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PROVIDING OF POGO STABILITY OF THE CYCLONE-4M LAUNCH VEHICLE

Low-frequency longitudinal (POGO) oscillations of liquid launch vehicles is a phenomenon inherent to almost all liquid rockets. POGO oscillations of launch vehicles can lead to various emergencies: damages of the rocket structure and liquid propellant propulsion system, unacceptable malfunctions of the rocket control system. The use of liquid-propellant rocket engines with an oxidizer-rich staged combustion cycle for the first stage of launch vehicles can introduce a number of features into the POGO stability analysis. First of all, in this case, longitudinal vibrations of launch vehicles can occur due to the low-frequency instability of a liquid propulsion system at frequencies associated with the dynamics of the circuit of the turbopump — gas generator - gas duct. Another feature of these engines is the manifestation of a significant maximum of the module of the engine dynamic pressure gain in the low frequency range (up to 10 Hz), which can lead to POGO instability of the launch vehicle even in the initial part of its flight with significant values of the rocket structure generalized masses for the lower modes of launch vehicle natural vibrations.

To predict the POGO stability of the currently designed Cyclone-4M two-staged launch vehicle, the mathematical model of the low-frequency dynamics of the “propulsion system — rocket structure” system has been developed. The model describes the interac-

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tion of the launch vehicle elastic structure longitudinal vibrations with low-frequency processes in the main propulsion system. The developed mathematical model contains a mathematical description of the low-frequency dynamics of the RD-874 main propulsion system (includes four RD-870 engines with oxidizer-rich staged combustion cycle), the launch vehicle structure, and feed lines. As a result of a theoretical analysis of the POGO stability, based on the developed mathematical model, using the Nyquist criterion, it was found that the "propulsion system — rocket structure" dynamic system is unstable with respect to the first mode of the structure longitudinal vibrations at the initial flight time interval (5 s, 70 s). This instability is caused not only by the convergence of the first oscillation frequency of the liquid in the oxidizer feed line and the natural frequency of the first mode of the longitudinal vibrations of the Cyclone-4M launch vehicle structure but also by a significant increase in the (6 Hz — 9 Hz) frequency range of the dynamic gain of the RD-870 engine. It leads to POGO instability of the Cyclone-4M launch vehicle in the indicated interval of the flight time. This pattern of the POGO-instability development was discovered for the first time, and it can be noted as a characteristic feature of this dynamic phenomenon for rocket engines with an oxidizer-rich staged combustion cycle.

To provide the launch vehicle POGO stability, it is proposed to install POGO suppressor in the oxidizer feed lines of the main propulsion system. A mathematical model of the low-frequency dynamics of a POGO suppressor with a bellows gas-liquid separation was developed, and the suppressor parameters were determined. The approach to determining the POGO suppressor parameters to provide the POGO stability of liquid propellant LVs was developed: a rational choice of the POGO suppressor design parameters was carried out based on the conditions of the amplitude stabilization of the "propulsion system with POGO suppressors — rocket structure" open dynamic system.

***Keywords**: POGO stability of launch vehicle, cavitation phenomena in pumps, bellows type POGO suppressor, dynamic gains of oxidizer-rich staged rocket engine, Nyquist stability criterion.*

INTRODUCTION

When creating new or upgrading existing space launch vehicles (LVs), an analysis of longitudinal (POGO) stability is performed. Liquid LV most often lose longitudinal stability due to the dynamic interaction of the LV structure and the main liquid rocket propulsion system during the flight in the case when the frequencies of the lower longitudinal structural oscillation modes converge (coincide) with the frequencies of dynamic processes in the propulsion system.

Longitudinal oscillations of the launch vehicle are unacceptable both for the liquid propellant rocket engines and the LV control system, and for the launch vehicle structure, including the spacecraft launched into working orbits with complex and expensive apparatuses and sensitive equipment. Vibrational loads on the LV and spacecraft during the LV longitudinal instability can exceed the permissible level and lead to failure situations (examples of such unsuccessful LV launches are described, for example, in [6, 7, 10]). In the American aerospace papers, this phenomenon is called POGO oscillations (e. g. [12]).

