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V. Z. GRISTCHAK [1], Professor, Dr. of Techn. Sci., Professor, Honored Worker of Science and Technology of Ukraine, Acad. Int. Acad. Sci. Higher School, Member of the Bureau of the Presidium of the Ukrainian Society of Mechanical Engineers
ORCID: https://orcid.org/0000-0001-8685-3191
E-mail: HRYSHCHAK.V.Z@mnu.one
D. V. HRYSHCHAK [2], Deputy Minister for Digital Development, Digital Transformations and Digitalization, Candidate of Techn. Sci., Honorary practitioner of the space industry of Ukraine
ORCID: https://orcid.org/0000-0001-2345-6789
E-mail: GDVspace@gmail.com
N. M. DYACHENKO [3], Associate Professor of the Department of Fundamental and Applied Mathematics, Candidate of Phys. and Mathem. Sci., Associate Professor
ORCID: https://orcid.org/0000-0001-5284-4502
E-mail: dyachenkonata69@gmail.com
A. F. SANIN [4], Head at Department of Rocket Space and Innovative Technologies, Dr. of Techn. Sci., Professor, Laureate of the State Award of Ukraine in the Field of Education
ORCID: https://orcid.org/0000-0002-5614-3882
E-mail: afedsa60@gmail.com
K. M. SUKHYY [5], Rector of Ukrainian State University of Chemical Technology, Dr. of Techn. Sci., Professor, Academician of the Academy of Sciences of the Higher School of Ukraine
ORCID: http://orcid.org/0000-0002-4585-8268
E-mail: sukhyy@gmail.com

[1] Dnipro University of Technology
19, Dmytra Yavornytskoho Ave., Dnipro, 49005 Ukraine
[2] The Ministry of Strategic Industries
21-23, Ivan Franko Str., Kyiv, 01054 Ukraine
[3] Zaporizhzhia National University
66, Zhukovskoho Str., Zaporizhzhia, 69600 Ukraine
[4] Oles Honchar Dnipro National University
72, Gagarina Ave., Dnipro, 49010 Ukraine
[5] Ukrainian State University of Chemical Technology
8, Gagarin Ave., Dnipro, 49005 Ukraine

BIFURCATION STATE AND RATIONAL DESIGN OF THREE-LAYER REINFORCED COMPOUND CONE-CYLINDER SHELL STRUCTURE UNDER COMBINED LOADING

An analytical-numerical approach to solving the problem of state bifurcation in terms of local and overall stability of a three-layer cone-cylinder shell structure discretely supported by intermediate rings, in particular of modern launch vehicles, under static combined

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loading by external pressure, axial forces, and torque is proposed in the paper taking into account the stiffness parameters of the intermediate rings in the plane of the initial curvature and for torsion.

Corresponding solving equations for the problem are ordinary differential equations of the sixth order (for a cylindrical compartment with constant coefficients and for a conical one with variable coefficients along the axial coordinate). Differential relations that determine the conditions of conjugation through the intermediate ring are used.

For the numerical solution, the finite difference method is used with central finite differences of the third and second order at the inner points of the shell determination segments and at its ends, respectively, and the second order differences with one step backward or forward at the conjugation points through the ring.

The agreement of the calculation results with the known data for three-layer conical and cylindrical shells is shown, as well as in the limiting case, it is done when passing to a single-layer compound cone-cylinder structure.

For the considered class of cone-cylinder shell structures, boundary surfaces are constructed that separate the stability region of the structure being under study, depending on the geometric and stiffness parameters of the compartments, reinforcing elements, and the external load condition.

The external load effect on the parameter of the post-critical wave formation for the structure under investigation is studied, providing the visualization of the deformation behavior.

The analysis of the calculation results has shown that this approach to solving the problem of bifurcation and equistability of the compound structure compartments in relation to the local and overall forms of protrusion allows choosing rational geometric and stiffness parameters of the shell components and force elements in terms of improving the weight characteristics of the structure.

Keywords: *three-layer shell, compound cone-cylinder structure, combined loading, ring, boundary surface, local and overall forms of protrusion, visualization of post-critical forms of buckling.*

INTRODUCTION

Shell structures are the force elements of building structures, the chemical industry, aircraft for various purposes, aviation and rocket-space systems, and other industries. While designing the modern thin-walled shell structures, considerable attention is paid to the problems of stability and strength under the action of external normal pressure, axial force, and torque. To improve operational characteristics with minimal material consumption, composite materials [20, 21] or three-layer ones, consisting of two outer rigid layers and an inner filler layer of low stiffness and density [1, 2, 6, 7, 23], are used. In particular, papers [1—14, 16—19, 22—24] are devoted to the study of the stability of homogeneous and multi-layer shell structures. An overview of publications in this field is contained in the paper [6].

Recently, considerable attention has been paid to theoretical and experimental studies of compound cone-cylinder structures [3, 4, 11—14, 16, 18, 19, 22, 24]. The study of buckling is accompanied by the construction of boundary surfaces separating the region of structure stability [2—4, 9, 10, 14, 18, 19] and visualization of the behavior of wave formation of shells in a critical state [4, 9, 11, 14, 16, 18].

An essential means of increasing the shell structure stability is its reinforcement with frontal and intermediate rings [3—5, 8—13, 16—19, 22]. By means of a numerical experiment in papers [3, 4, 10], the influence of both stiffness characteristics of rings (in the plane of the initial curvature and from its plane) on the stability of shell structures is shown, demonstrating the specifics of the influence of each of the parameters.

The peculiarity of this paper is the development of an analytical-numerical approach to solving the problems of static stability of reinforced compound three-layer shell structures of the cone-cylinder type under combined loading with a continuation of the research described in the paper [2], in which the solving differential equation and a solution to the problem of stability of a three-layer conical shell under the action of uniform external pressure, axial compressive forces and torsion are obtained. The purpose of the paper is to study the interaction of local and overall buckling of a three-layer compound cone-cylinder structure under the joint action of three load parameters capable of causing buckling, analysis of wave formation, especially under the influence of reinforcing rings, with the purpose of rational design and reducing the weight characteristics of the structure under study.

PROBLEM FORMULATION AND SOLVING EQUATIONS

In the monograph [1], a complete system of basic differential equations in partial derivatives is ob-

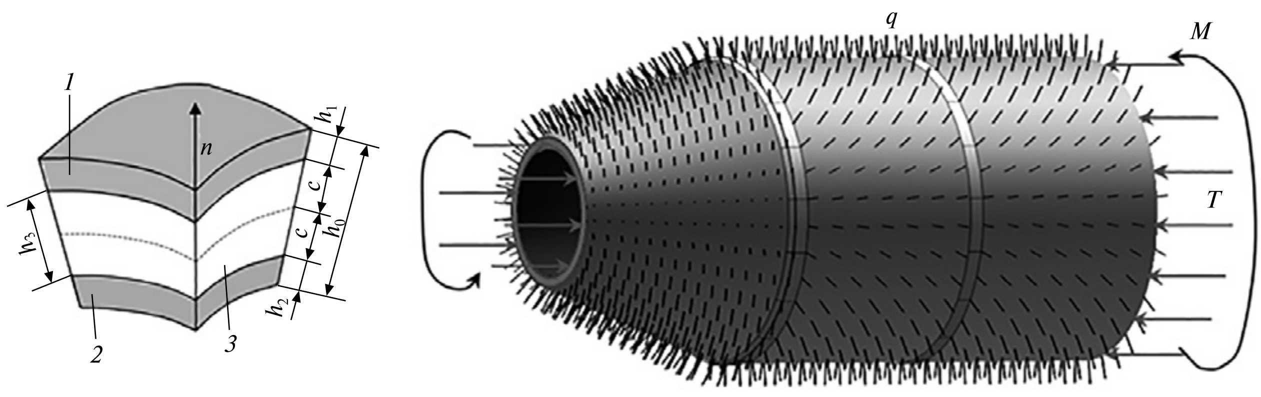


Figure 1. Three-layer compound shell structure under combined loading

tained to study the stability of three-layer shells using the Kirchhoff-Love hypotheses for the bearing layers, the incompressibility of the filler material, as well as the reduced Poisson ratio and elasticity modulus.

