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V. Z. GRISTCHAK[1]
ORCID: https://orcid.org/0000-0001-8685-3191
E-mail: grk@znu.edu.ua
D. V. HRYSHCHAK[2]
ORCID: https://orcid.org/0000-0001-6474-6926
E-mail: d.hryshchak@mspu.gov.ua
N. M. DYACHENKO[1]
ORCID: https://orcid.org/0000-0001-5284-4502
V. V. BABUROV[3]
ORCID: https://orcid.org/0000-0001-7202-5133

[1] Zaporizhzhia National University
66, Zhukovsky Str., Zaporizhzhia, 69600 Ukraine
[2] The Ministry for Strategic Industry of Ukraine
21-23, Ivana Franka Str., Kyiv, 01054 Ukraine
[3] Yuzhnoye State Design Office named M. K. Yangel
3, Kryvorizka Str., Dnipro, 49008 Ukraine

THE INFLUENCE OF THE GAUSSIAN CURVATURE SIGN OF THE COMPOUND SHELL STRUCTURE'S MIDDLE SURFACE ON LOCAL AND OVERALL BUCKLING UNDER COMBINED LOADING

The buckling problem of an elastic compound shell structure with a variable Gaussian curvature of the middle surface, especially the middle surface meridian curvature sign, under the action of external pressure and axial loading is considered. In continuation of the previous research of the authors, this paper is devoted, in particular, to examining the influence of the negative Gaussian curvature sign of one of its compartments on stability.

The solution is based on using the method of finite differences for basic stability equations of each compartment in the case when one of them can have a negative curvature of the meridian, taking into account the discreteness of the intermediate rib location and their rigidity from the initial curvature plane as well. The obtained solution allows visualizing the buckling modes under various combinations of external loading and identifying rational, according to overall buckling modes, geometric and rigidity parameters of the system being investigated.

Keywords: *buckling, shells, compound "barrel-ogive" structure, Gaussian curvature sign of the middle surface, rational design, combined loading.*

INTRODUCTION

The study of shell structures with a complex geometric shape of the middle surface for stability is dictated by the needs of modern engineering, aircraft, rocket and space technology, and internal trends in developing the mechanics of thin-walled shell systems. The selection of the effective forms of shell compartments and reinforcing elements depends on the purpose

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of the designed structure and the nature of external loading [10, 12].

A significant part of theoretical and experimental studies is devoted to the composite "cylinder-cone" type structures with zero Gaussian curvature [1, 3, 4, 12, 13]. Recently, the researchers have paid special attention to the problems of the compound shell structures' stability with the positive [5—7, 14] and the negative [2, 9, 11] Gaussian curvature. As for the visualization of buckling modes of shell structures, a reference should be made to [8, 11, 12, 14].

The purpose of this paper is to study the influence of the Gaussian curvature sign of the middle surface of the compound "barrel-ogive" shell structure on the local and overall stability under the combined action of uniform external pressure and axial loads with the determination of rational rigidity characteristics of the reinforcing ribs and visualization of the buckling modes.

THE PROBLEM STATEMENT AND RESOLVENT EQUATIONS

Following [6, 7], and adhering to the terminology introduced in these papers, in the case of the compartment convexity, which corresponds to a positive sign of the middle surface curvature, the structure is to be referred to as a "barrel" or an "ogive", respectively. To simplify further generalizing computations, the prefix "pseudo" (a "pseudo-ogive" or a "pseudo-barrel") is added to the name of the corresponding shell compartment in the case of a negative curvature of the meridian.

We consider this composite shell structure with constant thickness h, elasticity modulus E, and Poisson's ratio ν. The class of medium-length shells has been considered.

In the elastic region of the material deformation, this structure is generally under the influence of external pressure q and axial (compressive or tensile) force T. Moreover, the prevailing effect of the external pressure in relation to axial compression is assumed, which leads to the buckling modes corresponding to the formation of one half-wave in the longitudinal direction and n waves in the circumferential one, along with this $n^2 >> 1$ [14].