To date, research centers in the USA, Russia, France, Ukraine, and China have completed a number of works on the problem of the POGO stability of liquid launch vehicles (in particular, [1, 3, 13, 16, 24, 25]). Among the fundamental works on the liquid rockets POGO stability, one can distinguish a monograph by M. Natanzon [10], methodological articles, and NASA guidance to eliminating the POGO instability of liquid rockets by S. Rubin [12, 20, 21]. The main mechanism of longitudinal instability of liquid launch vehicles is described in the works of M. Natanzon and Sh. Rubin, mathematical models of the low-frequency dynamics of aggregates and systems of rocket engines are developed for the analysis of the POGO stability of LV and methods for studying the vehicle stability in the frequency and time domain. M. Natanzon showed the main method used in the theoretical forecast of the LV POGO stability is the mathematical modeling of launch vehicle dynamics due to the fact of the dynamic interaction of the rocket engine and the rocket launcher cannot be reproduced under various experimental studies in Earth conditions.

The POGO stability of liquid rockets is studied mainly for rocket engines designed without oxidizer-rich staged combustion cycle scheme (in particular, in [3, 13, 14, 17, 23—25]). The results of studies of the POGO stability of these liquid rockets showed the development of POGO oscillations occurred during the operation of these main engines in the final interval of the LV flight time, characterized by relatively small values of the generalized mass of the lower modes of the LV structure. The use of liquid-propellant rocket engines with an oxidizer-rich staged combustion cycle as the LV first stage main engines can introduce a number of features into the study of the POGO stability of the launch vehicle. First of all, for this case, this is the low-frequency instability of the liquid propulsion system arising at frequencies associated with the low-frequency dynamics of the engine circuit of the turbopump — gas generator — gas duct (as shown in [5]) based on mathematical modeling and fire tests results of this type of engines at throttle modes. This phenomenon can lead to POGO oscillations due to this instability. Another distinctive feature of this type of liquid-propellant rocket engines is the presence of a significant maximum modulus of the engine dynamic gain (for example, see work [14]) at the low-frequency range (from 3 Hz up to 10 Hz). It can lead to the LV POGO instability even in the initial interval of the LV flight path. Providing the LV POGO stability by installing POGO suppressors can be significantly complicated by the cavitation oscillations in the feed system of the rocket engine, as follows from [15].

Fig. 1. The Cyclone-4M LV layout

Thus, the POGO instability of the LV for liquid rocket engines designed with an oxidizer-rich staged combustion cycle, as a physical phenomenon, can manifest itself in a unique form. In this regard, in order to ensure the POGO stability of the Cyclone-4M launch vehicle currently under development, the above-mentioned dynamic phenomena in systems of main rocket engines with an oxidizer-rich staged combustion cycle should be investigated first.

The work aims to determine the features of POGO oscillations in the Cyclone-4M LV being developed uses liquid-propellant rocket engines with an oxidizer-rich staged combustion cycle and to develop an approach to determining the parameters of the POGO suppressor system to ensure the POGO stability of liquid LVs.

The Cyclone-4M LV is being designed by the Yuzhnoye State Design Office to meet the growing demand for commercial missions to put on spacecraft in low Earth and solar-synchronous orbits [11]. It is the two-staged liquid rocket of the middle class with a tandem stage layout scheme (Fig. 1). The preliminary value of LV starting weight (without payload) is about 2597.6 kN. The design of the first stage of the Cyclone-4M LV was carried out using the results of the development of systems and assemblies of Zenit family liquid launch vehicles [11].

1. POGO STABILITY ANALYSIS OF THE CYCLONE-4M LAUNCH VEHICLE

Theoretical prediction of the liquid launch vehicle POGO stability is based on mathematical modeling of the interaction of the LV structure elastic longitudinal oscillations and low-frequency processes in the LV propulsion system and investigation of the stability of closed "propulsion system — LV structure" dynamic system.