Referring to the three-layer walls of the shells (Figure 1), one should follow the notation in the monograph [1]: h_j, ν_j, and E_j are the thicknesses, Poisson ratios, and moduli of elasticity of the outer layer ($j=1$), inner layer ($j=2$), and filler ($j=3$), h_0 is the shell wall thickness. The walls of a three-layer shell are formed from isotropic bearing layers and transversely isotropic filler. The following parameters are introduced:

$$\nu=\sum_{j=1}^{3}\frac{E_j h_j \nu_j}{1-\nu_j^2}\left(\sum_{j=1}^{3}\frac{E_j h_j}{1-\nu_j^2}\right)^{-1},$$

$$E=\frac{1-\nu^2}{h_0}\left(\sum_{j=1}^{3}\frac{E_j h_j}{1-\nu_j^2}\right)$$

are the reduced Poisson ratios and modulus of elasticity,

$$K_{cone}=\frac{h_0^2}{\beta l_1^2},\ K_{cyl}=\frac{h_0^2}{\beta L^2}\ —$$

are shear parameters of the conical and cylindrical compartment of the structure, respectively.

The formulas for calculating the parameters as Θ, ϑ, β, D are given in the monograph [1] and specified in the paper [2].

A linear axisymmetric stability problem of a thin elastic three-layer shell structure of the cone-cylinder type is considered, being under the simultaneous action of three load parameters capable of causing buckling: external pressure (q), axial compressive force (T), and torsion (M). In accordance with the applied theory of thin shells, the case is considered when the change in the stress-strain state in the annular direction significantly exceeds the change in the stress-strain state in the axial direction, provided that the wave number is $n^2 \gg 1$.

For a compound structure, we introduce the notation: $\bar{s}$ and s are the coordinates along the generatrix of cylindrical and conical shells, respectively; y is the coordinate along the arc of the cylinder, φ is the arc coordinate along the parallel of the cone. Parameters of the shell structure compartments are as follows: α is the angle at the base of the cone, l_0 and l_1 are the distances from the cone vertex to smaller and larger bases of the truncated cone, L and R are the generatrix height and the radius of the cylinder. The compartments are docked with the larger base of the cone (Figure 1).

Neglecting the summands that contained ε to the power of not less than the second, in the paper [2], a solvable differential equation of the sixth order with variable coefficients for the conical shell has been derived. Regarding the deflection function $\Psi_{cone}(x)$, this equation is applied in the following form:

$$a_0\Psi_{cone}+a_1\Psi'_{cone}+a_2\Psi''_{cone}+a_3\Psi'''_{cone}+ +a_4\Psi^{IV}_{cone}+a_5\Psi^{V}_{cone}+a_6\Psi^{VI}_{cone}=0, \qquad (1)$$

where

$$a_0=\frac{p^5\vartheta\Theta}{x^5}+\left(-\frac{20\vartheta}{x^2}+\frac{1}{K_{cone}}+3\vartheta\gamma^2_{cone}\right)\times$$

$$\times\frac{\Theta\varepsilon p^4}{x^3}-\frac{\chi_{cone}p^4}{x^2}-\frac{\lambda_{cone}\gamma_{cone}p^{7/2}}{x^4}+$$

$$+\left(-\gamma_{cone}^2+\frac{6}{x^2}\right)\frac{\eta_{cone}p^3}{x^2}+$$

$$+\left(-\frac{3\gamma_{cone}^2}{2}-\frac{\gamma_{cone}^2 l_0^{\,2}}{2l_1^{\,2}x^2}+\frac{1}{x^2}-\frac{1}{K_{cone}}-\frac{3l_0^{\,2}}{l_1^{\,2}x^4}\right)\chi_{cone}\varepsilon p^3+$$

$$+\left(-\frac{2}{x^2}-1-\gamma_{cone}^2\right)\frac{\gamma_{cone}\,\lambda_{cone}\,p^{5/2}}{x^2}+$$

$$+\left(-\frac{2}{x^2}-\frac{1}{K_{cone}}-\gamma_{cone}^2\right)\gamma_{cone}^2\eta_{cone}\varepsilon\,p^2+$$

$$+\left(-\frac{6\gamma_{cone}^2}{x}+\frac{12}{x^3}+x\,\gamma_{cone}^4\right)p+$$

$$+\left(x^3\gamma_{conr}^6-4x\gamma_{cone}^4+\frac{x^3\gamma_{cone}^4}{K_{cone}}-\frac{6x\gamma_{cone}^2}{K_{cone}}\right)\varepsilon;$$

$$a_1=\frac{9\varepsilon p^4\vartheta\,\Theta}{x^4}+\left(\frac{2\varepsilon\chi_{cone}l_0^{\,2}}{l_1^{\,2}}-4\eta_{cone}\right)\frac{p^3}{x^3}+$$

$$+\left(3\gamma_{cone}^2-\frac{2}{x^2}\right)\frac{\eta_{cone}\varepsilon p^2}{x}+\left(2\gamma_{cone}^2-\frac{12}{x^2}\right)p+$$

$$+\left(4\gamma_{cone}^2-9x^2\gamma_{cone}^4-\frac{6x^2\gamma_{cone}^2}{K_{cone}}\right)\varepsilon\,;$$

$$a_2=-\frac{3p^4\vartheta\,\Theta\varepsilon}{x^3}+\left(\frac{3}{2}-\frac{l_0^{\,2}}{2l_1^{\,2}x^2}\right)\chi_{cone}\varepsilon p^3+$$

$$+\eta_{cone}\varepsilon p^3+\frac{3\varepsilon\lambda_{cone}\gamma_{cone}p^{5/2}}{x^2}+$$

$$+\left(\frac{6}{x}-2x\gamma_{cone}^2\right)p+\left(6\gamma_{cone}^2+\frac{2}{x^2}+\frac{1}{K_{cone}}\right)\varepsilon\eta_{cone}p^2+$$

$$+\left(-7x^3\gamma_{cone}^4-\frac{2x^3\gamma_{cone}^2}{K_{cone}}+44x\gamma_{cone}^2+\frac{6x}{K_{cone}}\right)\varepsilon\,;$$

$$a_3=-\frac{\eta_{cone}\varepsilon p^2}{x}-2p+\left(40\gamma_{cone}^2x^2+\frac{6x^2}{K_{cone}}\right)\varepsilon\,;$$

$$a_4=-\eta_{cone}\varepsilon p^2+px+\left(-8x+7x^3\gamma_{cone}^2+\frac{x^3}{K_{cone}}\right)\varepsilon\,,$$

$$a_5=-7x^2\varepsilon\,,\quad a_6=-x^3\varepsilon$$

with the appropriate notation signs:

$$\chi_{cone}=\frac{q\,l_1}{Eh_0\,\varepsilon^3\,\mathrm{tg}^3\alpha}\,,$$

$$\eta_{cone}=\frac{T\cos\alpha}{2\pi Eh_0\,\varepsilon^2 l_1\sin^3\alpha}\,,$$

$$\lambda_{cone}=\frac{M}{2\pi\,l_1^2 E\,h_0\,\varepsilon^{5/2}\sin^2\alpha}$$

are dimensionless efforts;

$$x=\frac{s}{l_1}\quad\delta=\frac{n^2}{\cos^2\alpha}\,,$$

$$p=\varepsilon\delta\,,$$

$$\varepsilon=\sqrt{\frac{h_0\,\mathrm{ctg}\,\alpha}{l_1\sqrt{12\left(1-\nu^2\right)}}}\,,$$

$$\gamma_{cone}=\frac{n}{\cos\alpha}\gamma_1\,,$$

γ_1 is the tangent of the angle of the wave crest inclination to the generatrix.

