https://doi.org/10.15407/knit2024.04.024
UDC 681.518.5

N. YEFYMENKO, Dr. Sci. in Tech., Associate Professor. Winner of the Award of the Cabinet of Ministers of Ukraine for the development and implementation of innovative technologies (2014)
ORCID 0000-0001-7760-8610
E-mail: nefimenko@gmail.com
R. KUDERMETOV, Head of Dept., Ph. D. in Tech., Associate Professor
ORCID 0000-0002-6766-9780
E-mail: kudermetov@gmail.com

National University „Zaporizhzhia Polytechnic"
64 Zhukovskoho Str., Zaporizhzhia, 69093 Ukraine

DYNAMIC MODEL OF VECTOR MOTION AND ITS APPLICATION IN SPACECRAFT UNIAXIAL ORIENTATION PROBLEMS

The object of study is the spacecraft attitude control system. The subject of the study is the quaternion dynamic equation of motion of an arbitrary normalized vector and methods for constructing on its basis algorithms to control the spacecraft's uniaxial orientation. In this work, a new dynamic model of vector motion in a body-fixed frame is obtained, its properties are investigated, and methods for solving uniaxial orientation problems using this model are considered. This model application significantly simplifies the synthesis control task, which, in this case, is reduced to control synthesis for a system that is a set of second-order integrating links. In many cases, the synthesis problem has an analytical solution for such systems. The resulting control algorithms are much simpler to implement than the ones obtained using the traditional model. The results of numerical simulation, which confirm the effectiveness of the proposed algorithm, are presented.

***Keywords**: spacecraft, uniaxial orientation, terminal reorientation, quaternion, stabilization, angular velocity.*

1. INTRODUCTION

There are operating modes of a spacecraft that do not require triaxial orientation. Examples of such modes are the mode of spacecraft onboard battery recharging when only orientation to the Sun is necessary; the emergency mode when only two actuators are operational and triaxial orientation is impossible; the mode of pointing the telescope's optical axis in inertial space, etc. If triaxial orientation is not required or impossible, uniaxial orientation mode is used. A feature of the uniaxial orientation mode is that the spacecraft orientation is determined up to an arbitrary rotation around the axis relative to which the orientation is required. In this case, the spacecraft can rotate around this axis at an arbitrary speed.

The issues of spacecraft uniaxial orientation algorithms' construction have been considered in many works. In [5], the possibility of using a uniaxial solar orientation mode for a satellite with a solar sail in a close-to-circular orbit at an altitude of 900 km is considered. The authors of [3] studied the control parameters' optimization for the satellite uniaxial

Цитування: Yefymenko N., Kudermetov R. Dynamic model of vector motion and its application in spacecraft uniaxial orientation problems. *Space Science and Technology*. 2024. **30**, No. 4 (149). P. 24—33. https://doi.org/10.15407/knit2024.04.024

orientation using jet engines. The purpose of control is to determine the certain axis rotation trajectory from an arbitrary initial position to a given one. A genetic algorithm is used to find the optimal values of the control parameters to minimize the number of jet engine actuations. In [9], the authors considered the problem of uniaxial orientation in the inertial coordinate system of a spacecraft with one faulty reaction wheel in the presence of residual angular momentum of the spacecraft. In [1], a new uniaxial orientation control law to move an optical sensor, jet engine nozzle, or antenna to a given position after the failure of one of the reaction wheels was proposed. Thus, the problem of uniaxial orientation remains relevant today.

There are two types of uniaxial orientation problems: the uniaxial stabilization problem and the terminal reorientation problem. To synthesize algorithms of the first type, the method of Lyapunov functions is more often used [4, 10]. The disadvantage of this approach is the difficulty of obtaining a preliminary estimate of the accuracy and dynamic characteristics of the algorithm. To get these estimates, numerical modeling is necessary. The solution to the problem of the synthesis algorithm for spacecraft a terminal reorientation is usually sought as a solution to an optimization problem. Usually, a model is used for this purpose in which the Euler equation describes the dynamics [6], and the kinematic equation describes the kinematic motion of the vector [2]. This model's advantages are the absence of calculation peculiarities and state vector minimal redundancy. But this model is nonlinear, so the solution to the optimization problem can be found numerically only, which is not always allowable for on-board algorithms. This problem can be simplified by using the quaternion differential equation proposed in [8], where the authors developed a dynamic quaternion model and considered the problems of stabilization and terminal reorientation for the case of triaxial orientation to construct the control. As shown in this work, the use of the quaternion model significantly simplifies the problem of control synthesis, which, in this case, is reduced to control synthesis for a system that is a set of second-order integrating links. In many cases, for such systems, the synthesis problem has an analytical solution. The resulting control algorithms are much simpler to implement than the algorithms obtained using the traditional model.

In this work, this approach was further developed, and a dynamic model of vector motion was obtained, similar to the dynamic model from [8], its properties were investigated, and methods for solving the problem of uniaxial stabilization and uniaxial terminal reorientation using this model were considered.

2. DYNAMIC MODEL OF VECTOR MOTION

Let $\boldsymbol{q}_B \in \mathbb{R}^3$ be an arbitrary normalized vector $\boldsymbol{q}$, given by projections onto the axes of the body-fixed frame $\boldsymbol{B}$. Denote by $\boldsymbol{Q}_B$ the quaternion mapping of the vector $\boldsymbol{q}_B$. In that case,

$$\operatorname{scal}\left(\boldsymbol{Q}_B\right) = 0 , \tag{1}$$

$$\operatorname{vect}\left(\boldsymbol{Q}_B\right) = \boldsymbol{q}_B , \tag{2}$$

where $\operatorname{scal}(.)$ and $\operatorname{vect}(.)$ are the designations of the scalar and vector parts of the quaternion.

Since $\boldsymbol{Q}_B$ is a normalized quaternion, the following quaternion equation is valid for it

$$\tilde{\boldsymbol{Q}}_B \circ \boldsymbol{Q}_B = 1 , \tag{3}$$

where $\tilde{\boldsymbol{Q}}_B$ is the conjugate of quaternion $\boldsymbol{Q}_B$, and $\circ$ is the quaternion multiplication operator.

Differentiating equality (3) twice with respect to time yields

$$\ddot{\tilde{\boldsymbol{Q}}}_B \circ \boldsymbol{Q}_B + 2\dot{\tilde{\boldsymbol{Q}}}_B \circ \dot{\boldsymbol{Q}}_B + \tilde{\boldsymbol{Q}}_B \circ \ddot{\boldsymbol{Q}}_B = 0 . \tag{4}$$

From (4), it follows

$$\operatorname{scal}\left(\tilde{\boldsymbol{Q}}_B \circ \ddot{\boldsymbol{Q}}_B\right) = -\left\|\dot{\boldsymbol{Q}}_B\right\|^2 . \tag{5}$$

Thus, the quaternion $\tilde{\boldsymbol{Q}}_B \circ \ddot{\boldsymbol{Q}}_B$ has the following form:

$$\tilde{\boldsymbol{Q}}_B \circ \ddot{\boldsymbol{Q}}_B = -\left\|\dot{\boldsymbol{Q}}_B\right\|^2 + \boldsymbol{f} , \tag{6}$$

where $\boldsymbol{f}$ is an arbitrary vector representing the vector part of the $\tilde{\boldsymbol{Q}}_B \circ \ddot{\boldsymbol{Q}}_B$. Solving this equation for $\ddot{\boldsymbol{Q}}_B$ gives

$$\ddot{\boldsymbol{Q}}_B = \boldsymbol{Q}_B \circ \boldsymbol{f} - \left\|\dot{\boldsymbol{Q}}_B\right\|^2 \boldsymbol{Q}_B . \tag{7}$$

