https://doi.org/10.15407/knit2025.01.017
UDC 62-932.4

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MODELING THE STABILITY OF A THROTTLE HYDRAULIC DRIVE OF ROCKET AND SPACE SYSTEMS UNDER RANDOM LOAD AND STOCHASTIC PARAMETERS

The use of a throttle hydraulic drive in rocket and space technology is promising due to its simplicity, reliability in operation, and low metal consumption. It has been determined that vibrations occur in the hydraulic system of rocket and space equipment under external influence. They cause unstable movement of working units, and, as a result, additional vibrations occur on the actuator. Determining the stability of the throttle hydraulic drive is relevant. This will ensure that the rocket and technical system maintains specified equilibrium states or types of motion. The article solves an important scientific and technical problem of increasing the accuracy of identifying the state of a hydraulic drive with throttle control under the action of a stochastic load in rocket and technical systems. A mathematical model of the operation of a hydraulic drive with throttle control is developed based on its calculation scheme. A generalized method for mathematical modeling of the probability of system stability for the mathematical expectation under a random load and in the presence of one random parameter, namely, the modulus of elasticity of the working fluid, is developed. Linearization of viscous friction forces was performed, and the schedule of the standard deviation of the random modulus of elasticity of the working fluid in the Taylor series was used. A solution to the mathematical model in the form of differential equations using a technique based on statistical linearization and expansion in the Taylor series was proposed. In this case, the stability condition of the hydraulic system is determined by the mathematical expectation in the form of the Hurwitz criterion. The stability condition of the hydraulic drive with throttle control is determined based on the probability of system stability, where the value of the random external load is specified in the form of mathematical expectation and variance.

Keywords: *hydraulic drive; stochastic parameters; pressure; elastic modulus; stability; rocket and space technology.*

Цитування: Ivanchuk Y. V., Yarovyi A. A., Liman V. V., Ozeranskyi V. S., Kozlovskyi O. A. Modeling the stability of a throttle hydraulic drive of rocket and space systems under random load and stochastic parameters. *Space Science and Technology.* 2025. **31**, No. 1 (152). P. 17—26. https://doi.org/10.15407/knit2025.01.017

INTRODUCTION

Hydraulic automation equipment is widely applied in the rocket and space industry [12]. In particular, hydraulic lifting mechanisms are exclusively used in the lifting and installation units of space launch complexes to transfer a space rocket from a horizontal to a vertical position [3]. The use of hydraulic systems for controlling various machines and devices in spacecraft testing facilities allows for creating external working load forces with a wide range of dynamic parameter adjustmentsat the experimental facility [18]. In modern rocket and technical systems with a high degree of cycle automation, it is necessary to implement many different types of motion. A compact hydraulic cylinder is easily built into mechanisms and connected by pipelines to a pump unit. The use of a hydraulic drive opens up wide opportunities for cycle automation, control, and optimization of work processes and is also easily upgraded. Throttle control based on the use of elements with variable flow sections in control circuits is promising for the hydraulic drive of rocket and space technology [9]. These elements regulate the flow rate of the working medium and, if necessary, change the direction of flows. In hydraulic systems with throttle control, the power source provides control circuits for the working environment with a small change in pressure in the pressure line at virtually unlimited costs [16].

The growing requirements for automatic control systems in rocket and space technology have led to an in-depth study of the properties of hydraulic devices and the development of calculation methods for systems built using these devices. It has been determined that vibrations occur in the hydraulic system of rocket and space equipment under external influence. This causes unstable movement of the working units, and, as a consequence, additional vibrations occur on the actuator [10]. In particular, when random vibrations occur, the change in the pressure drop in the cavities of the hydraulic cylinder can be so large that it significantly affects the instantaneous values of the flow rate of the working fluid flowing through the distributor. All this leads to a general decrease in the reliability of rocket and technical systems and, consequently, can result in emergency situations, both on the launch pad and in outer space [10, 22]. Therefore, when designing hydraulic systems, it is necessary to solve the synthesis problem. It consists of choosing the system structure, its parameters, and the design of the elements in such a way that both resistance to random stochastic loads (vibrations, impacts from space debris) and the necessary quality indicators of the control processes are ensured [7, 18].

The aim of the work is to increase the accuracy of identifying the state of a hydraulic drive with throttle control under the action of a stochastic load in rocket-technical systems by developing and implementing new, more effective methods of mathematical modeling. This will allow for achieving a number of high-quality practical results: increasing the reliability of determining the operating characteristics when designing a hydraulic drive; the possibility of developing rocket-technical systems with improved operational characteristics; and reducing the time for testing rocket and space technologies, etc.

To achieve the set goals, it is necessary to solve the following tasks:

• develop a mathematical model of the operation of a hydraulic drive with throttle control under the action of a stochastic load;

• develop a method for solving a mathematical model of the dynamics of a hydraulic drive with throttle control, which will allow determining the stability of the system under the action of a stochastic load;

• using the developed method, determine the stability of a given motion mode under the action of an external stochastic load.

LITERATURE REVIEW

The work of [22] presents a graph-analytical method for determining the stability region of pressure pulse generators. In this method, unlike existing ones, the Hurwitz stability criterion is used for a linear, nonhomogeneous differential equation of the third order to represent the mathematical model of the motion of the shut-off element in the pulsator valve. This allows us to determine the energy relationships of the drive for the occurrence of different types of oscillatory processes.