The coordinates along the generatrix of the cylindrical and conical surfaces are marked with $\overline{s}$ and s, respectively, the arc coordinate for the cylinder is y, and the angular coordinate along the parallel of the conies is ϕ. The middle surface of each shell structure's compartment is the surface of rotation with the following functions of a parallel circle radius in the cross section, which is perpendicular to the axis of rotation [5—7] :

– for the generalized barrel-shaped compartment:

$$r = R\left(1 + C_{bar}\sin\frac{\pi\overline{s}}{L}\right), \qquad (1)$$

where L is the distance between the bases, R is the compartment base radius, C_{bar} is the relative deviation of the shell generatrix from the cylinder;

– for the generalized ogive-type compartment:

$$r = \cos\alpha\left[s + C_{og}l_1\sin\frac{\pi(s-l_0)}{l_1-l_0}\right], \qquad (2)$$

where l_0 and l_1 are the distances along the axis Os to smaller and larger bases, α is the cone angle at the base, C_{og} is the relative deviation of the shell generatrix from the cone.

The restrictions on the shell parameters are attained in [5—7], and the approximate values of the main curvature radii are obtained on the basis of equations (1) and (2):

– for the generalized barrel-shaped compartment:

$$\tilde{R}_1 = -\frac{(1+(r')^2)^{3/2}}{r''} \approx \frac{L^2}{RC_{bar}\pi^2\sin\dfrac{\pi\overline{s}}{L}},$$

$$\tilde{R}_2 = r\sqrt{1+(r')^2} \approx R\left(1 + C_{bar}\sin\frac{\pi\overline{s}}{L}\right),$$

– for the generalized ogive-type compartment:

$$\tilde{\tilde{R}}_1 \approx \frac{\left(l_1-l_0\right)^2}{C_{og}l_1\pi^2\cos\alpha\sin\alpha\sin\Omega},$$

$$\tilde{\tilde{R}}_2 \approx \operatorname{ctg}\alpha(s + C_{og}l_1\sin\Omega),$$

$$\Omega = \frac{\pi(s-l_0)}{l_1-l_0}.$$

The signs of parameters C_{bar} and C_{og} determine the Gaussian curvature sign $\kappa = 1/(R_1R_2)$ for the middle surface of the corresponding structure compartment.

In [5—7], the resolvent differential equations of the main stress-strain state are derived as to the deflection functions for each compartment:

$$a_4(\bar{x})\, W_{bar}^{IV}(\bar{x}) + a_3(\bar{x})\, W_{bar}'''(\bar{x}) + a_2(\bar{x})\, W_{bar}''(\bar{x}) +$$

$$+ a_1(\bar{x}) W_{bar}'(\bar{x}) + a_0(\bar{x}) W_{bar}(\bar{x}) = 0 \,, \qquad (3)$$

$$b_4(x) W_{og}^{IV}(x) + b_3(x)\, W_{og}'''(x) + b_2(x)\, W_{og}''(x) +$$

$$+ b_1(x)\, W_{og}'(x) + b_0(x)\, W_{og}(x) = 0 \,, \qquad (4)$$

where $\bar{x} = \bar{s}/L$, $x = s/l_1$ and the variable coefficients of equations, being as follows $a_i(\bar{x}),\ i = \overline{1,4}$, $b_j(x),\ j = \overline{1,4}$, depend on the geometric characteristics of shells and external loads.

These papers also describe the specifics of applying the finite difference method to solve equations (3), (4), and the matrix method of the initial parameter to take into account the discreteness of the location of the intermediate rib, including the docking one:

$$W_{og}(1) = W_{bar}(0) \,,\ W_{og}'(1) = W_{bar}'(0) \,, \qquad (5)$$

$$W_{og}''(1) + G_2^* W_{og}'(1) = W_{bar}''(0) \,,$$

$$W_{og}'''(1) - G_1^* W_{og}(1) = W_{bar}'''(0) \,, \qquad (6)$$

where G_1^* and G_2^* are the dimensionless rigidity parameters of the ribs in the plane of the initial curvature and from this plane, respectively. In this case, the matching condition of the generalized ogive and barrel-shaped sections is determined by the equality of tangent angles β to the middle meridians of the sections. Therefore, the docking rib location can be considered locally conical. In this context, the rigidity of the ribs is determined by the formulas:

$$G_1^* = G_1 \frac{1}{\cos^3\beta} \,,\ G_2^* = G_2 \frac{1}{\cos\beta} \,,$$

$$G_1 = \frac{n^4 (n^2 - 1)^2 (EJ)_x^{ring}}{Eh\, R^3} \,,$$

$$G_2 = \frac{n^2 (n^2 - 1)^2 (EJ)_z^{ring}}{EhR^3 (n^2 + 1)} \,.$$

Here J_x^{ring}, J_z^{ring} are the moments of inertia under bending the rib in the plane of the initial curvature and, accordingly, from its plane.

In the process of studying the cylinder-cone structure, we shall follow the idea of [3, 4], where the docking rib is broken into two parts corresponding to its cylindrical and conical component, and, as a result, the rigidity is considered as

$$G_{cyl,1} = \frac{G_1}{2} \,,\ G_{cyl,2} = \frac{G_2}{2} \,,$$

$$G_{cone,1} = \frac{G_1}{2} \frac{1}{\cos^3\alpha} \,,\ G_{cone,2} = \frac{G_1}{2} \frac{1}{\cos\alpha} \,.$$

In this context, matching conditions (5) by means of the rib are persisted, and the conditions (6) are rewritten as

$$W_{og}''(1) + G_{cone,2} W_{og}'(1) = W_{bar}''(0) + G_{cyl,2} W_{bar}'(0) \,,$$

$$W_{og}'''(1) - G_{cone,1} W_{og}(1) = W_{bar}'''(0) + G_{cyl,1} W_{bar}(0) \,.$$

In the case of the "cylinder-cone" structure, the following formulas are taken

$$W_{bar} = W_{cyl} \,,\ W_{og} = W_{cone} \,.$$

ANALYSIS OF NUMERICAL RESULTS

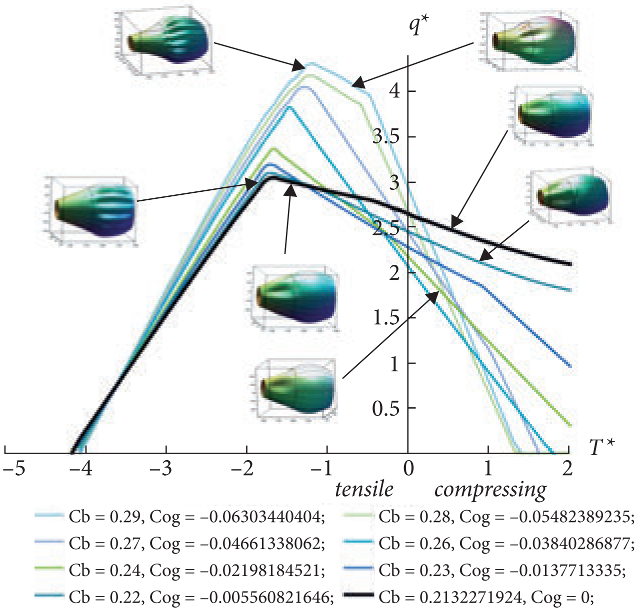
The compound shell structure has been selected for the numerical implementation, having the following characteristics: $h = 3 \cdot 10^{-3}$ m, $E = 6.87 \cdot 10^{10}$ Pa, $\nu = 0.32$. For the generalized ogive section, one has defined $l_1 = 1.82$ m, and $L = 2.5R$ has been accepted for the barrel-shaped section. Calculations have been carried out for the case of boundary conditions corresponding to the hinged support of the ends.

In [5, 7], the parameters are specified for the equally stable "barrel-ogive" structure for the case where $l_0 = 0.45\, l_1$, $\alpha = 75°$. They correspond to the shell, and its ogive and barrel-shaped compartments have approximately equal values of critical pressures under the conditions of the hinged support of the ends. Such a selection is implemented by varying the parameters of the high profile of these compartments. The corresponding parameter values are $C_{og} = 0.06258642604$; $C_{bar} = 0.137$.