Let us introduce the following quaternion:

$$\boldsymbol{U} = \boldsymbol{Q}_B \circ \boldsymbol{f} . \tag{8}$$

In this quaternion, the scalar part u_0 and the vector part $\boldsymbol{u}$ give

$$\boldsymbol{U} = \mathrm{u}_0 + \boldsymbol{u} . \tag{9}$$

Taking into account (8), the quaternion equation (7) can be written as follows:

$$\ddot{\boldsymbol{Q}}_B = \boldsymbol{U} - \left\|\dot{\boldsymbol{Q}}_B\right\|^2 \boldsymbol{Q}_B \tag{10}$$

where $\boldsymbol{U}$ is a quaternion, by setting which one can form the required character of the change in the projections of the vector $\boldsymbol{q}_B$ on the axes of the body-fixed frame $\boldsymbol{B}$. The quaternion $\boldsymbol{U}$ can be interpreted as a control quaternion, and equation (10) as a dynamic equation of the motion of vector relative to the body-fixed frame in quaternion form, where quaternion mappings of the $\boldsymbol{Q}_B$ and its derivative are used as the state vector components. Since $\boldsymbol{f}$ is a vector, it follows from (8) that quaternion $\boldsymbol{U}$ must satisfy the constraint

$$\operatorname{scal}\left(\tilde{\boldsymbol{Q}}_B \circ \boldsymbol{U}\right) = 0. \tag{11}$$

From (11) and (1), it follows that

$$\boldsymbol{q}_B^T \boldsymbol{u} = 0\,. \tag{12}$$

Let us write equation (10) in vector form

$$\ddot{\boldsymbol{q}}_B = \boldsymbol{u} - \left\|\dot{\boldsymbol{q}}_B\right\|^2 \boldsymbol{q}_B\,. \tag{13}$$

The control $\boldsymbol{u}$ can be represented as

$$\boldsymbol{u} = -\boldsymbol{q}_B \times \left(\boldsymbol{q}_B \times \boldsymbol{\mu}_B\right). \tag{14}$$

In this case, relation (12) will be satisfied for any $\boldsymbol{\mu}_B$. Taking into account (14), equation can be written as

$$\ddot{\boldsymbol{q}}_B = -\boldsymbol{q}_B \times \left(\boldsymbol{q}_B \times \boldsymbol{\mu}_B\right) - \left\|\dot{\boldsymbol{q}}_B\right\|^2 \boldsymbol{q}_B\,. \tag{15}$$

Let us decompose the left side of equation (15) into two components: perpendicular to $\boldsymbol{q}_B$ and parallel to $\boldsymbol{q}_B$

$$-\boldsymbol{q}_B \times (\boldsymbol{q}_B \times \ddot{\boldsymbol{q}}_B) + \boldsymbol{q}_B \boldsymbol{q}_B^T \ddot{\boldsymbol{q}}_B = -\boldsymbol{q}_B \times (\boldsymbol{q}_B \times \boldsymbol{\mu}_B) - \left\|\dot{\boldsymbol{q}}_B\right\|^2 \boldsymbol{q}_B\,. \tag{16}$$

The transformations of (16), taking into account (12) and (13), give

$$-\boldsymbol{q}_B \times \left[\boldsymbol{q}_B \times \left(\ddot{\boldsymbol{q}}_B - \boldsymbol{\mu}_B\right)\right] = \\ = -\left(\boldsymbol{q}_B^T \ddot{\boldsymbol{q}}_B^T + \left\|\dot{\boldsymbol{q}}_B\right\|^2\right)\boldsymbol{q}_B = \\ = -\left(-\left\|\dot{\boldsymbol{q}}_B\right\|^2 + \left\|\dot{\boldsymbol{q}}_B\right\|^2\right)\boldsymbol{q}_B = 0\,. \tag{17}$$

It follows from (17) that

$$\ddot{\boldsymbol{q}}_B = \boldsymbol{\mu}_B + \alpha \boldsymbol{q}_B\,, \tag{18}$$

where α is an arbitrary parameter.

Parameter α is arbitrary, so it can be set equal to zero ($\alpha = 0$) when solving various problems of vector $\boldsymbol{q}_B$ motion control. In this case, the dynamics model (18) takes a simple form

$$\ddot{\boldsymbol{q}}_B = \boldsymbol{\mu}_B\,. \tag{19}$$

This is a linear equation with constant coefficients and has a simple form, which makes it possible to apply well-developed methods of the theory of linear systems with constant coefficients to find $\boldsymbol{\mu}_B$.

The control $\boldsymbol{u}$ is virtual, and the actual control is a torque $\boldsymbol{M}_u$. Therefore, when using equation (13) to solve various problems of attitude control, it is necessary to know the dependence of the control torque $\boldsymbol{M}_u$ on the control vector $\boldsymbol{u}$ and the inverse dependence of the control vector $\boldsymbol{u}$ on the control torque $\boldsymbol{M}_u$. Let us find these dependencies. The following equations are valid for the $\boldsymbol{q}_B$ vector's velocity and acceleration:

$$\dot{\boldsymbol{q}}_B = -\boldsymbol{\omega}_B^{BR} \times \boldsymbol{q}_B + \dot{\tilde{\boldsymbol{q}}}_B\,, \tag{20}$$

$$\dot{\tilde{\boldsymbol{q}}}_B = \tilde{\boldsymbol{\Lambda}}_{RB} \circ \dot{\boldsymbol{q}}_R \circ \boldsymbol{\Lambda}_{RB}\,, \tag{21}$$

$$\ddot{\boldsymbol{q}}_B = -\dot{\boldsymbol{\omega}}_B^{BR} \times \boldsymbol{q}_B + \boldsymbol{p}\,, \tag{22}$$

$$\boldsymbol{p} = -\boldsymbol{\omega}_B^{BR} \times \left(\dot{\boldsymbol{q}}_B + \dot{\tilde{\boldsymbol{q}}}_B\right) + \ddot{\tilde{\boldsymbol{q}}}_B\,, \tag{23}$$

$$\ddot{\tilde{\boldsymbol{q}}}_B = \tilde{\boldsymbol{\Lambda}}_{RB} \circ \ddot{\boldsymbol{q}}_R \circ \boldsymbol{\Lambda}_{RB}\,, \tag{24}$$

where the relative angular velocity vector $\boldsymbol{\omega}_B^{BR}$ is determined from the equation

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BR} = -\boldsymbol{\omega}_B^{BI} \times \left(\boldsymbol{J}\boldsymbol{\omega}_B^{BI}\right) - \\ -\boldsymbol{J}\left(\boldsymbol{\omega}_B^{RI} \times \boldsymbol{\omega}_B^{BR} + \tilde{\boldsymbol{\Lambda}}_{RB} \circ \dot{\boldsymbol{\omega}}_R^{RI} \circ \boldsymbol{\Lambda}_{RB}\right) + \boldsymbol{M}_u\,. \tag{25}$$

In expressions (20)—(25), $\boldsymbol{R}$ is a reference frame in which the spacecraft motion is considered. It is assumed that the rotation angular velocity $\boldsymbol{\omega}_R^{RI}(t)$ of the frame $\boldsymbol{R}$ relative to the inertial coordinate system $\boldsymbol{J}$ is a known function of time, which has a time derivative $\dot{\boldsymbol{\omega}}_R^{RI}(t)$; $\boldsymbol{q}_R$ is $\boldsymbol{q}$ vector, given by projections on the reference frame $\boldsymbol{R}$ axes; $\boldsymbol{\Lambda}_{RB}$ is the transition quaternion from reference frame $\boldsymbol{R}$ to body-fixed frame $\boldsymbol{B}$; $\boldsymbol{\omega}_B^{BR}$ is the angular velocity of the frame $\boldsymbol{B}$ rotation relative to the reference frame $\boldsymbol{R}$, given by projections on the axes of the body-fixed frame $\boldsymbol{B}$; $\boldsymbol{\omega}_B^{BI}$ is the spacecraft's absolute angular velocity of rotation, provided by projections on the body-fixed frame $\boldsymbol{B}$ axes; $\boldsymbol{J}$ is the spacecraft inertia tensor.