The practical implementation of such an approach is possible only for mathematical models of mainly low dimensionality and describes the properties of objects under the action of deterministic loads. This restricts the applicability of mathematical modeling results, which do not consider the effects of all transient processes in the hydraulic link [26]. All this results in the accumulation of redundant, unrealized systems of technological movements [6].

In the works of [1, 23], a statistical linearization method was proposed that, unlike the existing ones, uses complex amplitudes and the probability integral. This allows for solving linear non-homogeneous differential equations of the second order of oscillatory systems in stochastic mathematical models of hydraulic systems using the spectral form of amplitude-frequency characteristics. In these models, the functions of the force interaction of the working elements of the pulse drive are a stationary, ordinary, random process. The method used does not take into account the stochastic change in the properties of the working fluid as an element of the energy carrier of the hydraulic system. The consequence of this is the numerical instability (fluctuations) of the solution of the mathematical model in the transient modes of operation of the hydraulic drive.

The physical parameters of the energy carrier (working fluid) and the design characteristics of the hydraulic drive have a significant impact on increasing the speed, energy saturation, and compactness of the hydraulic system. This necessitates the creation of mathematical models as systems of differential equations based on an artificial dynamic model with reduced coefficients [4, 27]. The reduced coefficients reflect the elastic-viscous properties of the hydraulic circuit with subsequent linearization of its dynamic characteristics. This, in turn, leads to the ignoring of wave processes [7] in the hydraulic drive. This approach to modeling does not allow for determining the stability of specified equilibrium states or movements of the hydraulic system.

The state of a hydraulic system can be stable or unstable depending on the characteristics and parameters of the elements that comprise it [21]. Since stability is the ability of a system to maintain specified equilibrium states or ensure specified types of movement, determining stability is, therefore, an urgent task when designing hydraulic systems.

MATERIALS AND METHODS

Figure 1 shows the hydraulic diagram of the hydraulic drive with throttle adjustment [5] for the movement of the executive body of hydraulic cylinder *1*. The hydraulic cylinder rod *1* is connected to the reduced mass *m* of the external load via a spring *3* with a stiffness of c'_k. The reduced mass *2* contacts the guide *9*. A damping piston *10* is placed on the upper

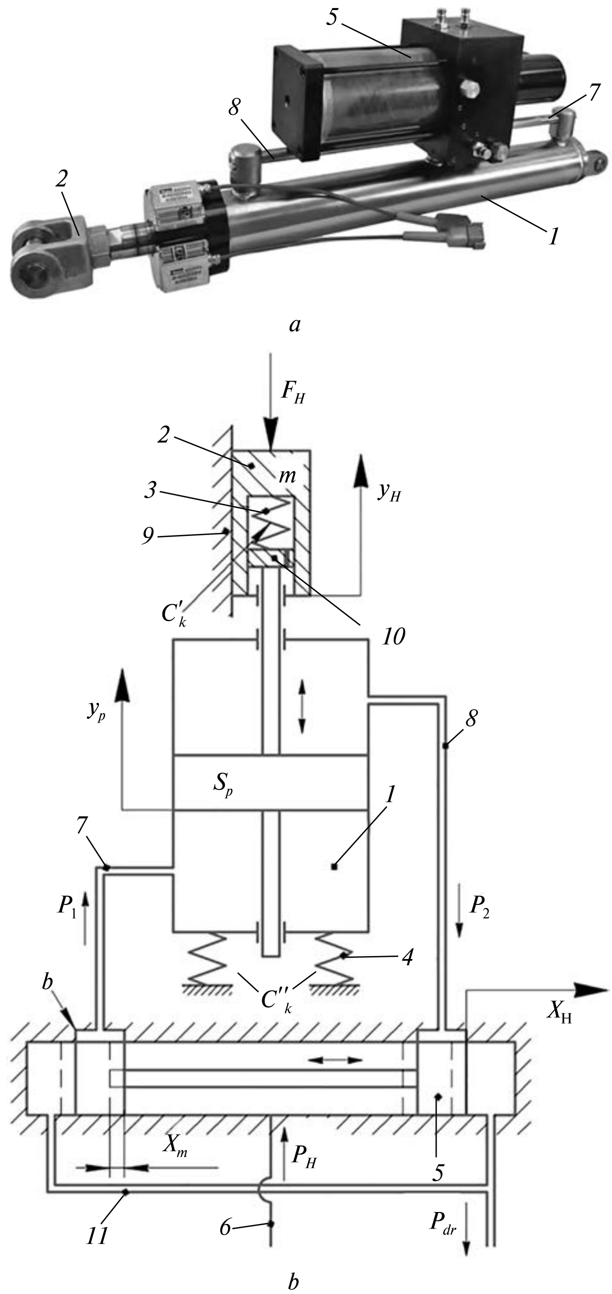

Figure 1. Hydraulic drive with throttle control: *a* — general look, *b* — calculation scheme

end of hydraulic cylinder rod *1*. The hydraulic cylinder *1* is attached to the base via springs *4* with a stiffness of c_k''. In hydraulic cylinder *1*, the working fluid is pumped through hydraulic line *7* by the hydraulic pump via pressure hydraulic line *6* and throttle distributor *5*. The working fluid from hydraulic cylinder *1* is drained into the tank via hydraulic line *8* through throttle distributor *5* and drain hydraulic line *11*.