To obtain the solving equation for the cylindrical compartment, an approach similar to the case of a conical shell, given in [2], is used. First, the system of stability equations for the cylindrical three-layer shell of medium length is considered [1]:

$$\begin{cases} D\left(1-\dfrac{\vartheta h_0^2}{\beta}\Delta\right)\Delta^2\psi_{cyl}+\dfrac{1}{R}\cdot\dfrac{\partial^2}{\partial s^2}F_{cyl}= \\ \quad=-\left(\dfrac{T}{2\pi R}\dfrac{\partial^2 w_{cyl}}{\partial s^2}+q\,R\dfrac{\partial^2 w_{cyl}}{\partial y^2}-\dfrac{M}{\pi R^2}\dfrac{\partial^2 w_{cyl}}{\partial s\,\partial y}\right), \\ \Delta^4 F_{cyl}=\dfrac{Eh_0}{R}\dfrac{\partial^2}{\partial s^2}\left(1-\dfrac{h_0^2}{\beta}\Delta\right)\psi_{cyl}, \end{cases} \qquad (2)$$

where

$$w_{cyl}=\left(1-\frac{h_0^2}{\beta}\Delta\right)\psi_{cyl}$$

is the dependence between the functions w_{cyl} and ψ_{cyl}, Δ is the Laplace operator defined by the formula:

$$\Delta=\frac{\partial^2}{\partial s^2}+\frac{\partial^2}{\partial y^2}\,.$$

Dimensionless coordinates,

$$\bar{x} = \frac{\bar{s}}{L}, \ \bar{\varphi} = \frac{y}{R},$$

dimensionless parameters of the external load are introduced:

$$\eta_{cyl} = \frac{T}{2\pi E h_0^2},$$

$$\chi_{cyl} = \frac{qR^2}{E h_0^2},$$

$$\lambda_{cyl} = \frac{M}{2\pi E R h_0^2}.$$

The required functions ψ_{cyl} and F_{cyl} are presented in the following form:

$$\psi_{cyl}(\bar{x},\bar{\varphi}) = \Psi_{cyl}(\bar{x}) \cdot \cos\left(\gamma_{cyl}(1-\bar{x}) + n\bar{\varphi}\right),$$

$$F_{cyl}(\bar{x},\bar{\varphi}) = \Phi_{cyl}(\bar{x})\, E h_0^2 \cdot \cos\left(\gamma_{cyl}(1-\bar{x}) + n\bar{\varphi}\right),$$

where

$$\gamma_{cyl} = \frac{nL}{R}\gamma_1.$$

After the transformations, the system equation (2) is integrated along the coordinate $\bar{\varphi}$ by the Galerkin method from 0 to 2π, and the parameters

$$k = \frac{L}{R}, \ \varepsilon_1 = \frac{h_0}{R}, \ \omega = 12\left(1-\nu^2\right)$$

are introduced. The function $\Phi_{cyl}(\bar{x})$ is excluded, and the summands containing the factor

$$\frac{\varepsilon_1}{k^4 n^2}$$

are neglected. As a result, the solvable differential equation of the main stress state with respect to the deflection function of the cylindrical shell is obtained:

$$b_0\Psi_{cyl} + b_2\Psi''_{cyl} + b_4\Psi^{IV}_{cyl} + b_6\Psi^{VI}_{cyl} = 0, \qquad (3)$$

where

$$b_0 = \left(k^2n^2 + 2\gamma_{cyl}^2\right)\Bigg\{-\chi_{cyl}\left(K_{cyl}n^2 + \frac{K_{cyl}\gamma_{cyl}^2+1}{k^2}\right) - \eta_{cyl}\left(\frac{K_{cyl}}{k^2} + \frac{K_{cyl}\gamma_{cyl}^2+1}{k^4n^2}\right) - 2\lambda_{cyl}\gamma_{cyl}\left(\frac{K_{cyl}n}{k} + \frac{K_{cyl}\gamma_{cyl}^2+1}{k^3n}\right) +$$

$$\frac{\gamma_{cyl}^4}{k^4n^4\varepsilon_1}\left(\left[k^2n^2+1\right]K_{cyl}+1\right) + \frac{\Theta\varepsilon_1}{k^6n^2\omega}\left(k^2n^2+\gamma_{cyl}^2\right)^2\left(K_{cyl}\vartheta\left[k^2n^2+\gamma_{cyl}^2\right]+1\right)\Bigg\},$$

$$b_2 = \chi_{cyl}\left(K_{cyl}\left[4\frac{\gamma_{cyl}^2}{k^2}+3n^2\right]+\frac{2}{k^2}\right) - \frac{2\gamma_{cyl}^2}{k^4n^4\varepsilon_1} - \frac{2\varepsilon_1\Theta}{\omega}\left(\frac{7\gamma_{cyl}^4}{k^6n^2}+\frac{7\gamma_{cyl}^2}{k^4}+\frac{2n^2}{k^2}\right) +$$

$$+\eta_{cyl}\left(K_{cyl}\left[\frac{14\gamma_{cyl}^2}{k^4n^2}+\frac{10\gamma_{cyl}^2}{k^2}+n^2\right]+\frac{4\gamma_{cyl}^2}{k^4n^2}+\frac{1}{k^2}\right) - \frac{K_{cyl}}{k^4n^4\varepsilon_1}\left(7\gamma_{cyl}^4+2\gamma_{cyl}^2k^2n^2\right) +$$

$$+\lambda_{cyl}\left(\frac{K_{cyl}}{k}\left[10n\,\gamma_{cyl}+\frac{16\gamma_{cyl}^3}{k^2n}\right]+\frac{4\gamma_{cyl}}{k^3n}\right) - \frac{K_{cyl}\vartheta\varepsilon_1\Theta}{\omega}\left(\frac{30\,n^2\gamma_{cyl}^2}{k}+\frac{32\gamma_{cyl}^6}{k^6n^2}+\frac{57\gamma_{cyl}^4}{k^4}+5n^4\right),$$

$$b_4 = -2\chi_{cyl}\frac{K_{cyl}}{k^2} - \frac{\eta_{cyl}}{k^2}\left(K_{cyl}\left[3+\frac{14\gamma_{cyl}^2}{k^2n^2}\right]+\frac{2}{k^2n^2}\right) - 12\lambda_{cyl}\frac{\gamma_{cyl}K_{cyl}}{k^3n} + K_{cyl}\left(\frac{1}{k^2n^2}+\frac{7\gamma_{cyl}^2}{k^4n^4\varepsilon_1}\right) +$$

$$+K_{cyl}\frac{\vartheta\varepsilon_1\Theta}{\omega}\left(\frac{60\gamma_{cyl}^4}{k^6n^2}+\frac{57\gamma_{cyl}^2}{k^4}+\frac{9n^2}{k^2}\right) + \frac{1}{k^4n^4\varepsilon_1} + \frac{\varepsilon_1\Theta}{\omega}\left(\frac{14\gamma_{cyl}^2}{k^6n^2}+\frac{5}{k^4}\right),$$

$$b_6 = \frac{2\eta_{cyl}K_{cyl}}{k^4n^2} - \frac{K_{cyl}\varepsilon_1\Theta\vartheta}{\omega k^4}\left(7+\frac{32\gamma_{cyl}^2}{k^2n^2}\right) - \frac{K_{cyl}}{\varepsilon_1k^4n^4} - \frac{2\varepsilon_1\Theta}{\omega k^6n^2}.$$