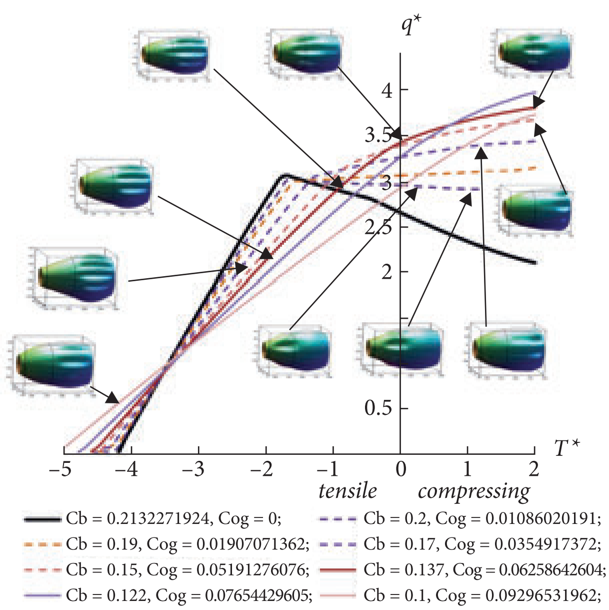
Equally buckling shells, in particular, are characterized by a total loss of stability, both with the consideration of the intermediate ribs. The mentioned effect will be visualized below.

We will introduce the dimensionless loads such as axial force and critical pressure for consideration:

$$T^* = \frac{T}{Eh^2} \,,\ q^* = \frac{q}{q_{cyl}} \,,$$



a



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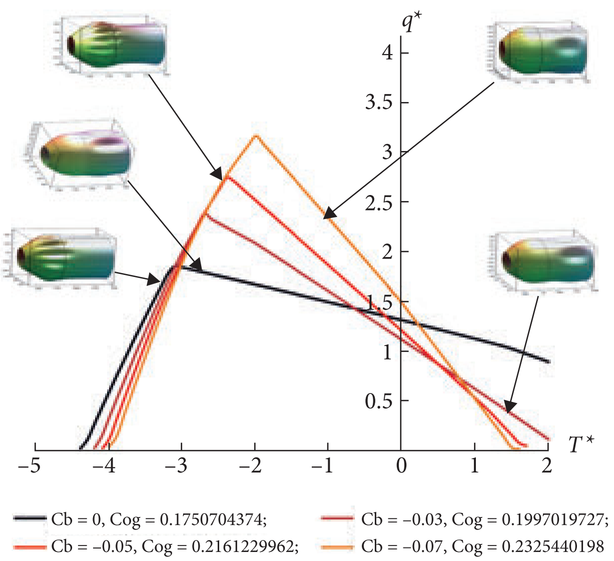

c

Fig. 1. Buckling loads with visualization

where

$$q_{cyl} = 0.92\,E\left(\frac{h}{R}\right)^{5/2}\frac{R}{L}$$

is the classical value of the critical pressure for a cylindrical shell [14].

COMPARATIVE ANALYSIS FOR THE EFFECTIVENESS OF COMPOUND STRUCTURES WITH COMPARTMENTS HAVING DIFFERENT CURVATURE SIGNS OF THE MIDDLE SURFACES

Figure 1 shows the dependency graphs of the critical pressure q^* from the axial (compressive or tensile) force T^* for the shells with different values of relative deviations from the cylinder C_{bar} and the barrel C_{og}. In all cases, the structure is reinforced with a docking rib with the rigidity parameters as follows: $G_1 = 5000$, $G_2 = 10$. The figure is provided with the illustrations of visualization of characteristic cases of buckling modes:

– Figure 1 (*a*) corresponds to the case of a negative Gaussian curvature of the generalized ogive compartment, that is, for the “barrel-pseudo-ogive” structure. At the same time, a heavy black line corresponds to the “barrel-cone” structure;

– Figure 1 (*b*) is the case of a positive curvature of both compartments (the “barrel-ogive” structure). The dependence for the “barrel-cone” type system is presented as well;

– Figure 1 (*c*) is the case of a negative curvature of the generalized barrel-shaped compartment (the “pseudo-barrel-ogive” structure). A heavy black curve corresponds to the “cylinder-ogive” system.