The mathematical model of the hydraulic drive with throttle control (see Fig. 1, *b*) is represented by a system of equations:

$$S_P \frac{d}{dt}(y_P - y_C) + \frac{V}{E}\frac{dp_F}{dt} = Q_1 - rp_F\,,$$

$$m\frac{d^2 y_H}{dt^2} = c_k'(y_P - y_H) + \eta' \frac{d}{dt}(y_P - y_H) -$$

$$-\eta \frac{dy_P}{dt} - k_P y_H - F_H - F_{fr}\,\mathrm{sign}\frac{dy_H}{dt}\,, \qquad (1)$$

$$p_F S_P = c_k'(y_P - y_H) + \eta' \frac{d}{dt}(y_P - y_H)\,,$$

$$-p_F S_P = c_k''(y_P - y_H) - \eta'' \frac{dy_C}{dt},$$

where

$$Q_1 = x_H \mu b \sqrt{\frac{1}{2}\left| p_H - p_F \mathrm{sign} x \right|}\,\mathrm{sign}\Delta p\,,$$

$$\Delta p = p_P - p_F\,,\;\; p_{\Pi} = p_H - p_{dr}\,,\;\; p_F = p_1 - p_2\,,$$

$$x_H = \begin{cases} x & |x| \le x_m, \\ x_m \mathrm{sign} x & |x| > x_m, \end{cases}$$

η' is the coefficient of viscous friction between the rod of hydraulic cylinder *1* and the reduced mass *2*, η'' is the coefficient of viscous friction between the piston and the walls of hydraulic cylinder *1*, η is the coefficient of viscous damping of the piston of hydraulic cylinder *10* of the reduced mass *2*, V is the volume of the pressure cavity of the hydraulic drive, S_P is the area of the piston of hydraulic cylinder *1*, r is the loss coefficient of the hydraulic drive, k_P is the coefficient of positional load on the initial link 2 of the hydraulic drive, P_p is the difference between the supply pressure P_H from the hydraulic pump and the drain pressure p_{dr}, p_F is the difference in pressure on the piston of hydraulic cylinder *1*, y_H is the displacement of the reduced mass *2*, y_P is the displacement of the piston of hydraulic cylinder *1*, E is the reduced modulus of elasticity (random variable), F_H is the external load on reduced mass *2* (random stationary function), F_{fr} is the dry friction force between the reduced mass *2* and guide *9*, b is the width of the throttle opening of hydraulic distributor *5*, x_H is the displacement of the piston of throt tle hydraulic distributor *5*, x_m is the maximum displacement of the piston of throttle hydraulic distributor *5*, p_1 is the discharge pressure of hydraulic pump *1*, p_2 is the drain pressure of hydraulic pump *1*, μ is the flow coefficient of throttle hydraulic distributor *5*.

Taking the external disturbance as $x = 0$, the system of equations (1) can be written in the following form [17]:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\eta_{fr}(\sigma) + \mathbf{B}F_H\,,\; s = x_2\,, \qquad (2)$$

where

$$\mathbf{A} = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ -a_2 & -a_3 & a_4 & 0 & a_1 \\ -\gamma & -\theta_1 & -\beta & \theta & \gamma \\ 0 & 0 & -a & -b & 0 \\ c & 1 & e & 0 & -c \end{vmatrix},$$

$$\mathbf{B} = \begin{vmatrix} 0 \\ -\frac{1}{m} \\ 0 \\ 0 \\ 0 \end{vmatrix}, \qquad \mathbf{x} = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{vmatrix},$$

$$a = \frac{S_P}{\eta''}\,,\; b = \frac{c_k''}{\eta''}\,,\; c = \frac{c_k'}{\eta'}\,,\; e = \frac{S_{\Pi}}{\eta'}\,,$$

$$a_1 = \frac{c_k' - c\eta'}{m}\,,\; a_2 = \frac{c_k' - c\eta' + k_P}{m}\,,$$

$$a_3 = \frac{F_{fr}}{m}\,,\; a_4 = \frac{e\eta'}{m}\,,$$

$$\alpha = \frac{2E}{V}\,,\;\; \beta = \alpha r + \alpha(e + a)S_P\,,$$

$$\theta_1 = \alpha S_P\,,\;\; \gamma = \theta_1 c\,,\;\; \theta = \theta_1 c\,,$$

$$x_1 = y_H\,,\; x_2 = \frac{dy_H}{dt}\,,\; x_3 = p_F\,,$$

$$x_4 = y_C\,,\; x_5 = y_P\,.$$

It is necessary to determine the probability of system stability by the mathematical expectation under random load and in the presence of one random parameter. Let us assume that the load $F_H(t)$ is a random stationary function with a known mathematical expectation m_{PH}, variance $D_{PH,}$ and correlation function $K_{PH}(\tau)$.

The reduced modulus of elasticity of the system E is considered a random parameter. To justify this assumption, it is necessary to note that the modulus of elasticity E depends on a number of factors: pressure and rate of pressure change [15], gas-air phase content [22], temperature of the air working fluid, etc. Considering that these factors are independent of each other and some of them are random variables, the value of the modulus of elasticity of the working fluid E is taken as a random variable with a normal probability density distribution and a given mathematical expectation m_E and variance D_E.