The development of the mathematical model of the dynamical system "propulsion system — LV structure" was carried out in the low-frequency range, considerable for the problem under the study. The upper limit of this range was 50 Hz. Developing the mathematical model of the "propulsion system — LV structure" dynamical system, the aggregation principle [22] was used. At the same time, the feed lines of the propulsion system, the engine, and the LV structure with propellant tanks were the main components in the studied system. So, the developed mathematical model included three blocks of equations. The first block describes the low-frequency dynamic processes in the liquid rocket engine (LRE), the second block describes the fluid dynamics in the feed lines, and the third describes the longitudinal elastic oscillations of the LV structure. At the same time, the mathematical model of the LRE low-frequency dynamics was developed by the models of LRE aggregates and by the models of the hydraulic and gas connecting lines.

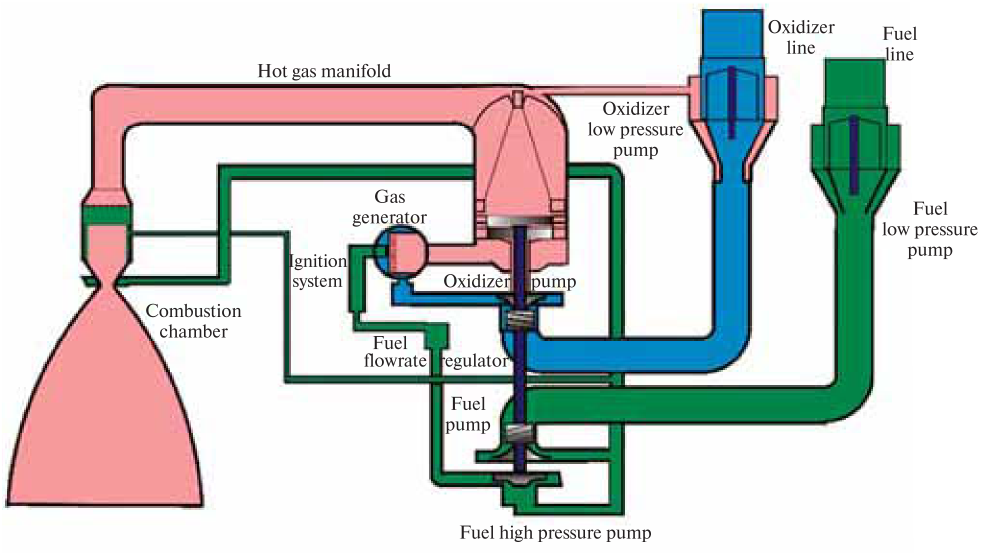

Fig. 2. Simple schematic of hydraulic and gas flowrates in the RD-870 LRE

1.1. Mathematical modeling of the low-frequency dynamics of LRE. Prediction of the dynamic characteristics of the main LRE is the central problem in the theoretical analysis of the LV POGO stability.

The first stage main LRE (RD-870) is designed according to the oxidizer-reach staged combustion cycle scheme with liquid oxygen and kerosene components. Figure 2 shows a simplified schematic of the engine liquid and gas flows. The thrust of the first stage RD-874 main liquid rocket propulsion engine system (LRPE) is 3178 kN near the Earth's surface and 3554 kN in the vacuum [11]. This LRPE consists of the four RD-870 engines.

The mathematical model of RD-870 LRE low-frequency dynamics is developed using theoretical advances in scientific works [7, 8, 14, 15, 22]. The model includes the dynamics equations of the main engine elements: liquid and gas lines, turbopumps (among other factors, the equations of cavitating oxidizer and fuel pumps' dynamics), the LRE gas generator, the LRE combustion chamber, the fuel flow regulator and other LRE elements. A system of linearized differential equations, describing the LRE low-frequency dynamics, is represented as follows [7]:

$$\sum_{i=1}^{n}[a_{\kappa i}\delta\dot{x}_i + b_{\kappa i}\delta x_i + c_{\kappa i}\delta x_i(t-\tau_{\kappa i})] = d_\kappa \delta y_\kappa , \quad (1)$$

$$\kappa = 1,\dots,\ n ,$$

where $\delta x_i, \delta y_\kappa$ are the deviations of engine parameters and forced actions; $a_{\kappa i}, b_{\kappa i}, c_{\kappa i}$ are the system coefficients, depending on the design and operating engine parameters; $\tau_{\kappa i}$ is the time delay ($\tau_{\kappa i} \neq 0$ only in the equations of the LRE gas lines low-frequency dynamics).