Formally, expressions (20)—(24) are the result obtained by double differentiation of the relation $\boldsymbol{q}_B = \tilde{\boldsymbol{\Lambda}}_{RB} \circ \boldsymbol{q}_R \circ \boldsymbol{\Lambda}_{RB}$. Let us decompose the vector $\boldsymbol{p}$ into two components: a component perpendicular to the vector $\boldsymbol{q}_B$ and a component parallel to the vector $\boldsymbol{q}_B$:

$$\boldsymbol{p} = -\boldsymbol{q}_B \times \left(\boldsymbol{q}_B \times \boldsymbol{p}\right) + \boldsymbol{q}_B \boldsymbol{q}_B^T \boldsymbol{p}\,. \tag{26}$$

According to equations (13) and (22)

$$\boldsymbol{q}_B^T\boldsymbol{p} = \boldsymbol{q}_B^T\ddot{\boldsymbol{q}}_B = -\left\|\dot{\boldsymbol{q}}_B\right\|^2 . \quad (27)$$

Then, expression (26) can be written as follows

$$\boldsymbol{p} = -\boldsymbol{q}_B \times \left(\boldsymbol{q}_B \times \boldsymbol{p}\right) - \left\|\dot{\boldsymbol{q}}_B\right\|^2 \boldsymbol{q}_B . \quad (28)$$

Substituting (28) into (22) gives

$$\ddot{\boldsymbol{q}}_B = -\dot{\boldsymbol{\omega}}_B^{BR} \times \boldsymbol{q}_B + \boldsymbol{p} =$$
$$= \boldsymbol{q}_B \times \dot{\boldsymbol{\omega}}_B^{BR} - \boldsymbol{q}_B \times \left(\boldsymbol{q}_B \times \boldsymbol{p}\right) - \left\|\dot{\boldsymbol{q}}_B\right\|^2 \boldsymbol{q}_B =$$
$$= \boldsymbol{u} - \left\|\dot{\boldsymbol{q}}_B\right\|^2 \boldsymbol{q}_B . \quad (29)$$

From equality (29), it follows that

$$\boldsymbol{u} = \boldsymbol{q}_B \times \left(\dot{\boldsymbol{\omega}}_B^{BR} - \boldsymbol{q}_B \times \boldsymbol{p}\right) . \quad (30)$$

Since $\dot{\boldsymbol{\omega}}_B^{BR}$ is a function of the control torque $\boldsymbol{M}_u$, then (30) is the desired dependence of the vector $\boldsymbol{u}$ on the control torque vector $\boldsymbol{M}_u$. To find the dependence of the control torque vector $\boldsymbol{M}_u$ on the vector $\boldsymbol{u}$, consider expression (30). Let us define the vector $\dot{\boldsymbol{\omega}}_B^{BR}$ as follows:

$$\dot{\boldsymbol{\omega}}_B^{BR} = -\boldsymbol{q}_B \times \left(\boldsymbol{u} - \boldsymbol{p}\right) . \quad (31)$$

Substituting relation (31) into equation (30) taking into account (12) gives

$$\boldsymbol{u} = \boldsymbol{q}_B \times \left(\dot{\boldsymbol{\omega}}_B^{BR} - \boldsymbol{q}_B \times \boldsymbol{p}\right) = -\boldsymbol{q}_B \times (\boldsymbol{q}_B \times \boldsymbol{u}) = \boldsymbol{u} . \quad (32)$$

Since (32) is an identity, therefore, formula (31) is a dependence $\dot{\boldsymbol{\omega}}_B^{BR}$ on the virtual control $\boldsymbol{u}$. Solving equation (25) for $\boldsymbol{M}_u$ yields

$$\boldsymbol{M}_u = \boldsymbol{\omega}_B^{BI} \times \left(\boldsymbol{J}\boldsymbol{\omega}_B^{BI}\right) +$$
$$+\boldsymbol{J}\left(\boldsymbol{\omega}_B^{RI} \times \boldsymbol{\omega}_B^{BR} + \tilde{\boldsymbol{\Lambda}}_{RB} \circ \dot{\boldsymbol{\omega}}_R^{RI} \circ \boldsymbol{\Lambda}_{RB}\right) - \boldsymbol{J}\left(\boldsymbol{q}_B \times \left(\boldsymbol{\mu}_B - \boldsymbol{p}\right)\right) . \quad (33)$$

Formula (33) is the dependence of the real control torque $\boldsymbol{M}_u$ on the variable $\boldsymbol{\mu}_B$. In this case, the variable $\boldsymbol{\mu}_B$ is selected based on equation (19), depending on the requirements of the spacecraft's uniaxial orientation problem.

3. APPLICATION OF A DYNAMIC MODEL MOTION OF VECTOR IN SPACECRAFT ATTITUDE CONTROL PROBLEMS

3.1. ***The problem of a spacecraft's uniaxial stabilization.*** The problem of spacecraft's uniaxial stabilization is usually understood as the problem of synthesizing control laws that ensure the orientation of the fixed axis in a body-fixed frame along the direction specified in the reference coordinate system. In general, the statement of the uniaxial stabilization problem is formulated as follows. Let $\boldsymbol{q}_R$ and $\boldsymbol{e}_B$ be given unit vectors in the bases $\boldsymbol{R}$ and $\boldsymbol{B}$, respectively. It is assumed that on the spacecraft board, there is information about the projections of the vector $\boldsymbol{q}$ onto frame $\boldsymbol{B}$ axes in the form of a vector $\boldsymbol{q}_B$, and the vector $\boldsymbol{e}_B$ is constant. It is necessary to find the control torque $\boldsymbol{M}_u$ using information about the vector $\boldsymbol{q}_B$, angular velocity $\boldsymbol{\omega}_B^{BI}$, and orientation quaternion $\boldsymbol{\Lambda}_{RB}$, providing asymptotic stability to the equilibrium position $\boldsymbol{q}_B = \boldsymbol{e}_B$. The solution to the formulated problem is given by the following.

Theorem 1. *Let the spacecraft rotation motion be given by the equation (25). Denote the normalized vectors in the reference frame* $\boldsymbol{R}$ *and the body-fixed frame* $\boldsymbol{B}$ *by* $\boldsymbol{q}_R$ *and* $\boldsymbol{e}_B$*, respectively. Let there be information on the spacecraft board about the absolute angular velocity vector* $\boldsymbol{\omega}_B^{BI}$*, orientation quaternion* $\boldsymbol{\Lambda}_{RB}$*, and the vector* $\boldsymbol{q}$ *projections on the body-fixed frame* $\boldsymbol{B}$ *axes in the form of the vector* $\boldsymbol{q}_B$*; the vector* $\boldsymbol{e}_B$ *is constant, and its coordinates are given. Then, the control law* (33)*, where*

$$\boldsymbol{\mu}_B = -\boldsymbol{K}_1\boldsymbol{e} - \boldsymbol{K}_2\dot{\boldsymbol{q}}_B , \quad (34)$$

$$\boldsymbol{e} = \boldsymbol{q}_B - \boldsymbol{e}_B , \quad (35)$$

$$\dot{\boldsymbol{q}}_B = -\boldsymbol{\omega}_B^{BR} \times \boldsymbol{q}_B + \dot{\tilde{\boldsymbol{q}}}_B , \quad (36)$$

$$\dot{\tilde{\boldsymbol{q}}}_B = \tilde{\boldsymbol{\Lambda}}_{RB} \circ \dot{\boldsymbol{q}}_R \circ \boldsymbol{\Lambda}_{RB} , \quad (37)$$

$$\boldsymbol{K}_1 = \operatorname{diag}\left(k_{1i}\right), \boldsymbol{K}_2 = \operatorname{diag}\left(k_{2i}\right),$$
$$k_{1i} > 0, k_{2i} > 0, i = 1, 2, 3 \quad (38)$$

provides asymptotic stability to the equilibrium position $\boldsymbol{q}_B = \boldsymbol{e}_B$.