To solve the problem, it is necessary to apply an approach based on the statistical linearization method [2]. When m_{FH} = const, the mathematical expectation $m_x(t)$ in the established mode is determined from a system of nonlinear algebraic equations [20]:

$$\begin{gathered} m_{x_2} = 0, \\ -a_2 m_{x_1} - (a_3 + k_0 / m) m_{x_2} + a_4 m_{x_3} + \\ + a_1 m_{x_5} = \frac{m_{FH}}{m}, \\ -\gamma m_{x_1} - \theta m_{x_2} - \beta m_{x_3} - \theta m_{x_4} + \gamma m_{x_5} = 0, \\ -a m_{x_3} - b m_{x_4} = 0, \\ c m_{x_1} + m_{x_2} + e m_{x_3} - c m_{x_5} = 0, \end{gathered} \tag{3}$$

where

$$k_0(0, \sigma_{x_2}) = k_1(0, \sigma_{x_2}) =$$

$$= \eta_2 + \frac{2l}{\sigma_{x_2}\sqrt{2\pi}} - 2(\eta_1 + \eta_2)\Phi\left(\frac{\varepsilon}{\sigma_{x_2}}\right),$$

and parameters l_1, $\eta_1 = \text{tg}\eta'$, $\eta_2 = \text{tg}\eta''$ are known values determined from the nonlinear characteristic diagram Figure 2.

The solution of system (3) is

$$m_x = \alpha = \left| -\frac{m_{PH}}{m}, 0, 0, 0, -\frac{m_{PH}}{m} \right|.$$

Taking into account:

$$\beta = \text{grad} k_0(m_x, \sigma_x)\big|_{m_x} = \left| 0, \left.\frac{\partial k_0}{\partial m_{x_2}}\right|_{m_x}, 0, 0, 0 \right|$$

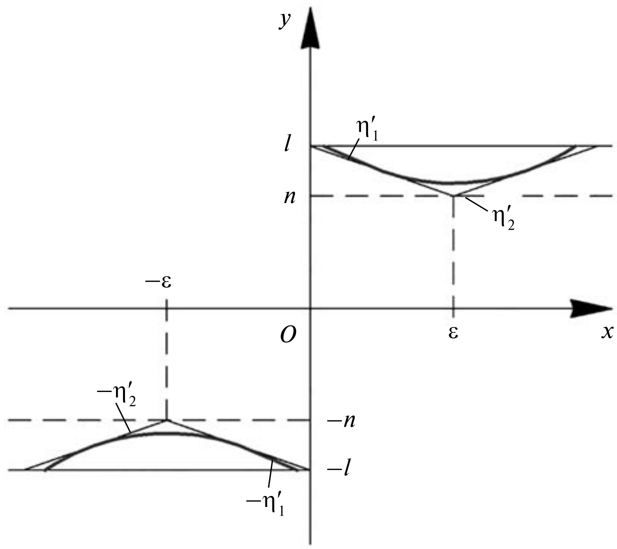

Figure 2. Diagram of the characteristics of a nonlinear element

and B, c^*, α, β^*= 0, we have the Hurwitz stability condition [24] for the following polynomial [4, 7]:

$$\Delta = | pE - A - Bk_0(0, \sigma_{x_2})c^* | =$$

$$= p^5 + d_1 p^4 + d_2 p^3 + d_3 p^2 + d_4 p + d_5, \tag{4}$$

where

$$\begin{gathered} d_1 = \mu_2 + a_3, \\ d_2 = \mu_1 + a_3\mu_2 + a_2 + a_4\theta_1 - a_1, \\ d_3 = \mu_0 + a_3\mu_1 + (a_2 - a_1)\mu_2 + a_4\theta_1(b + c) + a_1\theta c, \\ d_4 = a_3\mu_0 + (a_2 - a_1)\mu_1 + \theta b(a_4 c + a_1 e), \\ d_5 = (a_2 - a_1)\mu_0, \\ \mu_0 = b\beta c - a\theta c - \gamma e b, \\ \mu_2 = b + \beta + c, \\ \mu_1 = b\beta + cb + c\beta - a\theta - \gamma e, \\ a_3 \equiv a_3 + k_0/m . \end{gathered}$$

The stability condition according to Hurwitz [7, 24] for Δ:

$$d_1 > 0, \delta_2 > 0, \delta_3 > 0, \delta_4 > 0, d_5 > 0, \tag{5}$$

where

$$\delta_2 = d_1 d_2 - d_3, \ \delta_3 = d_3\delta_2 - d_1\delta_0,$$

$$\delta_4 = d_4\delta_0 - d_2 d_5\delta_2 - d_5\delta_0, \ \ \delta_0 = d_1 d_4 - d_5.$$