The engine dynamic properties are characterized by LRE amplitude-phase frequency characteristics, describing the steady-state harmonic forced oscillations of the system parameters occurring under the influence of external harmonic forcing. To determine the frequency responses of the investigated engine RD-870, the transformation to the form of the system (1) is made in the frequency domain. The resulting mathematical model is the system of algebraic equations with complex coefficients:

$$(j\omega a_{\kappa i} + b_{\kappa i} + c_{\kappa i}\,\mathrm{e}^{-j\omega\tau})\delta\overline{x}_i = d_\kappa \delta\overline{y}_\kappa . \quad (2)$$

The frequency responses are determined from the system (2) and represent the ratio of the complex amplitudes of the system parameters' oscillations to the complex amplitudes of the external forcing:

$$\frac{\delta\overline{x}_i}{\delta\overline{y}_\kappa} = W_i(j\omega) = \mathrm{Re}_i(\omega) + j\mathrm{Im}_i(\omega) = A_i(\omega)\,\mathrm{e}^{-j\varphi_i(\omega)} , \quad (3)$$

where $\mathrm{Re}_i(\omega)$ and $\mathrm{Im}_i(\omega)$ are the real and imaginary parts of the engine frequency response, $A_i(\omega)$ and $\varphi_j(\omega)$ are the amplitude and phase of the engine frequency response.

The main frequency responses of the RD-870 LRE necessary for analyzing the LV POGO stability are the input impedances and the engine pressure dynamic gains. They can be obtained based on the LRE calculated frequency responses (3).

The engine input impedances $Z_{in}^{ox}(j\omega)$ and $Z_{in}^{f}(j\omega)$ are the ratios of the complex amplitudes of pressure oscillations and a propellant flow rate at the LRE inlet in the oxidizer line and the fuel line correspondingly:

$$Z_{in}^{ox}(j\omega)=\frac{\delta\bar{p}_{in}^{ox}(j\omega)}{\delta\bar{G}_{in}^{ox}(j\omega)},\quad Z_{in}^{f}(j\omega)=\frac{\delta\bar{p}_{in}^{f}(j\omega)}{\delta\bar{G}_{in}^{f}(j\omega)}. \tag{4}$$

It sets the boundary conditions at the end of the feed lines of the oxidizer and fuel (from the engine side).

The rocket engine pressure dynamic gains in the oxidizer and the fuel channel ($W_{en}^{ox}(j\omega)$ and $W_{en}^{f}(j\omega)$) are the ratios of the complex amplitude of the pressure oscillations in the combustion chamber to the complex amplitude of the pressure oscillations at the inlet of the corresponding LRE propellant feed line:

$$W_{en}^{ox}(j\omega)=\frac{\delta\bar{p}_{cc}(j\omega)}{\delta\bar{p}_{in}^{ox}(j\omega)},\quad W_{en}^{f}(j\omega)=\frac{\delta\bar{p}_{cc}(j\omega)}{\delta\bar{p}_{in}^{f}(j\omega)}. \tag{5}$$

The dynamic gains of the first stage LV main engine RD-870 and the impedances at the inlet of the oxidizer and fuel low-pressure pumps were calculated taking into account cavitation phenomena in the pumps for various combinations of nominal and limiting (from the LRE operating range) pressure and temperature values at the engine inlet. The computation results — the input impedance modules and dynamic gains (by pressure) of the RD-870 engine at the LOX inlet and the fuel inlet of low-pressure pumps are presented in Fig. 3, 4. The numbers *1*, *2* indicate the curves for the following computation cases: curve *1* is a gain (or impedance) without taking into account pump cavitation; curve *2* is a gain (or impedance) taking into account pump cavitation at the nominal pressure and nominal temperature values.