Proof. Let us use the equation of motion of the vector $\boldsymbol{q}_B$ in the form (19). In this case, for the control error

$$\boldsymbol{e} = \boldsymbol{q}_B - \boldsymbol{e}_B , \quad (39)$$

the equation

$$\ddot{\boldsymbol{e}} = \boldsymbol{\mu}_B \quad (40)$$

is valid.

Let's define $\boldsymbol{\mu}_B$ as follows

$$\boldsymbol{\mu}_B = -\boldsymbol{K}_1\boldsymbol{e} - \boldsymbol{K}_2\dot{\boldsymbol{q}}_B , \quad (41)$$

$$\boldsymbol{K}_1 = \operatorname{diag}\left(k_{1i}\right), \boldsymbol{K}_2 = \operatorname{diag}\left(k_{2i}\right), \quad (42)$$

$$k_{1i} > 0, k_{2i} > 0, i = 1, 2, 3, \quad (43)$$

where $\dot{\boldsymbol{q}}_B$ is defined by expressions (20) and (21). A system of equations (40) is a system of three inde-

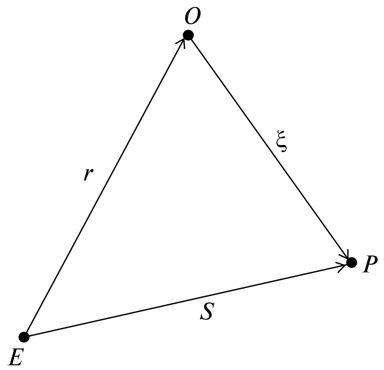

Figure 1. Geometric interpretation of pointing the on-board transmitter antenna to the GRS

pendent second-order integrating links. The inputs of these links are signals μ_{B_i}, $i = 1, 2, 3$. For the i-th link, it can be written the equation

$$\ddot{\boldsymbol{e}} = -k_{1i}\boldsymbol{e}_i - k_{2i}\dot{\boldsymbol{e}}_i \,. \tag{44}$$

Since (44) is a second-order linear equation, for it to be asymptotically stable, according to the Hurwitz stability criterion, it is necessary and sufficient conditions (44) to be satisfied. Thus, Theorem 1 is proved.

Example 1. Consider the problem of pointing a stationary antenna of an onboard transceiver to a ground-based information receiving station (GRS) by turning the spacecraft body. This problem arises when there is no line-of-sight "antenna-GRS" due to the constructive elements at the regular orientation of the spacecraft. Let us introduce the following notations (Figure 1): $\boldsymbol{E}$ is the Earth's centre of mass, $\boldsymbol{r}$ is the radius-vector that specifies the spacecraft centre of mass position $\boldsymbol{O}$ in orbit; $\boldsymbol{\xi} = \boldsymbol{s} - \boldsymbol{r}$ is the radius-vector drawn from the centre of mass of the spacecraft to a given point $\boldsymbol{P}$ on the Earth's surface, where the GRS is located.

Let $\boldsymbol{e}_B$ be a unit vector that determines the onboard transceiver electrical axis position in the body-fixed frame $\boldsymbol{B}$. It is necessary to find control torque $\boldsymbol{M}_u$, which provides asymptotic stability to the equilibrium position

$$\frac{\boldsymbol{\xi}_B}{\left\|\boldsymbol{\xi}_B\right\|} = \boldsymbol{e}_B \,.$$

Assume the following.

1. The position of the vector $\boldsymbol{e}_B$ relative to the body-fixed frame is known, and the equality $\dot{\boldsymbol{e}}_B = 0$ is true for it.

2. The following information is available on board the spacecraft:

- projections of the vector $\boldsymbol{\xi}$ on the body-fixed frame axes in the form of the vector $\boldsymbol{\xi}_B$;
- position and velocity of the spacecraft centre of mass in the Greenwich coordinate system $\boldsymbol{G}$ as vectors $\boldsymbol{r}_G$ and $\dot{\boldsymbol{r}}_G$;
- the spacecraft absolute angular velocity vector $\boldsymbol{\omega}_B^{BI}$ and the orientation quaternion $\boldsymbol{\Lambda}_{IB}$.

The problem solution. According to Fig. 1, in the frame $\boldsymbol{G}$, the following equations are valid:

$$\boldsymbol{\xi}_G = \boldsymbol{s}_G - \boldsymbol{r}_G \,, \tag{45}$$

$$\dot{\boldsymbol{\xi}}_G = -\dot{\boldsymbol{r}}_G \,, \tag{46}$$

$$\ddot{\boldsymbol{\xi}}_G = -\ddot{\boldsymbol{r}}_G \,. \tag{47}$$

In this case, the spacecraft's centre of mass motion is described by the equation [2]

$$\ddot{\boldsymbol{r}}_G = -\frac{\mu}{\left\|\boldsymbol{r}_G\right\|^3}\boldsymbol{r}_G + \boldsymbol{\omega}_G^{GI} \times \boldsymbol{\omega}_G^{GI} \times \boldsymbol{r}_G + 2\boldsymbol{\omega}_G^{GI} \times \dot{\boldsymbol{r}}_G \,. \tag{48}$$

In equation (48), $\mu = 3.986005 \times 10^{14}$ is the gravitational constant of the Earth; $\boldsymbol{\omega}_G^{GI}$ is the angular velocity of the Earth's rotation, given by projections on the frame $\boldsymbol{G}$ axes.

Let us denote by $\boldsymbol{q}$ the unit vector of the vector $\boldsymbol{\xi}$. Obviously, in the Greenwich coordinate system, the following equation holds for this vector:

$$\boldsymbol{q}_G = \frac{\boldsymbol{\xi}_G}{\left\|\boldsymbol{\xi}_G\right\|} \,. \tag{49}$$

Following equations (20)—(25), the expressions for calculating the projections of the velocity and acceleration of this vector on the axes of basis $\boldsymbol{B}$ have the form

$$\dot{\boldsymbol{q}}_B = -\boldsymbol{\omega}_B^{BG} \times \boldsymbol{q}_B + \dot{\tilde{\boldsymbol{q}}}_B \,, \tag{50}$$

$$\dot{\tilde{\boldsymbol{q}}}_B = \tilde{\boldsymbol{\Lambda}}_{GB} \circ \dot{\boldsymbol{q}}_G \circ \boldsymbol{\Lambda}_{GB} \,. \tag{51}$$

$$\ddot{\boldsymbol{q}}_B = -\dot{\boldsymbol{\omega}}_B^{BG} \times \boldsymbol{q}_B + \boldsymbol{p} \,, \tag{52}$$

$$\boldsymbol{J}\dot{\boldsymbol{\omega}}_B^{BG} = -\boldsymbol{\omega}_B^{BI} \times \left(\boldsymbol{J}\boldsymbol{\omega}_B^{BI}\right) - \boldsymbol{J}\left(\boldsymbol{\omega}_B^{GI} \times \boldsymbol{\omega}_B^{BG}\right) + \boldsymbol{M}_u \,, \tag{53}$$

$$\boldsymbol{p} = -\boldsymbol{\omega}_B^{BG} \times \left(\dot{\boldsymbol{q}}_B + \dot{\tilde{\boldsymbol{q}}}_B\right) + \ddot{\tilde{\boldsymbol{q}}}_B \,, \tag{54}$$

$$\ddot{\tilde{\boldsymbol{q}}}_B = \tilde{\boldsymbol{\Lambda}}_{GB} \circ \ddot{\boldsymbol{q}}_G \circ \boldsymbol{\Lambda}_{GB} \,. \tag{55}$$