The boundary conditions provide that the ends of the three-layer shell structure of the cone-cylinder type are freely supported, and there is also a diaphragm of infinite stiffness, which prevents the relative shift of the bearing layers along the edge of the shell [1]. Taking into account the fact that the shells are docked along the larger base of the cone, the indicated boundary conditions are written in the form:

$$\Psi_{cone} = \frac{\partial^2 \Psi_{cone}}{\partial x^2} = \frac{\partial^4 \Psi_{cone}}{\partial x^4} = 0 \text{ if } x = \frac{l_0}{l_1}. \qquad (4)$$

$$\Psi_{cyl} = \frac{\partial^2 \Psi_{cyl}}{\partial \overline{x}^2} = \frac{\partial^4 \Psi_{cyl}}{\partial \overline{x}^4} = 0 \text{ if } \overline{x} = 1. \qquad (5)$$

The finite difference method (FDM) is used to solve the problem [15]. For the ordinary differential equation of the sixth order for conical and cylindrical shells, FDM involves using the central finite differences of the third order at internal points x_k or $\overline{x}_k$ $(k = \overline{1, N-1})$ of splitting into N equal parts of the segments $\left[l_0/l_1;1\right]$ or $[0;1]$, corresponding to these shells. Such differential relations are written in [2]:

$$\Psi'(\tilde{x}_k) \approx \frac{1}{60H}(\Psi_{k+3} - 9\Psi_{k+2} + 45\Psi_{k+1} - 45\Psi_{k-1} + 9\Psi_{k-2} - \Psi_{k-3}),$$

$$\Psi''(\tilde{x}_k) \approx \frac{1}{180H^2}\left(2\Psi_{k+3} - 27\Psi_{k+2} + 270\Psi_{k+1} - 490\Psi_k + 270\Psi_{k-1} - 27\Psi_{k-2} + 2\Psi_{k-3}\right),$$

$$\Psi'''(\tilde{x}_k) \approx \frac{1}{8H^3}(-\Psi_{k+3} + 8\Psi_{k+2} - 13\Psi_{k+1} + 13\Psi_{k-1} - 8\Psi_{k-2} + \Psi_{k-3}),$$

$$\Psi^{(4)}(\tilde{x}_k) \approx \frac{1}{6H^4}(-\Psi_{k+3} + 12\Psi_{k+2} - 39\Psi_{k+1} + 56\Psi_k - 39\Psi_{k-1} + 12\Psi_{k-2} - \Psi_{k+3}),$$

$$\Psi^{(5)}(\tilde{x}_k) \approx \frac{1}{2H^5}(\Psi_{k+3} - 4\Psi_{k+2} + 5\Psi_{k+1} - 5\Psi_{k-1} + 4\Psi_{k-2} - \Psi_{k-3}),$$

$$\Psi^{(6)}(\tilde{x}_k) \approx \frac{1}{H^6}(\Psi_{k+3} - 6\Psi_{k+2} + 15\Psi_{k+1} - 20\Psi_k + 15\Psi_{k-1} - 6\Psi_{k-2} + \Psi_{k+3}),$$

where

$$\Psi(\tilde{x}_k) = \left(\Psi_{cone}\right)_k \text{ or } \Psi(\tilde{x}_k) = \left(\Psi_{cyl}\right)_k;$$

$$\tilde{x}_k = a + k\tilde{H}, \ k = \overline{1, N-1}; \ \tilde{H} = H_{cone} = \left(1 - l_0/l_1\right)/N$$

or

$$\tilde{H} = H_{cyl} = 1/N; \ a = l_0/l_1 \text{ or } a = 0.$$

Finite differences of the second order allow presenting the boundary conditions (4) and (5) in the form:

$$\left(\Psi_{cone}\right)_0 = 0, \ \left(\Psi_{cone}\right)_{-2} = -\left(\Psi_{cone}\right)_2,$$

$$\left(\Psi_{cone}\right)_{-1} = -\left(\Psi_{cone}\right)_1,$$

$$\left(\Psi_{cyl}\right)_N = 0, \ \left(\Psi_{cyl}\right)_{N+2} = -\left(\Psi_{cyl}\right)_{N-2},$$

$$\left(\Psi_{cyl}\right)_{N+1} = -\left(\Psi_{cyl}\right)_{N-1}.$$

The papers [3, 4, 10] contain details on considering the influence of the discrete principles of the intermediate rings' position, including the docking one. The stiffness parameters of the rings are taken in the form:

$$G^*_{cone,1} = G_1 \frac{1}{\cos^3\alpha},$$

$$G^*_{cone,2} = G_2 \frac{1}{\cos\alpha}; \qquad (6)$$

$$G^*_{cyl,1} = G_1, \ G^*_{cyl,2} = G_2;$$

$$G_1 = \frac{n^4\left(n^2 - 1\right)^2 (EJ)_x^{ring}}{Eh_0R^3},$$

$$G_2 = \frac{n^2\left(n^2 - 1\right)^2 (EJ)_z^{ring}}{Eh_0R^3\left(n^2 + 1\right)},$$

where J_x^{ring}, J_z^{ring} are the moments of inertia while bending the ring in the plane of the initial curvature and out of the plane (for torsion), respectively, $(EJ)_x^{ring}$, $(EJ)_z^{ring}$ are the corresponding stiffness characteristics of the ring.

Proceeding with the research laid out in [3], two docking rings at the edges of conical and cylindrical compartments with half stiffness are considered. The conditions of conjugation through the ring, in this case, will be as follows:

$$\Psi^{(i)}_{cone}(1) = \Psi^{(i)}_{cyl}(0) \text{ for all } i = 0; 4; 5; \qquad (7)$$

$$\Psi''_{cone}(1) + \frac{1}{2}G^*_{cone,2}\Psi'_{cone}(1) = \Psi''_{cyl}(0) + \frac{1}{2}G^*_{cyl,2}\Psi'_{cyl}(0); \qquad (8)$$

$$\Psi'''_{cone}(1) - \frac{1}{2}G^*_{cone,1}\Psi_{cone}(1) = \Psi'''_{cyl}(0) + \frac{1}{2}G^*_{cyl,1}\Psi_{cyl}(0), \qquad (9)$$

which are written by means of the second-order finite differences with one step backward and forward, respectively:

$$\Psi'_{cone,N} = \frac{1}{60H_{cone}}\left(-2\Psi_{cone,N-3} + 15\Psi_{cone,N-2} - 60\Psi_{cone,N-1} + 20\Psi_{cone,N} + 30\Psi_{cone,N+1} - 3\Psi_{cone,N+2}\right);$$

$$\Psi'_{cyl,0} = \frac{1}{60H_{cyl}}\left(3\Psi_{cyl,-2} - 30\Psi_{cyl,-1} - 20\Psi_{cyl,0} + \right.$$
$$\left. + 60\Psi_{cyl,1} - 15\Psi_{cyl,2} + 2\Psi_{cyl,3}\right);$$

$$\Psi''_{cone,N} = \frac{1}{12H^2_{cone}}\left(-\Psi_{cone,N-2} + 16\Psi_{cone,N-1} - 30\Psi_{cone,N} + \right.$$
$$\left. +16\Psi_{cone,N+1} - \Psi_{cone,N+2}\right);$$

$$\Psi''_{cyl,0} = \frac{1}{12H^2_{cyl}}(-\Psi_{cyl,-2} + 16\Psi_{cyl,-1} -$$
$$-30\Psi_{cyl,0} + 16\Psi_{cyl,1} - \Psi_{cyl,2});$$