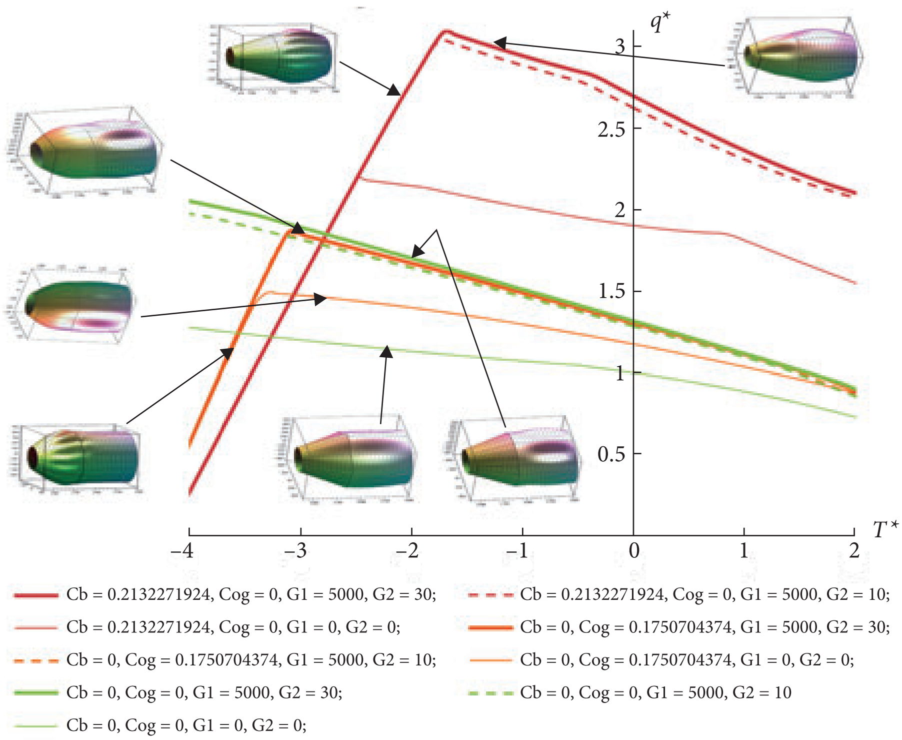

Fig. 2. Buckling loads and visualization of a structure with zero Gaussian curvature

Figure 2 shows the dependencies q^* on T^* for the structures in which at least one section (or both) has a zero Gaussian curvature. Cases of the structure' reinforcement with the docking rib having different rigidity parameters are considered.

Analysis of the results shows that under the combined action of uniform external pressure and axial compression, a structure having both compartments with the positive Gaussian curvature is more stable. With the combined effect of pressure and axial tension, it is advisable to introduce one of the compartments with the negative Gaussian curvature of the middle surface into the structure to increase the stability.

The visualization of the buckling modes shows that the general instability of the structure occurs when forming the buckling wave crest with the consideration of the rib for the equally stable "barrel-ogive" structure (C_{og} = 0.06258642604; C_{bar} = 0.137) under the action of uniform pressure and axial compression. Under axial tension for the parameters

$$C_{bar} = 0.28\,(0.29)\,,$$

$$C_{og} \approx -0.0548\,(-0.063)\,;$$

$$C_{bar} = -0.03\,(-0.05, -0.07)\,,$$

$$C_{og} \approx 0.2\,(0.216, 0.233)\,,$$

such a buckling mode is present in case of the negative curvature of the middle surface meridian on one of the compartments as well. Compound structures with just such parameters are the most effective according to the stability of behavior in the corresponding ranges of axial force variation.

THE EFFECT OF THE RIGIDITY PARAMETER OF A DOCKING RIB ON BUCKLING LOADS

The characteristic dependences of the effect of the rigidity parameters of the rib (in the plane G_1 and from the plane G_2 of its initial curvature) are shown in Fig. 2 for the shells with one of the sections with zero curvature of the middle surface. At the same time, the boundary curves are given in Fig. 3, which separate the stability region of the structure from the instability region (dependence q^* on T^*) for the shells with different Gaussian curvature signs of the compartments.