At the same time, derivatives $\dot{\boldsymbol{q}}_G$ and $\ddot{\boldsymbol{q}}_G$ are defined as follows

$$\dot{\boldsymbol{q}}_G = -\boldsymbol{q}_G \times \left(\boldsymbol{q}_G \times \frac{\dot{\boldsymbol{\xi}}_G}{\delta}\right), \tag{56}$$

$$\ddot{\boldsymbol{q}}_G = -\boldsymbol{q}_G \times \left(\boldsymbol{q}_G \times \left(\frac{\ddot{\boldsymbol{\xi}}_G}{\delta} - 2\frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{\xi}}_G \right) \right) - \left\| \dot{\boldsymbol{q}}_G \right\|^2 \boldsymbol{q}_G , \quad (57)$$

where

$$\delta = \left\| \boldsymbol{\xi}_G \right\|, \quad \dot{\delta} = \boldsymbol{q}_G^T \dot{\boldsymbol{\xi}}_G . \quad (58)$$

The deducing of formulas (56) and (57) are given in the Appendix.

To find the control $\boldsymbol{u}$ that ensures the asymptotic stability of the equilibrium position $\boldsymbol{q}_b = \boldsymbol{e}_b$, Theorem 1 is used. According to this theorem, the control law

$$\boldsymbol{\mu}_B = -\boldsymbol{K}_1 e - \boldsymbol{K}_2 \dot{\boldsymbol{q}}_B, \boldsymbol{K}_1 > 0, \boldsymbol{K}_2 > 0 \quad (59)$$

provides asymptotic stability to the equilibrium position $\boldsymbol{e} = 0$. In this case, the control $\boldsymbol{u}$ is determined by the expression (14) and the actual control torque $\boldsymbol{M}_u$ by the expression (33).

Simulation results. To analyze the qualitative features of the algorithm, the simulation of the proposed algorithm was carried out with the following initial data.

1. An inertial coordinate system was chosen as the coordinate system relative to which the spacecraft's angular motion was simulated.

2. The orbital elements at the start of the pointing process were as follows: $a = 7028$ km — the semi-major axis of the orbit, e = 0.001 — eccentricity of the orbit, i = 1.439897 rad — the orbital inclination, Ω = 1.523599 1/s — longitude of the ascending node of the orbit, ω = 0 rad — argument of periapsis, m = = 1.23 rad — mean anomaly at pointing start.

3. The motion of the centre of mass along the orbit in the Greenwich coordinate system was simulated by a system of differential equations (49).

4. The initial conditions for the rotational motion model were as follows:

$$\boldsymbol{\omega}_B^{BI}(t_0) = \begin{pmatrix} 0 \\ 0.001 \\ 0 \end{pmatrix},$$

$$\boldsymbol{\Lambda}_{IB}(t_0) = \begin{pmatrix} 0.670900566645541 \\ -0.077273729904976 \\ -0.1628015368791540 \\ -0.719316939833235 \end{pmatrix}.$$

5. The position of the GRS on the Earth's surface was given by a point with coordinates: longitude $\lambda = 60^{\circ}$, latitude $\varphi = 45^{\circ}$.

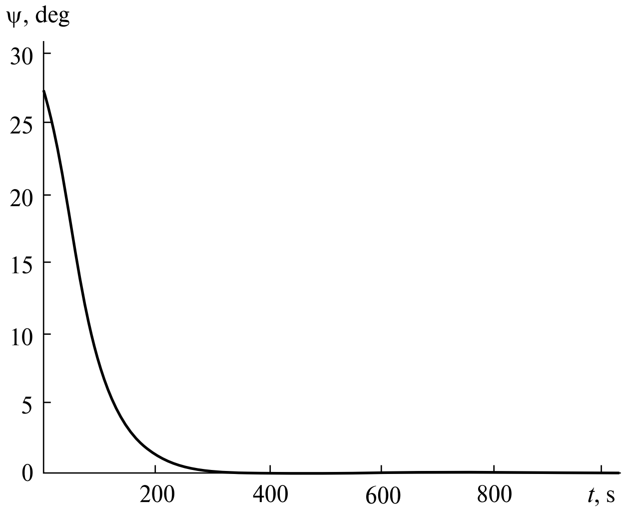

Figure 2. The Ψ estimation variation with time

6. The spacecraft's inertia tensor

$$\boldsymbol{J} = \begin{pmatrix} 195 & 0 & 0 \\ 0 & 121 & 0 \\ 0 & 0 & 189 \end{pmatrix}, \text{ kg m}^2.$$

7. The antenna electrical axis unit vector coordinates given by projections on of the body-fixed frame $\boldsymbol{B}$ axes, $\boldsymbol{e}_B = \left(0.25\, 0.43 - 0.87\right)^{\mathrm{T}}$.

8. The mutual position of the antenna electrical axis and the direction $\boldsymbol{\xi}$ was estimated by the formula

$$\Psi = \arccos\left(\frac{\boldsymbol{\xi}_B^T}{\left\| \boldsymbol{\xi}_B \right\|} \boldsymbol{e}_B \right). \quad (60)$$

Figure 2 shows a graph of the function $\Psi(t)$, and Figure 3 shows the graphs of time variations of coordinates of the vector $\dot{\boldsymbol{q}}_B$. The simulation results indicate the effectiveness of the proposed algorithm for uniaxial spacecraft stabilization.

3.2. *The problem of uniaxial terminal pointing.* Let us solve the problem of pointing in inertial space some fixed axis associated with the spacecraft to a given point. In the general case, this problem is formulated as follows: find the control law $\boldsymbol{u}$ that transfers the vector whose motion is described by (13) from the current state $\boldsymbol{q}_B(t_0)$, $\dot{\boldsymbol{q}}_B(t_0)$ at time t_0 to the required state $\boldsymbol{q}_B(t_1)$, $\dot{\boldsymbol{q}}_B(t_1)$ at time t_1. The times t_0 and t_1 are fixed. The solution to this problem is given by the following theorem.