$$\Psi'''_{cone,N} = \frac{1}{4H^3_{cone}}\left(\Psi_{cone,N-3} - 7\Psi_{cone,N-2} + 14\Psi_{cone,N-1} - \right.$$
$$\left. -10\Psi_{cone,N} + \Psi_{cone,N+1} + \Psi_{cone,N+2}\right);$$

$$\Psi'''_{cyl,0} = \frac{1}{4H^3_{cyl}}(-\Psi_{cyl,-2} - \Psi_{cyl,-1} + 10\Psi_{cyl,0} -$$
$$-14\Psi_{cyl,1} + 7\Psi_{cyl,2} - \Psi_{cyl,3});$$

$$\Psi^{IV}_{cone,N} = \frac{1}{H^4_{cone}}(\Psi_{cone,N-2} - 4\Psi_{cone,N-1} +$$
$$+ 6\Psi_{cone,N} - 4\Psi_{cone,N+1} + \Psi_{cone,N+2});$$

$$\Psi^{IV}_{cyl,0} = \frac{1}{H^4_{cyl}}\left(\Psi_{cyl,-2} - 4\Psi_{cyl,-1} + 6\Psi_{cyl,0} - 4\Psi_{cyl,1} + \Psi_{cyl,2}\right);$$

$$\Psi^{V}_{cone,N} = \frac{1}{H^5_{cone}}\left(-\Psi_{cone,N-3} + 5\Psi_{cone,N-2} - 10\Psi_{cone,N-1} + \right.$$
$$\left. +10\Psi_{cone,N} - 5\Psi_{cone,N+1} + \Psi_{cone,N+2}\right);$$

$$\Psi^{V}_{cyl,0} = \frac{1}{H^5_{cyl}}(\Psi_{cyl,-2} - 5\Psi_{cyl,-1} + 10\Psi_{cyl,0} -$$
$$- 10\Psi_{cyl,1} + 5\Psi_{cyl,2} - \Psi_{cyl,3}). \qquad (10)$$

In the case of placing the intermediate ring on a conical or cylindrical shell, the following conditions are applied:

$$\Psi^{(i)}_{shell}(x_{shell,left}) = \Psi^{(i)}_{shell}(x_{shell,right}) \text{ for all } i = 0; 4; 5; \qquad (11)$$

$$\Psi''_{shell}(x_{shell,left}) + G^*_{shell,2}\Psi'_{shell}(x_{shell,left}) = \Psi''_{cyl}(x_{shell,right}), \qquad (12)$$

$$\Psi'''_{shell}(x_{shell,left}) - G^*_{shell,1}\Psi_{shell}(x_{shell,left}) = \Psi'''_{cyl}(x_{shell,right}) \qquad (13)$$

for the corresponding shell type with finite differences similar to the (10). In formulas (11)—(13), the index "shell" specifies the shell type to which these formulas are applied, and the indices "left" or "right" at the axial coordinate determine the use of finite differences with one step backward or forward, respectively.

ANALYSIS OF NUMERICAL RESULTS

Numerical analysis has been carried out for the shells with parameters: $l_1 = 1.82$ m and $h_0 = 5\cdot10^{-3}$ m, the Poisson ratios of all layers $(\nu_1 = \nu_2 = \nu_3 = \nu = 0.3)$, and moduli of elasticity of outer layers ($E_1 = E_2 = 1.04\times10^{11}$ Pa). The dimensionless parameters such as

$$T^* = \frac{T}{Eh_0^2}, \; M^* = \frac{M}{2\pi E h_0^3},$$

and an auxiliary parameter of the external critical pressure

$$q^* = q\cdot10^{-5}[\text{Pa}]$$

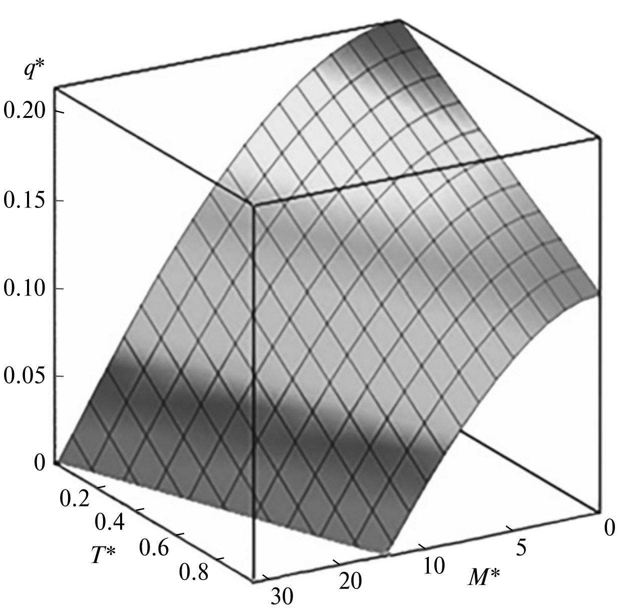
are introduced.

Comparative analysis with the known results. Table 1 shows the values of the critical pressure for cylindrical and conical shells, which allow comparing the results of the calculations with the known results of the monograph [1]. In addition, the listed values determine the critical pressure of local buckling of the compound cone-cylinder structure compartments.

To calculate the critical forces of overall buckling, a three-layer cone-cylinder shell structure is taken at first with the parameters: $E_3/E_1 = 0.01$, $L = 2.5R$, $l_0 = 0.4\,l_1$, $\alpha = 60^\circ$, $R = l_1\cos\alpha$. Table 2 contains the value of the critical pressure in the limiting case at a sufficiently small value of h_3/h_0 so that it would be possible to determine the limiting transition (at $h_3/h_0 \to 0$) to a single-layer shell. The details of the analytical calculations as to the specified boundary transition for the conical shell are contained in the paper [2]. The results of the numerical experiment given in Table 2 show the possibility of realizing the specified boundary transition for a compound shell structure.

The consistency of the results of this paper with the corresponding results of works [1, 3] indicates the accuracy of the proposed mathematical models and calculation algorithms.

Boundary surfaces of the compound shell structure under investigation. Figure 2 shows the boundary surface and curves separating the regions of stability



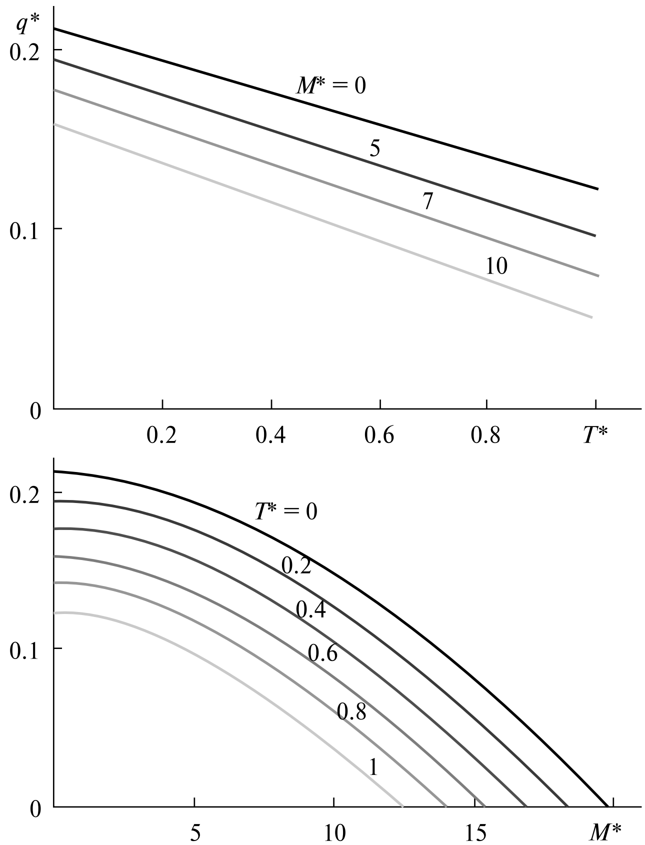


Figure 2. Boundary surface and boundary curves of stability for the three-layer unreinforced cone-cylinder shell structure