In addition to condition (5), the equation is used:

$$\sigma_{x_2}^2 = \int_{-\infty}^{+\infty} \left| \frac{L(i\omega)}{\Delta(i\omega)} \right| S_{P_H}(\omega) d\omega , \quad (6)$$

where $S_{P_H}(\omega)$ — special density of a stationary random function,

$$L(p) = -p(p^3 + \mu_2 p^2 + \mu_1 p + \mu_0) / m .$$

To solve equation (6) after integration, it is necessary to write it in implicit form:

$$\Phi_1(k_1(0,\sigma_{x_2}), \sigma_{x_2}(E)) = 0, \quad (7)$$

where from

$$\Phi_1(k_1(0,\sigma_{x_2}), \sigma_{x_2}(m_E)) = 0. \quad (8)$$

Then, in the region of the point, m_E is determined by Taylor's formula [11]:

$$\sigma_{x_2}(E) = \sigma_{x_2}(m_E) + \frac{\partial \sigma_{x_2}(m_E)}{\partial E}(E - m_E) +$$

$$+ \frac{1}{2} \frac{\partial^2 \sigma_{x_2}(m_E)}{\partial E^2}(E - m_E) + R_3, \quad (9)$$

where $\sigma_{x_2}(m_E)$ is determined from equation (8) and

$$\frac{\partial \sigma_{x_2}(m_E)}{\partial E} = - \left. \frac{\partial \Phi_1 / \partial E}{\partial \Phi_1 / \partial \sigma_{x_2}} \right|_{E=m_E}$$

$$\frac{\partial^2 \sigma_{x_2}(m_E)}{\partial E^2} = \left. \frac{\partial}{\partial E} \left(\frac{\partial \Phi_1 / \partial E}{\partial \Phi_1 / \partial \sigma_{x_2}} \right) \right|_{E=m_E} ,$$

R_3 — remainder term of the equation.

After determination $\sigma_{x_2} = \beta(E)$

$$\Sigma_{x_2} = M[\sigma_{x_2}] = \int_{-\infty}^{+\infty} \beta(E) f(E) dE , \quad (10)$$

$$\delta^2 = D[\sigma_{x_2}] = \int_{-\infty}^{+\infty} \beta^2(E) f(E) dE - \Sigma_{x_2}$$

and the stability region D based on inequalities (5), the probability of system stability is determined as:

$$P = \int_D f(E) dE ,$$

where

$$f(E) = \frac{1}{\sqrt{2}\sigma_E} \exp\left(-\frac{(E - m_E)^2}{2\sigma_E^2} \right).$$

RESULTS AND DISCUSSION

To implement the developed method of mathematical modeling of the stability of a throttle hydraulic drive (see Fig. 1, *b*), the following is adopted: $\eta' = \eta'' = 0$ N·s²/m, $c_k' = \infty$ N/m, $k_P = 0$ N/m, $c_k'' = 51.8 \cdot 10^5$ N/m, $m = 5 \cdot 10^3$ kg, $\eta = 2.34 \times 10^4$ N·s²/m, $V = 7.05 \cdot 10^{-3}$ m³, $S_P = 78.54 \cdot 10^{-4}$ m², $r = 2 \cdot 10^{-11}$ m⁵/(N·s).

Also, for the random modulus of elasticity, the value [2] is specified in the form of the mathematical expectation $m_E = 0.8 \cdot 10^9$ N/m², the standard deviation $\sigma_E = 0.07 \cdot 10^9$ N/m², and the probability density function is determined by the following formula:

$$f(E) = \frac{1}{\sqrt{2\pi}\sigma_E} \exp\left(-\frac{(E - m_E)^2}{2\sigma_E^2} \right).$$

For a random external load, the value [11] is also specified as a mathematical expectation $m_{P_H} = 3.0 \cdot 10^9$ N, a variance $D_{P_H} = 45.0 \cdot 10^6$ N², and the following formula determines the correlation function:

$$K_{P_H}(\tau) = D_{P_H} \exp\left(\frac{|\tau|}{0.62} \right).$$

The diagram of the characteristics of the external nonlinear load of viscous friction forces acting on hydraulic cylinder piston *1* (see Fig. 1) is shown in Figure 3.

For the given input data based on equations (2):

$$\mathbf{A} = \begin{vmatrix} -b & c \\ -\alpha & -\beta \end{vmatrix},$$

$$\mathbf{x} = \begin{vmatrix} x_2 \\ x_3 \end{vmatrix}, \quad \mathbf{B} = \begin{vmatrix} -1/m \\ 0 \end{vmatrix}, \quad \mathbf{c} = \begin{vmatrix} 1 \\ 0 \end{vmatrix},$$

$$b = \frac{\eta}{m} = \frac{2.34 \cdot 10^4}{5 \cdot 10^3} = 4.68 \ \text{N·s}^2/(\text{m·kg}),$$

$$\alpha = \frac{S_P}{d}, \quad c = \frac{S_P}{m} = \frac{78.54 \cdot 10^{-4}}{5 \cdot 10^3} = 15.708 \cdot 10^{-7} \ \text{m}^2/\text{kg},$$

$$\beta = \frac{r}{d}.$$

Taking $m_{x_2} = 0$, we obtain the conditional resistance according to Hurwitz in the form of [7, 20]:

$$d_1 = k_2 + b + \beta > 0 ,$$

$$d_2 = k_2\beta + b\beta + \alpha c , \quad (11)$$

where $k_2 \equiv k_0(0,\sigma_{x_2}) / m = k_1(0,\sigma_{x_2}) / m$.