From the analysis of the results, taking into account cavitation phenomena in the pumps led to a significant quantitative and qualitative changes in the input impedance modulus of the main LRE $Z_{in}^{ox}(j\omega)$ for the engine oxidizer channel. It is explained mainly by the effect of the cavitation cavities' elasticity in inducer pre-pumps (Fig. 3, *a*). The effect of pump cavitation formations on the resonant increasing of the RD-870 engine dynamic gain for the engine oxidizer channel $W_{en}^{ox}(j\omega)$ wasn't so significant as, for example, for a similar LRE with oxidizer-reach staged combustion scheme, given in [8]. For the LRE under research the calculated maximum of module $W_{en}^{ox}(j\omega)$ observed (see Fig. 3, *b*) at frequencies of 6—7 Hz (i. e. at oscillation frequency range less than

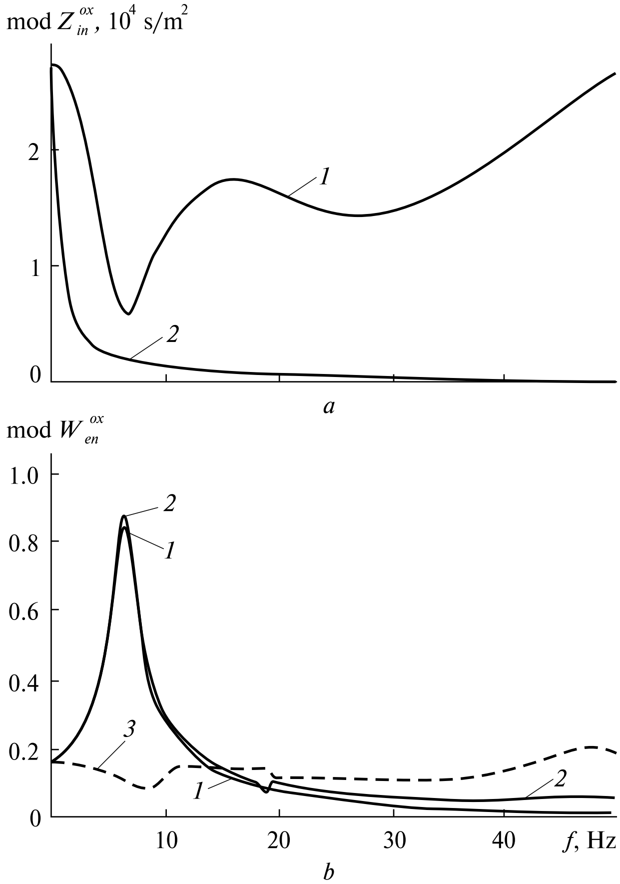

Fig. 3. Modules of the input impedance (*a*) and the pressure dynamic gains of the main LRE (*b*) for the oxidizer channel at LRE nominal pressure - temperature conditions (curve *1* is modules without taking into account pump cavitation, curve *2* is modules taking into account pump cavitation, curve *3* is module taking into account pump cavitation for an LRE gas-generator cycle engine)

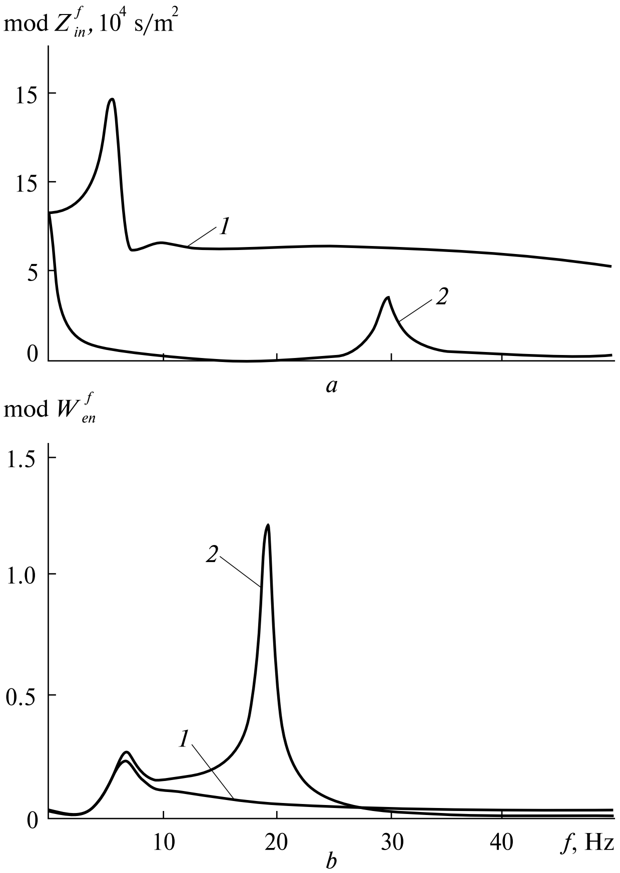

Fig. 4. Modules of the input impedance (*a*) and the pressure dynamic gain of the main LRE (*b*) for the engine fuel channel at the LRE nominal pressure - temperature conditions (curve *1* is modules without taking into account pump cavitation, curve *2* is modules taking into account pump cavitation)

