatrix}, \quad (10)$$

where $A_3 = (\xi_0^2 - 4\phi_0^2)^{1/2}$.

Substituting the elements of the matrix Eq. (10) into Eq. (7) for the case $\theta = 12^\circ$, $z = 1$, we obtain the geometric interpretation of the inhomogeneity η presented in Fig. 1, *a*. Figure 1, *a* shows that this class of media is characterized by:

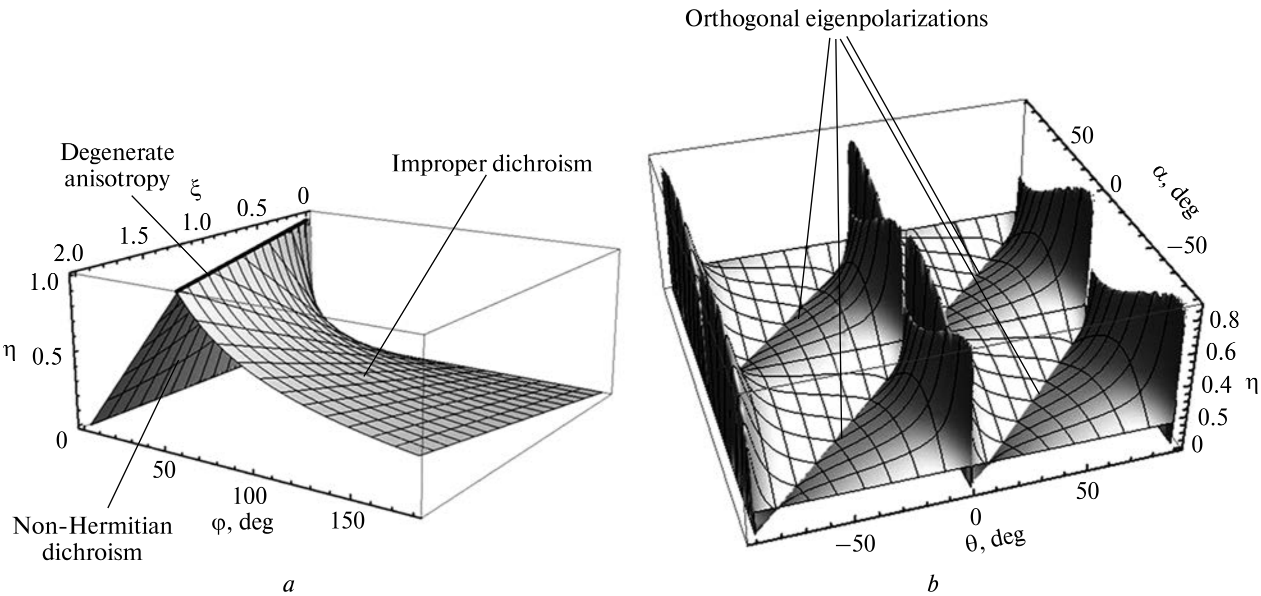

Figure 2. Dependence of inhomogeneity h on the anisotropy parameters: *a* — φ and ξ; *b* — α and θ

• improper dichroism ($\xi_0^2 < 2\phi_0^2$) — eigenpolarizations of the medium are absorbed equally and propagate with different phase velocities, while other input polarizations are absorbed differently but propagate with equal phase velocities,

• non-Hermitian dichroism ($\xi_0^2 > 2\phi_0^2$) — eigenpolarizations are nonorthogonal and are absorbed differently. The Jones matrix describing this class of media is neither Hermitian nor unitary, unlike the corresponding matrices for linear, circular, or elliptical amplitude (Hermitian dichroism) or phase (birefringence) anisotropy,

• degenerate case ($\xi_0^2 = 2\phi_0^2$ and $\eta = 1$) — in this case, the eigenpolarizations coincide.

Thus, from Fig. 1, *a*, we can see that this class of media is always characterized by nonorthogonal eigenpolarizations ($\eta = 0$).

Medium with linear phase and circular amplitude anisotropy. The Jones matrix for this class of media corresponds to the first partial Jones equivalence theorem [15]. From Eq.(1), we get:

$$\mathbf{T}^{LPCA} = \frac{\exp(-zi\delta_0/2)}{A_4} \times$$

$$\times \begin{bmatrix} A_4\cosh(zA_4/2) + i\delta_0\cos(2\alpha)\sinh(zA_4/2) & (\delta_0\sin(2\alpha) - r_0)\sinh(zA_4/2) \\ (\delta_0\sin(2\alpha) + r_0)\sinh(zA_4/2) & A_4\cosh(zA_4/2) - i\delta_0\cos(2\alpha)\sinh(zA_4/2) \end{bmatrix}, \quad (11)$$

where $A_4 = (r_0^2 - \delta_0^2)^{1/2}$.

After the substitution of elements of the matrix Eq. (11) into Eq. (7) for the case θ = 12°, *z* = 1, the graphical interpretation of the inhomogeneity η is presented in Fig. 1, *b*. Figure 1, *b* shows that this class of media is characterized by:

• improper dichroism ($r_0^2 < \delta_0^2$),

• nonhermitian dichroism ($r_0^2 > \delta_0^2$),

• degenerate aisotropy ($r_0^2 = \delta_0^2$ and $\eta = 1$).

Thus, from Fig. 1, *b* we can see that this class of media is always characterized by nonorthogonal eigenpolarizations as well.