Calculating the spectral density $S_{P_H}(\omega)$ through the correlation function $K_{P_H}(\tau)$ from equation (6), we obtain the following:

$$\sigma_{x_2}^2 = \frac{\sigma_{P_H}^2}{0.62\pi m} \int_{-\infty}^{+\infty} \left| \frac{i\omega + \beta}{(i\omega)^3 + i\omega d_1 + d_2} \right|^2 \frac{d\omega}{(1.6 + i\omega)^2} =$$

$$= \frac{2\sigma_{P_H}^2}{0.62 m^2} J^3 ,$$

where

$$J = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\frac{g_3(i\omega)}{h_3(i\omega)h_3(-i\omega)}\right|^2 d\omega,$$

$$g_3(z) = \beta^2 - z^2,$$

$$h(z) = z^3 + (d_1 + 1.6)z^2 + (d + 1.6d_1)z + 1.6d_2.$$

Finally, we get:

$$\Phi_1(k_2, \sigma_{x_2}, E) \equiv \sigma_{x_2}^2 (k_2 + b + \beta)(k_2\beta + b\beta + \alpha c)\times$$
$$\times[0.62(k_2\beta + b\beta + \alpha c) + k_2 + b + \beta + 1.6] -$$
$$-1.8[k_2\beta + b\beta + \alpha c + 0.62\beta^2(k_2 + b + \beta + 1.6)] = 0,$$

where

$$k_2 = \frac{k_1(0, \sigma_{x_2})}{m} = \frac{0.191}{\sigma_{x_2}} - 0.4\Phi\left(\frac{0.2}{\sigma_{x_2}}\right).$$

From the equation $\Phi_1(k_2, \sigma_{x_2}, m_E) = 0$, we determine $\sigma_{x_2}(m_E) = 1.37\cdot10^{-2}$ N/m^2. Based on expression (9), we determine:

$$\sigma_{x_2}(E) = 1.37\cdot10^{-2} - 1.8\cdot10^{-11}(E - m_E) -$$
$$-\frac{1}{2}0.355\cdot10^{-20}(E - m_E)^2. \qquad (12)$$

Using equations (10) and (12), we can obtain that:

$$\Sigma_{x_2} = 1.369\cdot10^{-2}, \quad \delta^2 = 1.894\cdot10^{-2}.$$

Then, based on Chebyshev's inequality [8, 26], we can obtain:

$$0.8439\cdot10^{-2} = \Sigma_{x_2} - 3\delta < \sigma_{x_2} < \Sigma_{x_2} + 3\delta =$$
$$= 1.894\cdot10^{-2}. \qquad (13)$$

The stability region D of the system is determined based on equation (11). To do this, it is necessary to approximate the function $k_2(0, \sigma_{x_2})$ by a linear function in the region of the point $M[\sigma_{x_2}] = 1.369\cdot10^{-2}$:

$$k(0, \sigma_{x_2}) \approx -1019 + 27.7\sigma_{x_2}. \qquad (14)$$

Substituting equation (14) into (11) and taking into account (12), we obtain:

$$D\{0 \le E < -2.9\cdot10^8\}.$$

Then the probability of stability of the system is:

$$P = \int_D f(E)dE =$$
$$= 1 - \frac{1}{\sqrt{2\pi D_E}}\int_{-2.9\cdot10^{-8}}^{0} \exp\left[-\frac{(E - m_E)^2}{2D_E}\right]dE =$$
$$= 1 - \Phi(0.47) = 0.82.$$

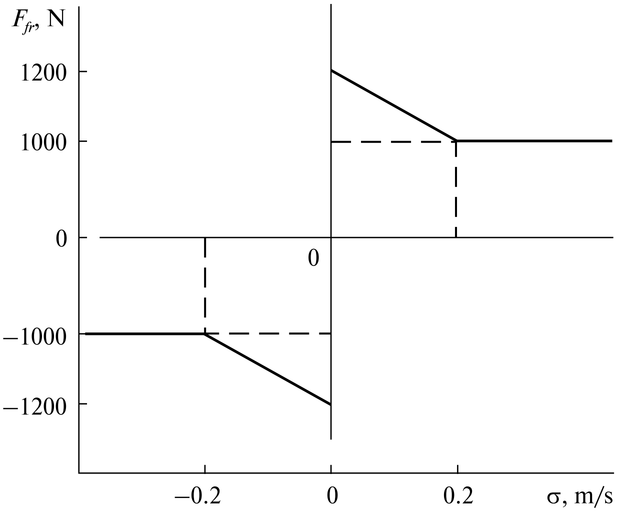

Figure 3. Diagram of the characteristics of the nonlinear element of the piston's viscous friction forces

Thus, a solution was obtained that provides a quantitative assessment of the probability of stability of the hydraulic system in Figure 1. The possibility of meeting the stability conditions of the hydraulic drive depends on the requirements for its speed. It also depends on the degree of damping created by friction forces both in the hydraulic cylinder and the load and on the permissible flow rate of fluid from the power supply system at the equilibrium state of the hydraulic drive [14]. If it is necessary to obtain a high-quality factor of the hydraulic drive in the presence of a large reduced mass *m* of the external load to the hydraulic cylinder rod and at small values of the viscous damping coefficient η, then additional measures are used to ensure stability. They are as follows: an overflow of working fluid is introduced between the cavities of hydraulic cylinder *1*, an elastic element *4* of hydraulic cylinder support 1 with a rigidity of c_k'' is used, and a damper is installed on the spool of the throttle distributor [19, 25].