10 Hz and dangerous to the self-oscillating POGO of launch vehicle) increased in 5 % (up to the value of 0.9).

At the same time, using the results presented in Fig. 4, it comes clear that taking into account cavitation phenomena in the low pressure and main fuel pump led to a five-fold increase in the modulus of the pressure dynamic gain of the LV first stage LRE for the engine fuel channel $W_{en}^f(j\omega)$ at an oscillation frequency of 18.0 Hz and to 7.0 % increase at an oscillation frequency of 7.0 Hz. In addition, taking into account cavitation led to a significant change in the input impedance module $Z_{in}^f(j\omega)$ of the main LRE at the fuel low-pressure pump inlet (Fig. 4, curve *1* corresponds to the computation case of without taking into account cavitation, curve *2* — to the computation case of taking into account cavitation for LRE nominal operating parameter values).

1.2. Mathematical modeling of cavitation phenomena in pumps. Taking into account cavitation phenomena in LRE pumps is fundamentally important for reliable prediction of the dynamic characteristics of LPRE and for predicting the LV POGO stability in general. It is known, e. g. from [10, 15], that cavitation in pumps with inducer leads to a noticeable decrease in the natural frequencies of fluid oscillations in the feed line compared to acoustic frequencies, and also leads to increase in the pressure dynamic gains of the LRE pumps during gain resonant increase (see Fig. 3, *b* and Fig. 4, *b*). As cavitation develops (in particular, with a decrease in pressure and flow rate at the inlet to the pump), the pump input impedance module decreases, and the pump dynamic gain modulus increases.

Accounting for cavitation phenomena in the low-pressure and main pumps of the RD-870 engine was developed according to the theory of cavitation oscillations in pump systems. This theory summarizes the results of numerous theoretical and experimental studies of cavitation oscillations in hydromechanical systems [15]. The basic input data for the numerical determination of the cavitation flow parameters in the flow part of the pumps are the geometric parameters of the pump — the inner and the outer diameter of the inducer, the angle of inducer blades, the number of blades, and the LRE operational parameters — pump rotation speed, flowrate coefficient, and pressure cavitation breakdown.

The basic equations in the mathematical model of the low-frequency dynamics of the cavitating pump according to [15] are follows:

- the equation of the dynamics of cavitation cavities in the flow part of inducer and centrifugal pump, obtained from the solution of the problem of non-steady-state cavitation flow past the flat plate cascade of inducer:

$$\delta P_1 = B_1 \delta P_1 + B_2 \delta G_1 + B_1 T_{cav} \frac{d\delta V_{cav}}{dt}, \qquad (6)$$

- equation of mass of fluid conservation in the flow part of the pump

$$\gamma \frac{d}{dt}(\delta V_{cav}) + \delta G_1 - \delta G_2 = 0, \qquad (7)$$

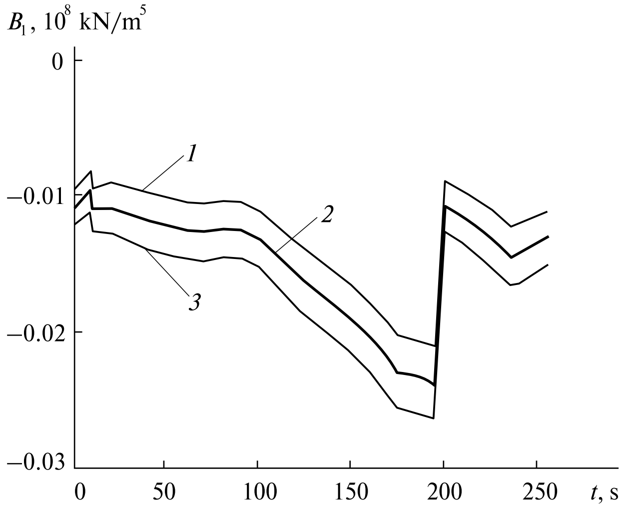

Fig. 5. Dependence of the elasticity B_1 cavitation cavities in the flow part of the oxidizer low-pressure pump on the LV flight for various temperature and pressure combinations at the pump inlet: at minimum pressure and maximum temperature (curve *1*), at nominal pressure and temperature (curve *2*); at maximum pressure and minimum temperature (curve *3*)