Table 1. **Comparison of the results of local buckling calculations with known results**

Based on $\alpha = 60^\circ$ at $T^* = 0$, $M^* = 0$								
$h_3/h_0 = 0.6$	Cylinder				Cone			
	q^*	n	q^*	n	q^*	n	q^*	n
	$k = 2.5$		$k = 1$		$l_0/l_1 = 0.4$		$l_0/l_1 = 0.65$	
According to equation (1)	0.60095	6	1.60654	9	1.85963	6	2.25755	9
Based on formulas from [6]	0.60075	6	1.60252	8	1.88719	6	2.24879	9
Based on $\alpha = 75^\circ$ at $T^* = 0$, $M^* = 0$								
$h_3/h_0 = 0.6$	$k = 1.5$		$k = 1$		$l_0/l_1 = 0.4$		$l_0/l_1 = 0.65$	
According to equation (3)	5.53545	6	8.88096	8	5.88415	4	7.17027	6
Based on formulas from [6]	5.51794	6	8.85829	8	5.93119	4	7.15235	5

Table 2. **Comparison of the results of overall buckling calculations at the limit transition to a single-layer cone-cylinder shell structure**

Cone-cylinder	$G_1 = 0$, $G_2 = 0$ (without reinforcement)		$G_1 = 5000$, $G_2 = 10$ for the docking ring	
	q^*	n	q^*	n
According to equations (1), (3), (7)—(9) at $h_3/h_0 = 0.01$	0.23506	4	1.03704	7
Based on equations from [**3**]	0.23124	4	1.05208	7

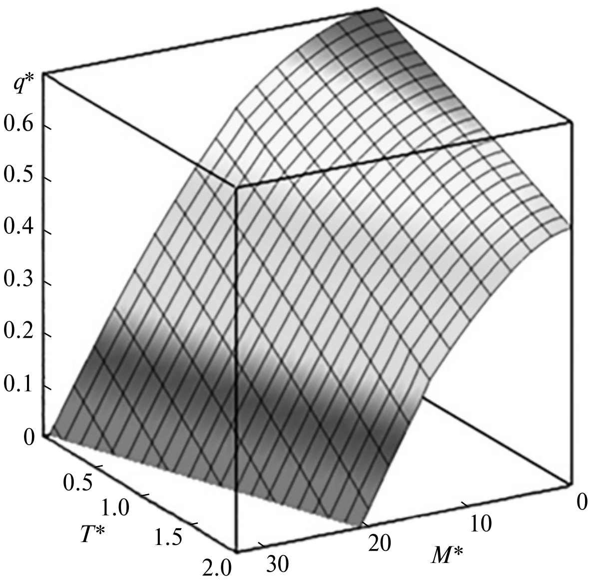


Figure 3. Boundary surface of the reinforced cone-cylinder structure

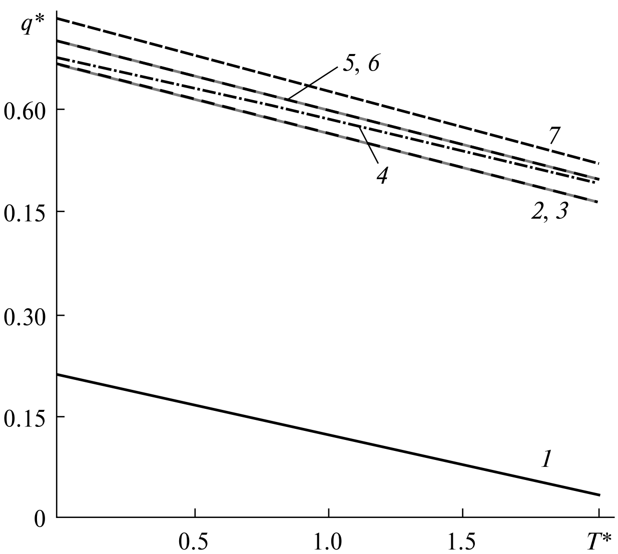


Figure 4. Dependence of critical pressure on axial force for different values of ring stiffness parameters

from the region of buckling for a three-layer unreinforced cone-cylinder shell structure with the abovementioned parameters at $h_3/h_0 = 0.6$. Figure 3 shows the boundary surface for the same structure reinforced with a docking ring with stiffness parameters as follows: $G_1 = 5000$, $G_2 = 10$.

For this type of construction, the boundary surfaces are convex. As one should expect, an increase in the axial compressive force and torque leads to a decrease in the critical external pressure.

The influence of ring stiffness parameters on the stability of the cone-cylinder structure. Figure 4 shows the graphs of critical pressure dependences from axial force for different values of stiffness parameters: a solid black line *1* corresponds to $G_1 = 0$, $G_2 = 0$; a black dash-dotted line *4* correlates to $G_1 = 100$, $G_2 = 10$; solid gray lines *2* and *5* are appropriate to $G_1 = 1000$, $G_2 = 0$, and $G_2 = 10$; black dashed lines 3, 6 and 7 correspond to $G_1 = 5000$, $G_2 = 0$, $G_2 = 10$, and $G_2 = 30$, respectively.

An increase in both stiffness parameters of the rings leads to an increase in the overall stability of the structure. However, increasing the stiffness parameter in the plane of the initial curvature of the ring not lead to an increase in critical forces, which allows choosing a rational value of stiffness to reduce the total mass of the structure. A similar conclusion has been made in the papers [3, 4, 9, 10].

Visualization of the post-critical deformation of the compound shell structure. For unreinforced and reinforced structures of the cone-cylinder type, Tables 3 and 4 show the parameters of the critical forces, as well as the visualization of the protrusion forms as a result of buckling.

Based on the data for local buckling of the structure, given in Table 1, the parameters of the compartments, which correspond to close values of the critical pressure, have been determined. Exactly these parameters are chosen for calculating the first two equally stable compartments in Table 4. The third structure has a lower critical pressure of the local buckling for the cylindrical compartment than the conical one and a distinct change in the waveform under the action of the axial force. Parameters of the fourth structure in Table 4 lead to the critical pressure of the cylindrical compartment greater than that of the conical one.

The general analysis of wave formation at buckling in Tables 3 and 4 shows that an increase in the tensile axial force and torsion leads to an elongation of the wave along the generatrix of the unreinforced shell, which corresponds to the increase in the critical external pressure, and the structure compression leads to the opposite effect.

The distribution of the wave along the reinforced non-equal stable structure is such that its part with the largest amplitude falls on the compartment that has a smaller value of the critical pressure of local buckling. Under the action of axial force, the amplitude decreases on the other compartment during stretching but increases during compression.

In the case of the equally stable structure with local and overall forms of protrusion, the wave crests are placed at the junction of the unreinforced structure or on the cylindrical compartment of the reinforced structure. If this occurs, the deformation of the conical compartment is insignificant.

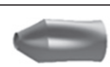
Analysis of the effective position (in terms of stability) of the intermediate ring. In each of the considered cases, equal values for pairs of stiffness parameters G_1, G_2 are chosen for all rings, taking into account the multipliers determined by the formulas (6), (8), (9), (12), (13). In Tables 5 and 6, stability characteristics are introduced, and forms of buckling are visualized depending on the position of the intermediate ring, which is determined by the coefficients of the generatrix division for the corresponding shell — k_{cone} and k_{cyl} .