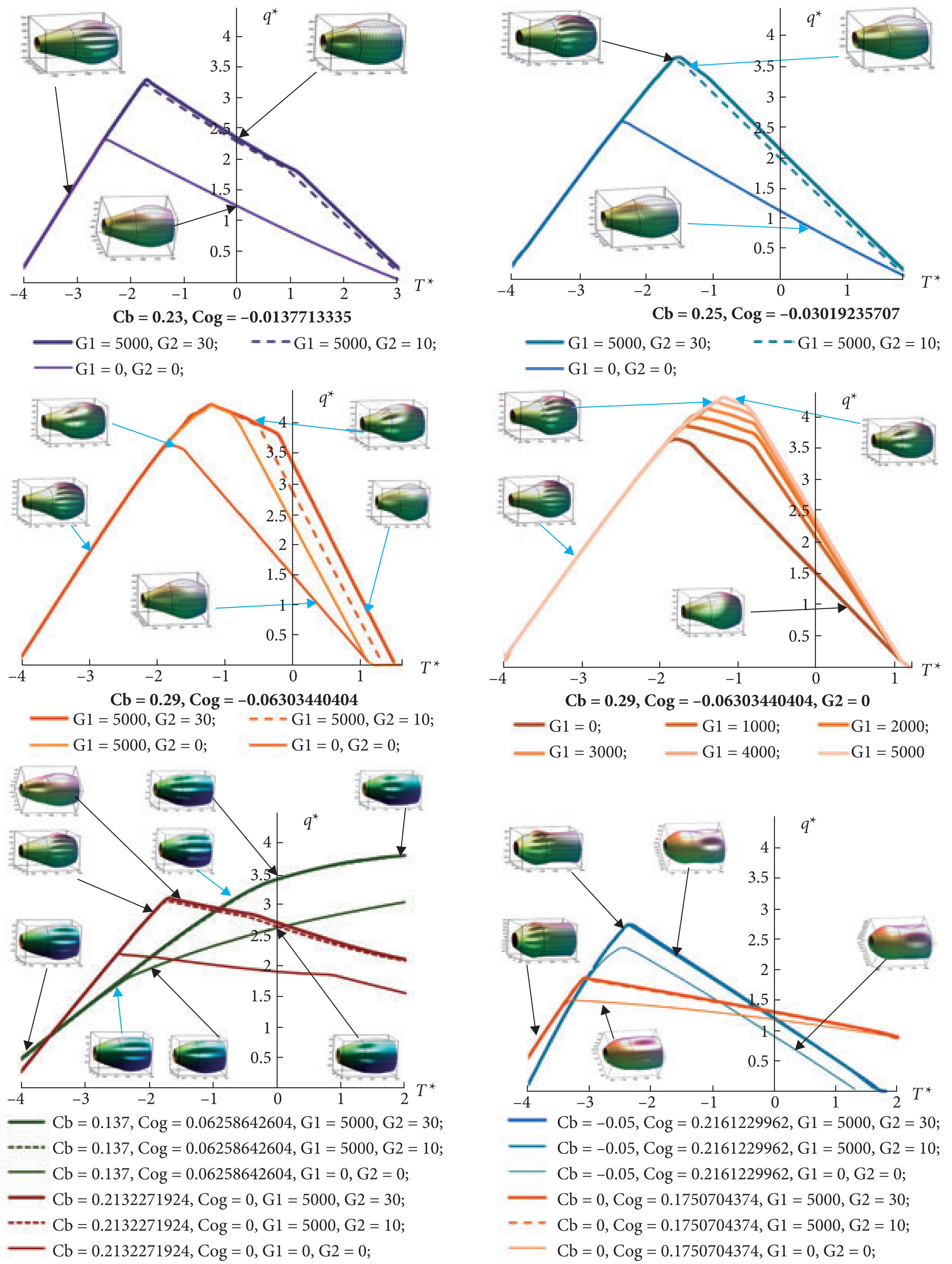

Fig. 3. Boundary buckling curves for the shells with different Gaussian curvature signs of the compartments

For the main part of the calculated structures, one has found a significant effect on the value of the critical loading for the rigidity parameter in the plane of the initial curvature G_1 and an insignificant effect of the rigidity parameter from the plane of the initial curvature G_2 in a certain range of the axial loading parameter $T^* > T_1$. For the parameter $T^* > -1$, the effect of the rigidity G_2 can be significant.