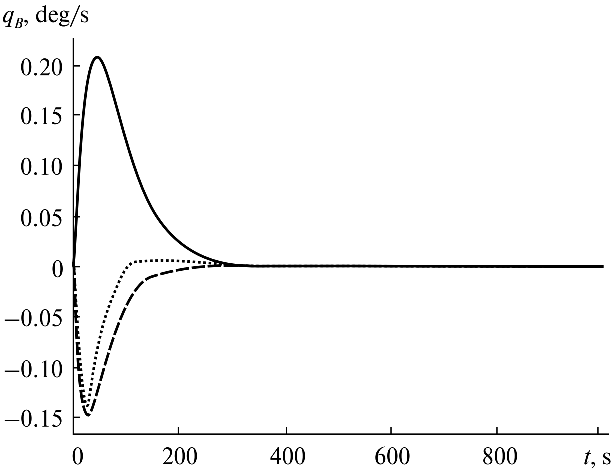

Figure 3. The spacecraft's angular velocities variation with time

Theorem 2. *Let the motion of vector* $\boldsymbol{q}_B$ *relative to the reference frame* $\boldsymbol{R}$ *be given by equation* (13). *Let us introduce an auxiliary vector* $\boldsymbol{x}$, *and the motion of this vector is described by the equation*

$$\ddot{\boldsymbol{x}}=\boldsymbol{\tau},\ \boldsymbol{x}\in\mathbb{R}^3 . \tag{61}$$

Suppose that for the fixed times t_0 *and* t_1, *the following boundary conditions are given for the vector* $\boldsymbol{x}$ *and its first derivative:*

$$\boldsymbol{x}(t_0)=\boldsymbol{q}_B(t_0),\ \dot{\boldsymbol{x}}(t_0)=\dot{\boldsymbol{q}}_B(t_0), \tag{62}$$

$$\boldsymbol{x}(t_1)=\boldsymbol{q}_B(t_1),\ \dot{\boldsymbol{x}}(t_1)=\dot{\boldsymbol{q}}_B(t_1), \tag{63}$$

and a control τ *is found that transfers the vector* $\boldsymbol{x}$ *and its first derivative to the position* (63).

Let us define the calculated trajectory $\hat{\boldsymbol{q}}_B(t)$ *for transferring the vector* $\boldsymbol{q}_B(t)$ *from the current position at time* t_0 *to a given position at time* t_1 *and a control that implements this motion as follows*

$$\hat{\boldsymbol{q}}_B(t)=\frac{\boldsymbol{x}}{\delta}\quad \delta=\|\boldsymbol{x}\|, \tag{64}$$

$$\dot{\hat{\boldsymbol{q}}}_B(t)=-\hat{\boldsymbol{q}}_B\times\left(\hat{\boldsymbol{q}}_B\times\frac{\dot{\boldsymbol{x}}}{\delta}\right), \tag{65}$$

$$\hat{\boldsymbol{u}}=\ddot{\hat{\boldsymbol{q}}}_B(t)=-\hat{\boldsymbol{q}}_B\times\left(\hat{\boldsymbol{q}}_B\times\left(\frac{\boldsymbol{\tau}}{\delta}-2\frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{x}}\right)\right)-\left\|\dot{\hat{\boldsymbol{q}}}_B\right\|^2\hat{\boldsymbol{q}}_B , \tag{66}$$

where the vectors $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$, *and* $\boldsymbol{\tau}$ *are defined by the expressions*

$$\boldsymbol{\tau}=\boldsymbol{C}_1(t-t_0)-\boldsymbol{C}_2, \tag{67}$$

$$\boldsymbol{C}_1=\frac{6}{(t_1-t_0)^2}\boldsymbol{y}_2-\frac{12}{(t_1-t_0)^3}\boldsymbol{y}_1, \tag{68}$$

$$\boldsymbol{C}_2=\boldsymbol{C}_1\frac{t_1-t_0}{2}-\frac{1}{t_1-t_0}\boldsymbol{y}_2 , \tag{69}$$

$$\boldsymbol{y}_1=\boldsymbol{q}_B(t_1)-\boldsymbol{q}_B(t_0)-\dot{\boldsymbol{q}}_B(t_0)(t_1-t_0), \tag{70}$$

$$\boldsymbol{y}_2=\dot{\boldsymbol{q}}_B(t_1)-\dot{\boldsymbol{q}}_B(t_0), \tag{71}$$

$$\dot{\boldsymbol{x}}(t)=\dot{\boldsymbol{q}}_B(t_0)+\boldsymbol{C}_1\frac{(t-t_0)^2}{2}-\boldsymbol{C}_2(t-t_0), \tag{72}$$

$$\boldsymbol{x}(t)=\boldsymbol{q}_B(t_0)+\dot{\boldsymbol{q}}_B(t_0)(t-t_0)+ +\boldsymbol{C}_1\frac{(t-t_0)^3}{3}-\boldsymbol{C}_2\frac{(t-t_0)^2}{2}. \tag{73}$$

Then the control

$$\boldsymbol{u}=-\boldsymbol{q}_B\times(\boldsymbol{q}_B\times\boldsymbol{\mu}_B), \tag{74}$$

$$\boldsymbol{\mu}_B=-\boldsymbol{K}_1\boldsymbol{e}-\boldsymbol{K}_2\dot{\boldsymbol{e}}+\hat{\boldsymbol{u}}, \tag{75}$$

$$\boldsymbol{e}=\boldsymbol{q}_B-\hat{\boldsymbol{q}}_B,\ \dot{\boldsymbol{e}}=\dot{\boldsymbol{q}}_B-\dot{\hat{\boldsymbol{q}}}_B \tag{76}$$

ensures the transfer of the vectors $\boldsymbol{q}_B$ *and* $\dot{\boldsymbol{q}}_B$ *from position* $\boldsymbol{q}_B(t_0)$, $\dot{\boldsymbol{q}}_B(t_0)$ *to the given position* $\boldsymbol{q}_B(t_1)$, $\dot{\boldsymbol{q}}_B(t_1)$ *in a fixed time* t_1-t_0.

Proof. To prove this theorem, equation (61) is used. Let us find the control law $\boldsymbol{\tau}(t)$ that transfers the system $\ddot{\boldsymbol{x}}=\boldsymbol{\tau}$ from the current state $\boldsymbol{x}(t_0)=\boldsymbol{q}_B(t_0)$, $\dot{\boldsymbol{x}}(t_0)=\dot{\boldsymbol{q}}_B(t_0)$ at time t_0 to the required state $\boldsymbol{x}(t_1)=\boldsymbol{q}_B(t_1)$, $\dot{\boldsymbol{x}}(t_1)=\dot{\boldsymbol{q}}_B(t_1)$ at time t_1 and providing a minimum for the functional

$$V=\frac{1}{2}\int_{t_0}^{t_1}\|\boldsymbol{\tau}\|^2 dt . \tag{77}$$

For this problem, there is an analytical solution defined by expressions (69)—(74) [7]. Let us define the calculated trajectory $\hat{\boldsymbol{q}}_B(t)$ as

$$\hat{\boldsymbol{q}}_B(t)=\frac{\boldsymbol{x}(t)}{\|\boldsymbol{x}(t)\|}. \tag{78}$$

Then, according to Appendix, for the derivative $\dot{\hat{\boldsymbol{q}}}_B(t)$ and control $\hat{\boldsymbol{u}}$, the following expressions are valid:

$$\dot{\hat{\boldsymbol{q}}}_B(t)=-\hat{\boldsymbol{q}}_B\times\left(\hat{\boldsymbol{q}}_B\times\frac{\dot{\boldsymbol{x}}}{\delta}\right), \tag{79}$$

$$\hat{\boldsymbol{u}}=\ddot{\hat{\boldsymbol{q}}}_B=-\hat{\boldsymbol{q}}_B\times\hat{\boldsymbol{q}}_B\times\left(\frac{\boldsymbol{\tau}}{\delta}-2\frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{x}}\right)-\left\|\dot{\hat{\boldsymbol{q}}}_B\right\|^2\hat{\boldsymbol{q}}_B . \tag{80}$$

Since

$$\boldsymbol{x}(t_0)=\boldsymbol{q}_B(t_0),\ \dot{\boldsymbol{x}}(t_0)=\dot{\boldsymbol{q}}_B(t_0),$$
$$\boldsymbol{x}(t_1)=\boldsymbol{q}_B(t_1),\ \dot{\boldsymbol{x}}(t_1)=\dot{\boldsymbol{q}}_B(t_1),$$

it is evident that the control $\hat{\boldsymbol{u}}$ will ensure the transfer of the vector $\hat{\boldsymbol{q}}_B$ from position $\boldsymbol{q}_B(t_0)$, $\dot{\boldsymbol{q}}_B(t_0)$ to a given position $\boldsymbol{q}_B(t_1)$, $\dot{\boldsymbol{q}}_B(t_1)$ in a fixed time $t_1 - t_0$.