• equation for determining the pressure at the outlet of the pump

$$\delta P_2 = A_N \cdot \delta n + \varepsilon \delta V_K + s\,\delta G_2 - I_H \frac{d}{dt}(\delta G_2) + \delta P_1, \quad (8)$$

where B_1, B_2, T_{cav}, V_{cav} are the parameters of the inducer pump cavitation flow (cavitation elasticity, cavitation resistance, the time constant of cavitation cavities and the volume of cavitation cavities in the pump), n is the pump rotation speed, P_1, P_2, G_1, G_2 are the pressure and weight flow rate at the input and at the outlet of the pump, I_H is the inertial resistance factor of the inducer and centrifugal pump, S, A_N are partial derivatives of the pressure head of the flow rate and pump rotation speed at the operating point.

Comparison of the numerous results of experimental and theoretical investigations of pumps of various sizes [18] showed that for values of the flow coeffcient in the range of 0.3—0.8, the hydrodynamic model of cavitating inducer centrifugal pumps makes it possible to describe the LRE low-frequency dynamics and to determine the oscillations frequencies and stability boundaries of pumping systems with satisfactory accuracy. Note that the nominal value of the flow coefficient of the LRE oxidizer low-pressure pump is 0.534.

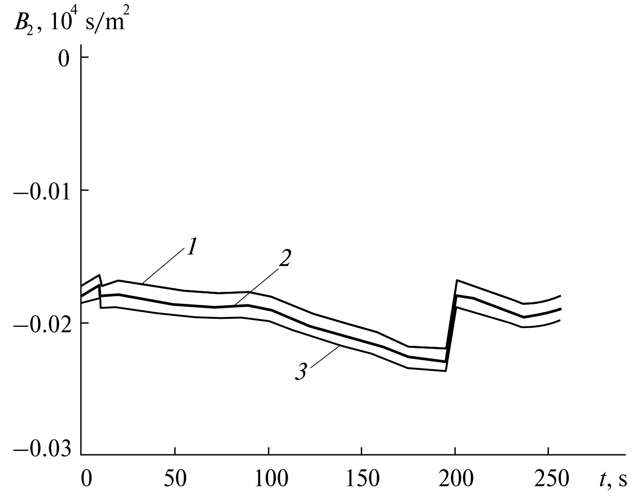

Fig. 6. Dependence of the negative cavitation resistance B_2 of the oxidizer low-pressure pump on the LV flight time for various temperature and pressure combinations at the pump inlet: at minimum pressure and maximum temperature (curve *1*), at nominal pressure and temperature (curve *2*); at maximum pressure and minimum temperature (curve *3*)

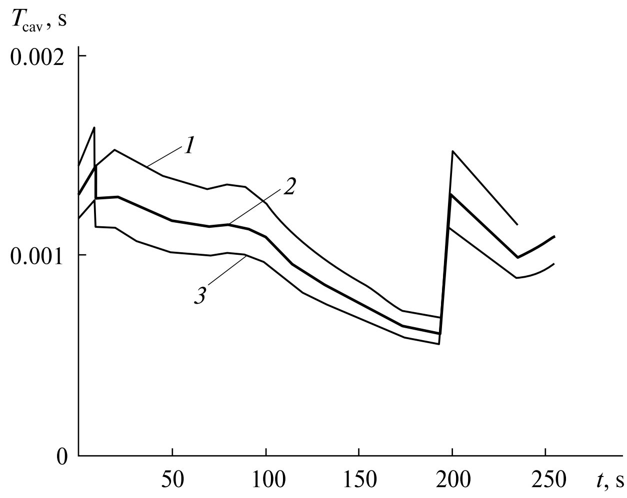

Fig. 7. Dependence of the cavitation time constant T_{cav} of the oxidizer low pressure pump on the LV flight time for various temperature and pressure combinations at the pump inlet: at minimum pressure and maximum temperature (curve *1*), at nominal pressure and temperature (curve *2*); at maximum pressure and minimum temperature (curve *3*)