CONCLUSIONS

Based on the theoretical studies performed, an important scientific and technical problem of increasing the accuracy of identifying the state of a hydraulic drive with throttle control under the action of a stochastic load in rocket and spacecraft systems was solved. This was achieved by developing a generalized method for mathematical modeling of the probability of system stability according to the mathematical expectation under random load and in the presence of one stochastic parameter.

A solution to the mathematical model in the form of differential equations is proposed using a technique based on statistical linearization through expansion in a Taylor series. In this case, the stability condition of the hydraulic system is determined by the mathematical expectation in the form of the Hurwitz criterion.

Using the developed method, the stability condition of a hydraulic drive with throttle control is determined. In this case, for a random external load, the value of which is specified as a mathematical expectation $m_{P_H} = 3.0 \cdot 10^9$ N and dispersion $D_{P_H} = 45.0 \cdot 10^6$ N^2, the probability of system stability is equal to $P = 0.82$.

REFERENCES

1. Debeleac C., Nastac S. (2014). Stochastic approaches of nonlinear model-based simulations for vibratory compaction process. *PAMM*, **14**, No. 1, 749—750. https://doi.org/10.1002/pamm.201410357.
2. Fossen T. I., Nijmeijer H. (2012). *Parametric resonance in dynamical systems*. New York: Springer, 342 p. https://doi.org/10.1007/978-1-4614-1043-0.
3. Freye C. E., Fit B. D., Billingsley M. C., Synovec R. E. (2016). Partial least squares analysis of rocket propulsion fuel data using diaphragm valve-based comprehensive two-dimensional gas chromatography coupled with flame ionization detection. *Talanta*, No. 153, 203—210. https://doi.org/10.1016/j.talanta.2016.03.016.
4. Hirmand M. R., Vahab M., Papoulia K. D., Khalili N. (2019). Robust simulation of dynamic fluid-driven fracture in naturally fractured impermeable media. *Computer Methods in Applied Mechanics and Engineering*, No. 357, 112—574. https://doi.org/10.1016/j.cma.2019.112574.
5. Iskovych-Lototsky R. D., Ivanchuk Y. V., Veselovska N. R., Surtel W., Sundetov S. (2018). Automatic system for modeling vibro-impact unloading bulk cargo on vehicles. *Proc. SPIE 10808: Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018*, 1080860. https://doi.org/10.1117/12.2501526.
6. Iskovych-Lototsky R. D., Ivanchuk Y. V., Veselovsky Y. P., Gromaszek K., Oralbekova A. (2018). Automatic system for modeling of working processes in pressure generators of hydraulic vibrating and vibro-impact machines. *Proc. SPIE 10808: Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018*, 1080850. https://doi.org/10.1117/12.2501532.
7. Iskovich-Lototsky R. D., Kots I. V., Ivanchuk Y. V., Ivashko Y. I., Gromaszek K., Mussabekova A., Kalimoldayev M. (2019). Terms of the stability for the control valve of the hydraulic impulse drive of vibrating and vibro-impact machines. *Przeglad Elektrotechniczny*, No. **4**(19), 19—23. https://doi.org/10.15199/48.2019.04.04.
8. Ivanchuk Y., Manzhilevskyy O., Belzetskyi R., Zamkovyi O., Pavlovych R. (2022). Modelling of piling technology by vibro-impact device with hydropulse drive. *Scientific Horizons*, **25**, No. 1, 9—20. https://doi.org/10.48077/scihor.25(1).2022.9-20.
9. Jiao X., Zhang J., Li W., Wang Y., Ma W., Zhao Y. (2023). Advances in spacecraft micro-vibration suppression methods. *Progress in Aerospace Sci.*, No. 138, 100898. https://doi.org/10.1016/j.paerosci.2023.100898.
10. Kang Sh., Yan H., Dong L., Li Ch. (2018). Finite-time adaptive sliding mode force control for electro-hydraulic load simulator based on improved GMS friction model. *Mechanical Systems and Signal Processing*, No. 102, 117—138. https://doi.org/10.1016/j.ymssp.2017.09.009.
11. Kartashova E. (2011). *Nonlinear resonance analysis: Theory, computation, applications*. New York: Cambridge Univ. Press, 287 p. https://doi.org/10.1017/CBO9780511779046.
12. Masum Mostafizur R. (2024). *Introduction to Space Launch and Transportation System*, 224 p. https://doi.org/10.13140/RG.2.2.10413.50408.
13. Muŕin J., KompiÓ V., KutiÓ V. (2011). *Computational modelling and advanced simulations*. Dordrecht: Springer, 312 p. https://doi.org/10.1007/978-94-007-0317-9.
14. New South Wales., Manly Hydraulics Laboratory, & Sydney Water. Hydraulic modelling of head loss in under-pressure cut-in connections for Sydney Water Corporation. Manly Vale. N.S.W: Manly Hydraulics Laboratory, 28.
15. Pang Y. L., Dai D. M. (2014). Simulation of Dynamic Compaction on Soft Soil Foundation. *Adv. Mater. Res.*, No. 989—994, 2373—2376. https://doi.org/10.4028/www.scientific.net/AMR.989-994.2373.
16. Pei X., Wang Xi., Liu J., Zhu M., Dan Zh., He Ai., Miao K., Zhang L., Xu Zh. (2023). A review of modeling, simulation, and control technologies of altitude ground test facilities for control application. *Chinese J. Aeronautics*, No. **36**(9), 38—62. https://doi.org/10.1016/j.cja.2023.07.014.