The results of the numerical analysis make it possible to draw conclusions about the possibility of rational designing of the reinforced compound shell structures with different Gaussian curvatures in terms of the equivalence of the local and overall buckling modes.

CONCLUSIONS

1. The hybrid analytical-numerical approach is proposed for the problem of the compound shell structure buckling under combined external loading with the influence of the Gaussian curvature sign of the compartment middle surface.

2. Shell structures with the negative Gaussian curvature of the surface on one of the compartments are effective under the combined action of uniform external pressure and axial tension, and structures having two compartments with the positive curvature are effective under axial compression.

3. The results of compound structure calculations are given in terms of equal stability of the compartments. In each case of combining the curvature signs of shell compartments, the parameters of the middle surface high profile of the compartments, which correspond to the visualized effect of an equal global buckling mode, are indicated both for axial compression and tension

4. The effect of increasing the docking rib rigidity allows determining the rational characteristics of the shell system being studied.

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В. З. Грищак[1], проф., д-р техн. наук, акад. АН ВО України,
акад. Міжнародної академії наук вищої школи, Заслужений діяч науки і техніки України
ORCID: https://orcid.org/0000-0001-8685-3191
E-mail: grk@znu.edu.ua
Д. В. Грищак[2], заст. директора департаменту — нач. Управління науково-технічного розвитку ОПК Департаменту стратегії реформування ОПК, Міністерство з питань стратегічних галузей промисловості України, канд. техн. наук
ORCID: https://orcid.org/0000-0001-6474-6926
E-mail: d.hryshchak@mspu.gov.ua
Н. М. Д'яченко[1], доцент, канд фіз.-мат. наук
ORCID: https://orcid.org/0000-0001-5284-4502
В. В. Бабуров[3], нач. сектору
ORCID: https://orcid.org/0000-0001-7202-5133

[1] Запорізький національний університет
вул. Жуковського 66, Запоріжжя, Україна, 69600
[2] Міністерство з питань стратегічних галузей промисловості України
вул. Івана Франка, 21-23, Київ, Україна, 01054
[3] Державне підприємство «Конструкторське бюро «Південне» ім. М. К. Янгеля»
вул. Криворізька 3, Дніпро, Україна, 49008

ВПЛИВ ЗНАКУ ГАУССОВОЇ КРИВИНИ СЕРЕДИННОЇ ПОВЕРХНІ СКЛАДЕНОЇ ОБОЛОНКИ НА ЛОКАЛЬНЕ ТА ЗАГАЛЬНЕ ВИПИНАННЯ ПРИ КОМБІНОВАНОМУ НАВАНТАЖЕННІ

Розглянуто проблему втрати стійкості пружної складеної оболонкової конструкції з різними знаками гауссової кривини серединної поверхні, особливо знаками кривини меридіана серединної поверхні, під впливом зовнішнього тиску та осьового зусилля. Робота продовжує попередні дослідження авторів та присвячена впливу негативного знаку гауссової кривини на одному з відсіків оболонкової конструкції на стійкість.

Розв'язування базується на застосуванні методу скінченних різниць для основних рівнянь стійкості кожного відсіку у випадку, коли один з них може бути з негативною кривиною меридіана, з урахуванням дискретності розміщення проміжних ребер та їхньої жорсткості з площини початкової кривини. Отриманий розв'язок дозволяє здійснити візуалізацію форми втрати стійкості для різних комбінацій зовнішнього навантаження та визначити раціональні з точки зору загальних форм втрати стійкості геометричні та жорсткісні параметри досліджуваної системи.

Ключові слова: втрата стійкості, оболонки, складена конструкція «бочка-оживало», знак гауссової кривини серединної поверхні, раціональна конструкція, комбіноване навантаження.