The first of the structures considered in Table 5 corresponds to the calculations, the results of which are shown in Figures 2 and 3 and in Table 3. For the structure reinforced with the docking ring between the compartments, the part of the wave with the largest amplitude is placed on the cylindrical compartment. When placing the intermediate ring on this compartment, the structure is strengthened, causing the shape of the wave to change depending on the ring position: the wave receives a bend after passing through the ring.

It should be noted that differences in critical wave amplitudes can affect the destructive processes of the structure as a result of stability loss. For the placement of the ring, which corresponds to the case of the highest critical pressure (k_{cyl} =0.53), the wave amplitudes on the cylindrical compartments are equal, corresponding to the most acceptable form of wave formation in comparison with the other forms given in Table 5 for the first structure.

The second structure in Table 5, if $k_{cyl} = 0.5$, can be related to the cone-cylinder-cylinder type with

Table 3. **Wave formation while buckling of the cone-cylinder structure with parameters:**
$L = 2.5R$, $l_0 = 0.4\, l_1$, $\alpha = 60°$, $h_3/h_0 = 0.6$, $E_3/E_1 = 0.01$

$G_1 = 0, G_2 = 0$						
	$M^* = 0$			$M^* = 10$		
						
T^*	–2	0	1	–0.5	0	0.5
n	4	4	4	4	4	4
q^*	0.38804	0.21213	0.12378	0.19926	0.14725	0.09319
γ_1	0	0	0	0.082	0.091	0.102
G_1 =5000, G_2 =10						
	M^* =0			M^* =19		
						
T^*	–3	0	1	–1.9	0	1.9
n	7	6	6	7	7	7
q^*	0.95675	0.69778	0.5962	0.68714	0.43207	0.09139
γ_1	0	0	0	0.139	0.194	0.266

compartments having close values of the critical pressure of the local stability loss. However, placing the ring in the middle of the cylindrical shell does not provide the highest critical pressure value.

A feature of the exhaustion of the third structure supporting capacity is the decrease in the critical pressure on the cylindrical compartment in comparison with the conical one, which requires its reinforcement. As a result, the structure belonging to the cone-cylinder-cylinder type is formed, and its cylindrical compartments, if $k_{cyl} = 0.5$, have a higher critical pressure of local buckling than the conical ones. The allocation of the intermediate ring on the cylindrical compartment in certain positions creates the wave with the largest amplitude on the conical compartment, in particular for placing the ring that corresponds to the highest critical pressure (k_{cyl} =0.44).

Table 4. **Visualization of wave formation while buckling for the cone-cylinder structure with different parameters of the compartments**

$L = R$, $l_0 = 0.4 l_1$, $\alpha = 60°$, $h_3/h_0 = 0.6$, $E_3/E_1 = 0.01$, $M^* = 0$						
	$G_1 = 0$, $G_2 = 0$			$G_1 = 5000$, $G_2 = 10$		
						
T^*	–3	0	0.5	–3	0	2
n	4	4	4	11	10	9
q^*	0.76185	0.21968	0.12888	2.61987	1.90788	1.31248
$L = 1.5R$, $l_0 = 0.4 l_1$, $\alpha = 75°$, $h_3/h_0 = 0.6$, $E_3/E_1 = 0.01$, $M^* = 0$						
	$G_1 = 0$, $G_2 = 0$			$G_1 = 5000$, $G_2 = 10$		
						
T^*	–3	0	2	–5	0	2
n	3	3	3	8	7	6
q^*	2.29854	1.32767	0.691701	10.4632	6.70476	2.57021
$L = R$, $l_0 = 0.65 l_1$, $\alpha = 60°$, $M^* = 0$						
	$G_1 = 0$, $G_2 = 0$			$G_1 = 5000$, $G_2 = 10$		
						
T^*	–3	0	2	–4	0	2
n	6	5	5	11	10	9
q^*	1.04366	0.58844	0.16165	2.81736	1.90454	1.27536
$L = R$, $l_0 = 0.4 l_1$, $\alpha = 75°$, $M^* = 0$						
	$G_1 = 0$, $G_2 = 0$			$G_1 = 5000$, $G_2 = 10$		
						
T^*	–4	0	2	–4	0	2
n	3	3	3	5	5	4
q^*	2.55862	1.33078	0.724467	10.51964	7.89206	6.10913

Table 5. **Analysis of the intermediate ring position on the cylindrical compartment of the cone-cylinder structure**

$L = 2.5R$, $l_0 = 0.4l_1$, $\alpha = 60°$, $M^* = 0$, $T^* = 0$, $G_1 = 5000$, $G_2 = 10$						
						
k_{cyl} n q^*	0.3 8 1.04245	0.4 8 1.18429	0.5 9 1.41725	0.53 9 **1.44803**	0.6 8 1.32368	0.7 8 1.13755
$L = 3R$, $l_0 = 0.4l_1$, a = 75°, $M^* = 0$, $T^* = 0$, $G_1 = 5000$, $G_2 = 10$						
	1 ring	2 rings: a docking one and an intermediate one				
						
k_{cyl} n q^*	— 5 3.12951	0.4 6 5.21565	0.5 7 6.26242	0.53 7 **6.48832**	0.6 7 6.04131	0.7 6 5.15638
$L = 2R$, $l_0 = 0.65l_1$, a = 75°, $M^* = 0$, $T^* = 0$, $G_1 = 5000$, $G_2 = 10$						
	1 ring	2 rings: a docking one and an intermediate one				
						
k_{cyl} n q^*	— 6 4.84541	0.4 8 8.38117	0.44 6 **8.78456**	0.5 6 8.73781	0.6 6 8.63729	0.7 7 8.18299

Table 6. **Analysis of the intermediate ring position on the conical compartment of the cone-cylinder structure**

$l_0 = 0.4l_1$, $M^* = 0$, $T^* = 0$, $G_1 = 5000$, $G_2 = 10$								
	$\alpha = 60°$					$\alpha = 75°$		
								
k_{cone} n q	0.5 10 2.51014	0.6 11 2.82568	0.74 12 3.38686	0.75 8 **3.39983**	0.76 8 3.34532	0.74 8 10.77841	0.745 5 **10.8289**	0.75 5 10.7422

$l_0 = 0.4l_1$, a = 75°, $M^* = 0$, $T^* = 0$, $G_1 = 5000$, $G_2 = 10$						
	1 ring	2 rings: a docking one and an intermediate one				
	$L = R$				$L = 2R$	
						
k_{cone} n q	— 5 7.89206	0.5 7 10.04077	0.7 8 11.08547	0.8 5 **11.17728**	$k_{cone} = 0.8$ 6 4.94667	$k_{cyl} = 0.39$ 5 **8.07674**

In the first part of Table 6, for the conical shell with different taper angles, the largest critical pressure values are highlighted in bold. As one can see, the specified values correspond to the form of buckling with the lowest wave number (n) and the most "smoothed" form of wave formation.

The second part of Table 6 contains calculations for the cone-cylinder structures, for which the critical pressure of local buckling of the cylindrical compartment, when $L = R$, is higher than the conical one. Therefore, it is reasonable to reinforce the conical compartment of such a structure. If $L = 2R$, then the cylindrical compartment has a smaller value of the critical pressure, and it is advisable to reinforce it with the intermediate ring. This is actually confirmed by the calculations of the overall buckling (for structures with different values of L/R, the highest critical pressure is highlighted in bold). For both considered structures, the highest critical pressure values correspond to the allocation of the intermediate ring closer to the junction of the compartments. It should be highlighted that in all cases, there is a consistency between the position of the intermediate ring, which corresponds to the highest critical pressure, and the behavior of wave formation as the most acceptable for preventing the possible destruction of the structure.