17. Perestiuk N. A., Plotnikov V. A., Samoilenko A. M., Skripnik N. V. (2011). Differential equations with impulse effects: Multivalued right-hand sides with discontinuities. Berlin: De Gruyter, 287 p. https://doi.org/10.1515/9783110218176.
18. Ponomarov O. (2019). Ground-based experimental testing of elements of automation of pneumatic-hydraulic systems of rocket and space technology. *J. Rocket-Space Technology*, **27**, No. 4, 58—61. https://doi.org/10.15421/451909.
19. Qiu Zh., Yuan X., Wang D., Fan S., Wang Q. (2023). Physical model driven fault diagnosis method for shield. *Machine hydraulic system. Measurement*, 220, 113436. https://doi.org/ 10.1016/j.measurement.2023.113436.
20. Ram M., Davim J. P. (2017). *Mathematics applied to engineering*. London, United Kingdom: Acad. Press, 457 p. https://doi.org/10.1016/C2015-0-06715-6.
21. Reinicke A. (2011). *Mechanical and hydraulic aspects of rock-proppant systems: Laboratory experiments and modelling approaches*. PhD Thesis, (Scientific Technical Report; 11/09), Potsdam: Deutsches GeoForschungsZentrum GFZ, 140 p. https://doi.org/10.2312/GFZ.b103-11098.
22. Shatokhin V. M., Sobol V. N., Wojcik W., Mussabekova A., Baitussupov D. (2019). Dynamical processes simulation of vibrational mounting devices and synthesis of their parameters. *Przeglad Elektrotechniczny*, **4**, No. 19, 86—92. https://doi.org/10.15199/48.2019.04.15.
23. Singh V. P. (2014). *Entropy theory in hydraulic engineering: An introduction*. Reston, Virginia: Amer. Soc. Civil Eng., 358 p. https://doi.org/10.1061/9780784412725.
24. Trompette P., Axisa F. (2005). *Modelling of Mechanical Systems*: *Structural Elements*. Butterworth-Heinemann, 306 p.
25. Urbanowicz K., Bergant A., Stosiak M., Karpenko M., Bogdevičius M. (2023). Developments in analytical wall shear stress modelling for water hammer phenomena. *J. Sound and Vibration*, **562**, 117848. https://doi.org/10.1016/j.jsv.2023.117848.
26. Woś P., Dindorf R. (2019). Modeling and identification of the hydraulic servo drive. *EPJ Web of Conf.*, 213:02100. https://doi.org/10.1051/epjconf/201921302100.
27. Zhang J., Ma Zh., Wang X., Wu Q., Zhang L. (2023). Transient vibration of shafting in coupled hydraulic-mechanical-electrical-structural system for hydropower station during start-up process. *Appl. Mathem. Model.*, No. 124, 860—880. https://doi.org/10.1016/j.apm.2023.08.041.

Стаття надійшла до редакції 26.10.2024 — *Received 26.10.2024*
Після доопрацювання 10.11.2024 — *Revised 10.11.2024*
Прийнято до друку 20.11.2024 — *Accepted 20.11.2024*

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МОДЕЛЮВАННЯ СТІЙКОСТІ ДРОСЕЛЬНОГО ГІДРОПРИВОДУ РАКЕТНО-КОСМІЧНИХ СИСТЕМ ПРИ ВИПАДКОВОМУ НАВАНТАЖЕННІ ТА СТОХАСТИЧНИХ ПАРАМЕТРАХ

Перспективним є застосування в ракетно-космічній техніці дросельного гідроприводу завдяки своїй простоті, надійності в експлуатації та невисокій металомісткості. При зовнішньому впливі в гідросистемі ракетно-космічного обладнання виникають вібрації, які призводять до нестабільного руху робочих вузлів, внаслідок чого виникають додаткові коливання на виконавчому органі. Актуальним є визначення умов стійкості роботи дросельного гідроприводу, що дозволить ракетно-технічній системі зберігати задані рівноважні стани або види руху. У роботі вирішено важливу науково-технічну проблему підвищення точності ідентифікації стану гідроприводу із дросельним регулюванням при дії стохастичного навантаження у ракетно-технічних системах. Розроблено математичну модель роботи гідроприводу із дросельним регулюванням на основі її розрахункової схеми. Розроблено узагальнений метод математичного моделювання ймовірності стійкості системи по математичному сподіванню при випадковому навантаженні і при наявності одного випадкового параметра, а саме модуля пружності робочої рідини. Було проведено лінеаризацію сил в'язкого тертя і використано розклад в ряд Тейлора значень стандартного відхилення параметра випадкового модуля пружності робочої рідини. Запропоновано розв'язок математичної моделі у вигляді диференціальних рівнянь із використанням методики на основі статистичної лінеаризації на основі розкладу в ряд Тейлора, де умова стійкості гідросистеми визначається за математичним сподіванням у вигляді критерію Гурвіца. Визначено умову стійкості гідроприводу із дросельним регулюванням на основі ймовірності стійкості системи, де значення випадкового зовнішнього навантаження задано у вигляді математичного сподівання і дисперсії.

***Ключові слова*:** гідропривод, стохастичні параметри, тиск, модуль пружності, стійкість, ракетно-космічна техніка.