The control built in this way is programmatic. With such control, the vector $\boldsymbol{q}_B(t)$ will move along a certain trajectory different from the calculated one. That is due to errors in the program control implementation and disturbance moments presence acting on the spacecraft. To stabilize the calculated trajectory, it is necessary to add a stabilizing control through feedback. To find this control, consider the equation for the relative motion of the vectors $\boldsymbol{q}_B(t)$ and $\hat{\boldsymbol{q}}_B(t)$. As said above, the motion of vector $\boldsymbol{q}_B(t)$ can be represented in the form (19). Subtracting equation (80) from equation (19) gives

$$\ddot{\boldsymbol{e}} = \boldsymbol{u}_s = \boldsymbol{\mu}_B - \hat{\boldsymbol{u}}. \qquad (81)$$

According to Theorem 1, the control

$$\boldsymbol{u}_s = -\boldsymbol{K}_1 \boldsymbol{e} - \boldsymbol{K}_2 \dot{\boldsymbol{e}} \qquad (82)$$

provides asymptotic stability for the position $\boldsymbol{e} = 0$. It follows from (81) that

$$\boldsymbol{\mu}_B = -\boldsymbol{K}_1 \boldsymbol{e} - \boldsymbol{K}_2 \dot{\boldsymbol{e}} + \hat{\boldsymbol{u}}, \qquad (83)$$

which completes the proof of Theorem 2.

Example 2. To analyze the qualitative features of the proposed terminal reorientation algorithm, the simulation of the reorientation process was carried out with the initial data from Example 1. In this case, the start time of the manoeuvre was taken $t_0 = 0$ s, and the end time $t_1 = 1000$ s. The boundary conditions were as follows

$$\boldsymbol{q}_B(t_0) = \begin{pmatrix} -0.162161484770419 \\ 0.635129976541619 \\ -0.755191078969618 \end{pmatrix},$$

$$\dot{\boldsymbol{q}}_B(t_0) = \begin{pmatrix} 0.296186315679564 \\ -0.828484883752114 \\ -0.760371266945204 \end{pmatrix} \times 10^{-4},$$

$$\boldsymbol{q}_B(t_1) = \begin{pmatrix} 0.250000000000000 \\ 0.433012701892219 \\ -0.866025403784439 \end{pmatrix},$$

$$\dot{\boldsymbol{q}}_B(t_1) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

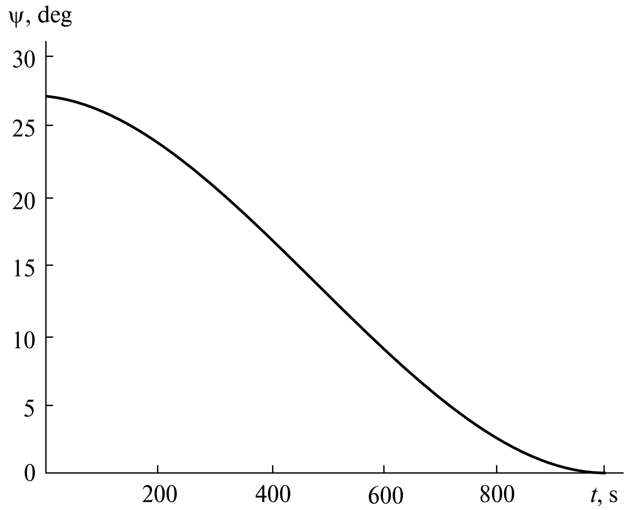

Figure 4. The Ψ estimation variation with time

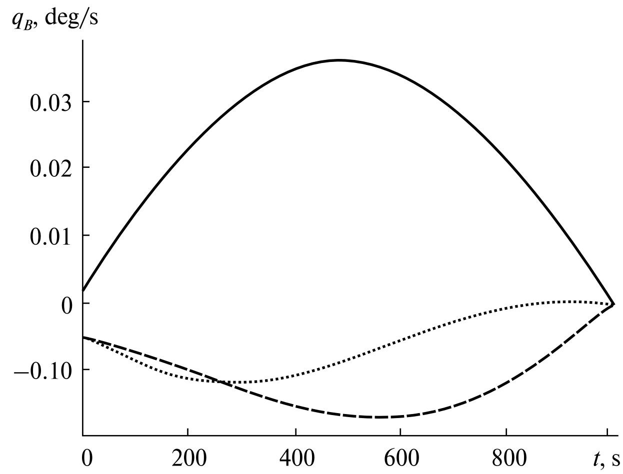

Figure 5. The spacecraft's angular velocities variation with time

Figure 4 shows a graph of the change in the function $\Psi(t)$, and Figure 5 shows the graphs of time variations of coordinates of the vector $\dot{\boldsymbol{q}}_B$. The simulation results indicate the efficiency of the proposed algorithm for the uniaxial orientation of the spacecraft.

4. CONCLUSION

For an arbitrary normalized vector, a dynamic model of motion in the associated coordinate system is obtained. This model significantly simplifies the problem of synthesizing control of the spacecraft's uniaxial orientation. In this case, the synthesis problem is reduced to constructing a control for a system with three second-order integrating links, and the synthesis problem has an analytical solution for such systems. The resulting control algorithms are much simpler to implement than the ones obtained using the traditional model. A new approach has been suggested for this model, which is a transformation of the right-hand side of the Euler dynamics equation into a new control vector $\boldsymbol{u} \in \mathbb{R}^3$. This allows for the concise representation of the right-hand side of the dynamics equation for the vector as a function of the spacecraft's angular motion parameters. The transformation found is reversible, allowing us to return to the original form of the right-hand side of the Euler dynamics equation and find the control torque $\boldsymbol{M}_u \in \mathbb{R}^3$, physically realized by the control system actuators. Based on the obtained model, two algorithms for constructing a spacecraft uniaxial orientation are proposed: an algorithm for spacecraft uniaxial stabilization and an algorithm for spacecraft uniaxial terminal reorientation. The application of the proposed model is demonstrated using two examples: solving the stabilization problem and the problem of the spacecraft's uniaxial terminal reorientation. When solving the stabilization problem, in contrast to the well-known works [4, 10], in which the direct Lyapunov method was used to construct the control, for the first time, it was possible to reduce the problem of finding the control $\boldsymbol{M}_u$ to the trivial problem of finding the control $\boldsymbol{\mu}_B$, which ensures asymptotic stability of the error equation $\ddot{\boldsymbol{e}} = \boldsymbol{\mu}_B$. This is a linear equation with constant coefficients, which makes it possible to apply well-developed methods of the theory of linear systems with constant coefficients. The numerical simulation results confirming the efficiency of the proposed algorithms have been presented.