The influence of the inner layer parameters on the stability of the three-layer structure. It should be noted that depending on the relative thickness and elasticity modulus of the inner layer, the reduced modulus of elasticity changes. Table 7 shows the values of critical pressure (q^*), reduced elasticity modulus (E), and dimensionless pressure q/E for various parameters h_3/h_0 and E_3/E_1 of the three-layer unreinforced compound shell structure.

The calculations have shown that an increase in the relative thickness of the layer at a fixed value of E_3/E_1 leads to a decrease in the critical pressure (q) but to an increase in the parameter q/E. A decrease in the value of E_3/E_1 ("softening" of the inner layer) in the range of values of h_3/h_0 from 0.1 to 0.4 leads to an increase in the external pressure, and in the range of h_3/h_0 from 0.5 to 0.9 causes a decrease in the value of q. However, for each fixed relative thickness of the inner layer h_3/h_0, its "softening" leads to an increase in the parameter values of q/E. The specified conclusion qualitatively corresponds to the analysis results of the paper [2].

CONCLUSIONS

1. The paper proposes a mathematical model and method of analytical-numerical approach for the stability research of a three-layer reinforced compound shell structure of the cone-cylinder type under combined external loading capable of causing the local and overall bifurcation states.

2. Boundary surfaces separating the region of stability for the structure reinforced with the transverse force elements depending on the parameters of the compartments of the studied system and the external load are constructed.

Table 7. **The influence of the inner layer parameters on the stability of the three-layer unreinforced cone-cylinder structure ($\alpha = 60°$, $l_0/l_1 = 0.4$, $k = 2.5$, $E_1 = E_2$, $h_1 = h_2$)**

h_3/h_0	$E_3/E_1 = 0.01$			$E_3/E_1 = 0.001$			$E_3/E_1 = 0.0001$		
	q^*, Па	E, 10^{-10} Па	q/E, 10^7	q^*, Па	E, 10^{-10} Па	q/E, 10^7	q^*, Па	E, 10^{-10} Па	q/E, 10^7
0.1	0.24137	9.3704	2.57592	0.24143	9.36104	2.57918	0.24144	9.360104	2.57951
0.2	0.24656	8.3408	2.95616	0.24667	8.32208	2.964154	0.24669	8.320208	2.96495
0.3	0.24806	7.3112	3.39296	0.24819	7.28312	3.407753	0.24820	7.280312	3.40923
0.4	0.24404	6.2816	3.88501	0.24413	6.24416	3.90968	0.24414	6.240416	3.91216
0.5	0.23266	5.252	4.43007	0.23265	5.20520	4.46952	0.23264	5.20052	4.47351
0.6	0.21213	4.2224	5.02395	0.21192	4.16624	5.08664	0.21191	4.160624	5.09301
0.7	0.18062	3.1928	5.65731	0.18013	3.12728	5.75983	0.18007	3.120728	5.77032
0.8	0.13635	2.1632	6.30344	0.13544	2.08832	6.48595	0.13535	2.080832	6.50492
0.9	0.07753	1.1336	6.83943	0.07608	1.04936	7.25036	0.07593	1.040936	7.29509

3. For local and overall forms of the compound shell structure protrusion, the issues of equistability of the structure spans and rational design related to the selection of stiffness parameters for the frontal and intermediate rings and their positions in the compartments of the three-layer compound structure are discussed to reduce the weight characteristics of the studied system.

4. The proposed approach and method of analyzing the stability of three-layer compound shell structures of the cone-cylinder type under the combined external load have allowed the realization of visualizing the forms of buckling and the determination of the parameters for rational design of structures for the new equipment.

5. A comparison of the research results with the existing published data is provided. The discrepancy does not exceed 4 %.

6. The proposed solutions and research results can be applied in engineering calculations, in particular in aviation and rocket-space systems.

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В. З. Грищак [1], проф., д-р техн. наук, проф., заслужений діяч науки і техніки України, акад. Міжнародної академії наук вищої школи, член бюро Президії Укр. тов-ва інженерів-механіків
Д. В. Грищак [2], заст. міністра з питань цифрового розвитку, цифрових трансформацій і цифровізації, канд. техн. наук, почесний працівник космічної галузі України
Н. М. Д'яченко [3], доцент кафедри фундаментальної та прикладної математики, канд. фіз.-мат. наук, доцент
А. Ф. Санін [4], зав. кафедри ракетно-космічних та інноваційних технологій, д-р техн. наук, проф., лауреат Державної премії України в галузі освіти
К. М. Сухий [5], ректор , д-р техн. наук, проф., акад. Академії наук вищої школи України

[1] Національний технічний університет «Дніпровська Політехніка»
Пр. Дмитра Яворницького 19, Дніпро, Україна, 49005
[2] Міністерство з питань стратегічних галузей промисловості України
вул. Івана Франка 21-23, Київ, Україна, 01054
[3] Запорізький національний університет
вул. Жуковського 66, Запоріжжя, Україна, 69600
[4] Дніпровський національний університет імені Олеся Гончара
Пр. Гагаріна 72, Дніпро, Україна, 49100
[5] Український державний хіміко-технологічний університет
Пр. Гагаріна 8, Дніпро, Україна, 49005

БІФУРКАЦІЙНИЙ СТАН ТА РАЦІОНАЛЬНЕ ПРОЄКТУВАННЯ ПІДКРІПЛЕНОЇ ТРИШАРОВОЇ СКЛАДЕНОЇ КОНСТРУКЦІЇ «КОНУС — ЦИЛІНДР» ПРИ КОМБІНОВАНОМУ НАВАНТАЖЕННІ

Запропоновано аналітико-чисельний підхід до розв'язання задачі біфуркації станів з точки зору локальної і загальної стійкості дискретно підкріпленої проміжними шпангоутами тришарової оболонкової конструкції конус — циліндр, зокрема сучасних ракет-носіїв, при статичному комбінованому навантаженні зовнішнім тиском, осьовими зусиллями і крутильним моментом з урахуванням параметрів жорсткості проміжних шпангоутів у площині початкової кривини і на крутіння. Розв'язувальними рівняннями проблеми є звичайні диференціальні рівняння шостого порядку (для циліндричного відсіку зі сталими коефіцієнтами, для конічного — зі змінними за осьовою координатою коефіцієнтами). Використано диференціальні співвідношення, що визначають умови спряження через проміжний шпангоут. Для чисельного розв'язання застосовується метод скінченних різниць з центральними скінченними різницями третього та другого порядку у внутрішніх точках відрізків визначення оболонок і на її торцях відповідно та різницями другого порядку з одним кроком назад або вперед у точках спряження через шпангоут.

Показано, що результати розрахунків узгоджуються з відомими даними для тришарових конічної та циліндричної оболонок, а також у граничному випадку — при переході до одношарової складеної конструкції конус — циліндр. Для розглянутого класу оболонкових конструкцій конус — циліндр побудовано граничні поверхні, що відокремлюють область стійкості досліджуваної конструкції залежно від геометричних і жорсткісних параметрів відсіків, підкріплювальних елементів і характеру зовнішнього навантаження. Досліджено вплив зовнішнього навантаження на параметр за критичного хвилеутворення досліджуваної конструкції з наданням візуалізації характеру деформування.

Аналіз результатів розрахунків показав, що даний підхід до розв'язання проблеми біфуркації і рівностійкості відсіків складеної конструкції по відношенню до локальної і загальної форм випинання дозволяє обирати раціональні геометричні параметри і параметри жорсткості складових оболонок і силових елементів з точки зору поліпшення вагових характеристик конструкції.

***Ключові слова*:** тришарова оболонка, складена конструкція «конус — циліндр», комбіноване навантаження, шпангоут, гранична поверхня, локальна і загальна форми випинання, візуалізація закритичних форм втрати стійкості.