Medium with linear phase and amplitude anisotropy. From Eq. (1) for the Jones matrix of this class of media, we get:

$$\mathbf{T}^{LPLA} = \frac{\exp(-z(i\delta_0 + \xi_0)/2)}{A_5} \times$$

$$\times \begin{bmatrix} A_5\cosh(zA_5/2) + B\sinh(zA_5/2) & B\sinh(zA_5/2) \\ B\sinh(zA_5/2) & A_5\cosh(zA_5/2) - B\sinh(zA_5/2) \end{bmatrix}, \quad (12)$$

where

$$A_5 = (r_0^2 - \delta_0^2)^{1/2}, \; B = i\delta_0 \cos(2\alpha) + \xi_0 \cos(2\theta) .$$

Substituting the elements of the matrix Eq. (12) in Eq. (7) for the case $\alpha = 55°$, $\theta = 10°$, $z = 1$, we obtain the graphical interpretation of the inhomogeneity η presented in Fig. 1, *c—d*. Figure 1, *c—d* shows that this class of media is characterized by:

- improper dichroism ($\xi_0^2 < \delta_0^2$),
- nonhermitian dichroism ($\xi_0^2 > \delta_0^2$),
- degenerate case ($\xi_0^2 = \delta_0^2$ and $\eta = 1$).

Thus, from Fig. 1, *c—d* we can see that this class of media is characterized by nonorthogonal eigenpolarizations, excluding the case of $\alpha = \theta$ when eigenpolarizations are orthogonal [18].

Arbitrary homogeneous anisotropic medium. This section summarizes the results obtained above.

Substituting the elements of the Jones matrix Eq.(5) in Eq.(7) for the case $\alpha = 12°$, $\theta = 23°$, $r = 0.8$, $z = 1$, and $\delta = 2r\phi \csc(2a)\sin(2\theta)/\xi$ gives the graphical interpretation of the inhomogeneity η, presented in Fig. 2. Figure 2 shows that this class of media is characterized by:

- improper dichroism ($\delta_0{}^2 < \xi_0{}^2 + r_0^2$),
- nonhermitian dichroism ($\delta_0{}^2 > \xi_0{}^2 + r_0^2$),
- degenerate case ($\delta_0{}^2 = \xi_0{}^2 + r_0^2$ and $\eta = 1$).

From Fig. 2, we can see that this class of media is characterized by orthogonal eigenpolarizations ($\eta =$ $= 0$) in the case of $\delta = 2\xi\phi / r$ and/or $\alpha = \theta$.

SUMMARY AND CONCLUSIONS

In this paper, we systematically analyzed the properties of complex anisotropy types (elliptical birefringence and Hermitian dichroism, improper dichroism, non-Hermitian dichroism, and degenerate anisotropy) by solving the spectral problem for the corresponding Jones matrices and the inhomogeneity parameter that arise from remote scattering. Such a comprehensive analysis, to our knowledge, has not been performed before. For a more detailed interpretation and clarity of the results, their geometric interpretation is presented.

The results enhance understanding of the polarization phenomena in electromagnetic scattering and form the basis for future studies in polarization diagnostics and remote sensing. An important and interesting consequence is that the results obtained in this paper will contribute to the development of more accurate, compact, and fast methods for measuring the polarization characteristics of natural objects (such as various types of vegetation, water surfaces, buildings, snow, ice, clouds, fogs, and aerosols, etc.).

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СПЕКТРАЛЬНА ЗАДАЧА ДЛЯ МАТРИЦІ ДЖОНСА У ДИСТАНЦІЙНОМУ РОЗСІЯННІ

Статтю присвячено вивченню анізотропії віддаленого розсіяння на основі спектральної задачі. Спектральна задача формулюється як визначення власних поляризацій і власних значень для матриці Джонса, шо описують оптичну анізотропію середовища. Матриці Джонса середовищ із складною анізотропією (середовища, що характеризуються декількома типами анізотропії) розглядаються в рамках однорідного (диференціального) підходу. Суть такого підходу полягає в тому, що анізотропія класу середовищ, який розглядається, не залежить від товщини цього середовища. Виконано аналіз матриць Джонса для випадку довільних однорідних середовищ (середовищ, що характеризуються усіма чотирма основними типами оптичної анізотропії: лінійна, циркулярна, фазова та амплітудна анізотропія) і середовищ, що характеризуються двома типами анізотропії як окремого випадку. Основним інструментом для такого аналізу був параметр неоднорідності середовища, що дозволяє охарактеризувати останнє як середовище з ортогональними або ж неортогональними власними поляризаціями. Виявлено особливості складних типів анізотропії (еліптичне подвійне променезаломлення та ермітів дихроїзм, невласний дихроїзм, неермітів дихроїзм, вироджена анізотропія) на основі параметра неоднорідності. Продемонстровано геометричну інтерпретацію власних поляризацій за допомогою параметра неоднорідності. Розраховано умови на параметри анізотропії, при яких зазначені вище складні типи анізотропії реалізуються в досліджуваних класах середовищ. Дослідження було мотивоване фундаментальними результатами ван де Хюлста і Ховеніра, які лягли в основу аналізу внутрішньої структури матриць Джонса і Мюллера. Отримані результати сприяють глибшому розумінню явищ поляризації в електромагнітному розсіюванні та створюють основу для майбутніх досліджень поляризаційної діагностики та дистанційного зондування.

Ключові слова: матриця Мюллера, матриця Джонса, фазова лінійна та циркулярна анізотропія, амплітудна лінійна та циркулярна анізотропія, спектральна задача.