APPENDIX. THE DEDUCING OF FORMULAE FOR THE FIRST AND SECOND DERIVATIVES OF THE UNIT VECTOR

The formula deducing for the first derivative of a unit vector of $\boldsymbol{\xi}_B$. Let $\boldsymbol{q}_B$ be the unit vector of vector $\boldsymbol{\xi}_B$. Denote by δ the modulus of the vector $\boldsymbol{\xi}_B$ and identity matrix of order 3 × 3 as $\boldsymbol{I}_3$. Then the time derivative of the unit vector is defined by the following expressions

$$\dot{\boldsymbol{q}}_B = \frac{d}{dt}\left(\frac{\boldsymbol{\xi}_B}{\|\boldsymbol{\xi}_B\|}\right) = \frac{\dot{\boldsymbol{\xi}}_B}{\delta} - \frac{\dot{\delta}}{\delta}\frac{\boldsymbol{\xi}_B}{\delta} = \frac{\dot{\boldsymbol{\xi}}_B}{\delta} - \boldsymbol{q}_B\boldsymbol{q}_B^T\frac{\dot{\boldsymbol{\xi}}_B}{\delta^2} =$$

$$= \left(\boldsymbol{I}_3 - \boldsymbol{q}_B\boldsymbol{q}_B^T\right)\frac{\dot{\boldsymbol{\xi}}_B}{\delta} = -\boldsymbol{q}_B \times \left(\boldsymbol{q}_B \times \frac{\dot{\boldsymbol{\xi}}_B}{\delta}\right). \quad \text{(A1)}$$

The formula deducing for the second-time derivative of a unit vector of $\boldsymbol{\xi}_B$. Differentiating (A1) to time gives

$$\ddot{\boldsymbol{q}}_B = \left(I_3 - \boldsymbol{q}_B\boldsymbol{q}_B^T\right)\left(\frac{\ddot{\boldsymbol{\xi}}_B}{\delta} - \frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{\xi}}_B\right) - \dot{\boldsymbol{q}}_B\boldsymbol{q}_B^T\frac{\dot{\boldsymbol{\xi}}_B}{\delta} - \boldsymbol{q}_B\dot{\boldsymbol{q}}_B^T\frac{\dot{\boldsymbol{\xi}}_B}{\delta} =$$

$$\left(I_3 - \boldsymbol{q}_B\boldsymbol{q}_B^T\right)\left(\frac{\ddot{\boldsymbol{\xi}}_B}{\delta} - \frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{\xi}}_B\right) - \left(I_3 - \boldsymbol{q}_B\boldsymbol{q}_B^T\right)\frac{\dot{\boldsymbol{\xi}}_B}{\delta}\boldsymbol{q}_B^T\frac{\dot{\boldsymbol{\xi}}_B}{\delta} -$$

$$-\boldsymbol{q}_B\frac{\dot{\boldsymbol{\xi}}_B^T}{\delta}\left(I_3 - \boldsymbol{q}_B\boldsymbol{q}_B^T\right)\frac{\dot{\boldsymbol{\xi}}_B}{\delta} =$$

$$\left(I_3 - q_Bq_B^T\right)\left(\frac{\ddot{\boldsymbol{\xi}}_B}{\delta} - \frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{\xi}}_B - \frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{\xi}}_B\right) - \left\|\dot{\boldsymbol{\xi}}_B\right\|^2\boldsymbol{\xi}_B =$$

$$= \left(I_3 - q_Bq_B^T\right)\left(\frac{\ddot{\boldsymbol{\xi}}_B}{\delta} - 2\frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{\xi}}_B\right) - \left\|\dot{\boldsymbol{\xi}}_B\right\|^2\boldsymbol{\xi}_B =$$

$$-\boldsymbol{q}_B \times \boldsymbol{q}_B \times \left(\frac{\ddot{\boldsymbol{\xi}}_B}{\delta} - 2\frac{\dot{\delta}}{\delta^2}\dot{\boldsymbol{\xi}}_B\right) - \left\|\dot{\boldsymbol{\xi}}_B\right\|^2\boldsymbol{\xi}_B. \quad \text{(A2)}$$

REFERENCES

1. Avanzini G., Zavoli A., De Matteis G., Giulietti F. (2022). Single axis pointing for underactuated spacecraft with a residual angular momentum. *Aerospace Sci. and Technology*, **124**, 107512. doi:10.1016/j.ast.2022.107512.
2. Curtis H. D. (2020). *Orbital Mechanics for Students* (4-th ed.). Butterworth-Heinemann, 792 p. doi:10.1016/B978-0-08-102133-0.00013-1
3. Khalili N., Ghorbanpour A. (2020). *Optimal tuning of single-axis satellite attitude control parameters using genetic algorithm*. Proc. ASME 2020 Dynamic Systems and Control Conf., Vol. 2. doi:10.1115/DSCC2020-3212
4. Lebedev D. V., Tkachenko A. I. (1991). *Inertial Control Systems. Algorithmic Aspects*. Naukova Dumka, 208 p. [in Russian].
5. Likhachev V. N., Sazonov V. V., Ul'yashin A. I. (2003). Single-axis solar orientation of a satellite of the Earth. *Cosmic Res.*, **41**(2), 159—170. doi:10.1023/A:1023387131144
6. Wittenburg J. (1977). *Dynamics of Systems of Rigid Bodies*. Vieweg+Teubner Verlag, 224 p. doi:10.1007/978-3-322-90942-8.
7. Yefymenko N. (2015). Synthesis of control algorithms of the spacecraft spatial reorientation with the use of dynamic equations of a solid body rotational motion in Rodrigo-Hamilton parameters. *J. Automation and Inform. Sci.*, **47**(6), 1—16. doi:10.1615/JAutomatInfScien.v47.i6.10
8. Yefymenko N., Kudermetov R. (2022). Quaternion models of a rigid body rotation motion and their application for spacecraft attitude control. *Acta Astronautica*, **194**, 76—82. doi:10.1016/j.actaastro.2022.01.029
9. Zavoli A., De Matteis G., Giulietti F., Avanzini G. (2017). Single-axis pointing of an underactuated spacecraft equipped with two reaction wheels. *J. Guidance, Control, and Dynamics*, **40**(6), 1465—1471. doi:10.2514/1.G002182
10. Zubov V. I. (1975). *Lectures on the Control Theory*. Nauka, 496 p. [in Russian].

Стаття надійшла до редакції 13.03.2024 — *Received 13.03.2024*
Після доопрацювання 13.03.2024 — *Revised 13.03.2024*
Прийнято до друку 02.05.2024 — *Accepted 02.05.2024*

М. Єфименко, д-р техн. наук, доцент. Лауреат Премії Кабінету міністрів України
за розроблення і впровадження інноваційних технологій (2014)
ORCID 0000-0001-7760-8610
E-mail: nefimenko@gmail.com
Р. Кудерметов, зав. каф., канд. техн. наук, доцент
ORCID 0000-0002-6766-9780
E-mail: kudermetov@gmail.com

Національний університет «Запорізька політехніка»
вул. Жуковського 64, Запоріжжя, Україна, 69093

ДИНАМІЧНА МОДЕЛЬ РУХУ ВЕКТОРА ТА ЇЇ ЗАСТОСУВАННЯ В ЗАДАЧАХ КЕРУВАННЯ ОДНОВІСНОЮ ОРІЄНТАЦІЄЮ КОСМІЧНОГО АПАРАТА

Об'єкт дослідження: система управління космічного апарата. Предмет дослідження: кватерніонне динамічне рівняння руху довільного нормованого вектора і методи побудови алгоритмів керування одновісною орієнтацією космічного апарата на його основі. У роботі отримано нову динамічну модель руху вектора у зв'язаній системі координат, досліджено її властивості та розглянуто методи вирішення задач одновісної орієнтації із застосуванням цієї моделі. При цьому задача синтезу зводиться до побудови керування для системи, що є сукупністю інтегрувальних ланок другого порядку. У багатьох випадках для таких систем задача синтезу має аналітичний розв'язок. Отримані при цьому алгоритми керування одновісною орієнтацією реалізуються значно простіше, ніж алгоритми, отримані при використанні традиційної моделі. Наведені результати чисельного моделювання, що підтверджують працездатність запропонованого алгоритму.

***Ключові слова**:* космічний апарат, одновісна орієнтація, термінальна переорієнтація, кватерніон, стабілізація, кутова швидкість.