In Fig. 5—8 as an example, the results of determining the cavitation flow parameters B_1, B_2, T_{cav}, V_{cav} for the oxidizer low-pressure pump are given, depending on the LV flight time. The indicated pa-

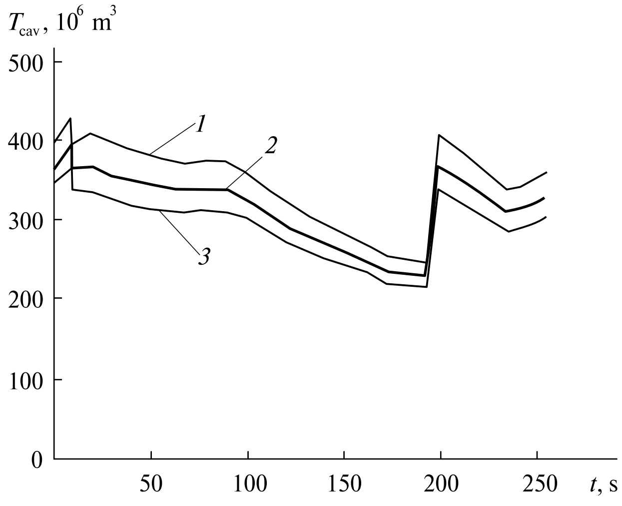

Fig. 8. Dependence of the total volume of cavitation cavities V_{cav} of the oxidizer low-pressure pump on the LV flight time for various temperature and pressure combinations at the pump inlet: at minimum pressure and maximum temperature (curve *1*), at nominal pressure and temperature (curve *2*); at maximum pressure and minimum temperature (curve *3*)

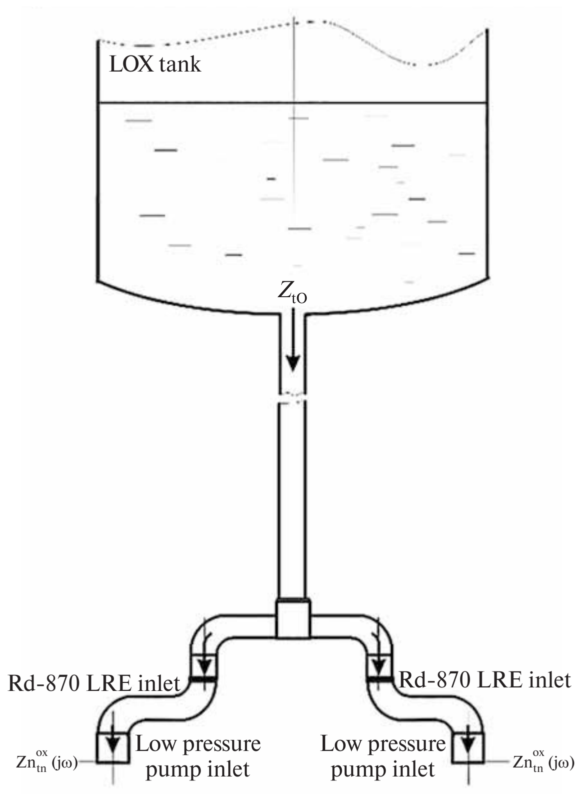

Fig. 9. Simple computation scheme of the oxidizer feed system of the LRE RD-874

rameters are computed for various combinations of the limiting (minimum and maximum) and nominal values of temperature and pressure of liquid oxygen from the LRE operating range. Curves *1*, *2*, *3* show the calculation results for the case of minimum pressure and maximum temperature, nominal pressure and temperature, and maximum pressure and minimum temperature accordingly.

The calculated values of the cavitation flow parameters in the oxidizer and fuel pumps were used to develop the system of equations (1). Based on the system of equations (1), the dynamic gain factors of the main LRE of the first stage Cyclone-4M LV (for the engine oxidizer and fuel lines) were determined. Besides, the input impedance values at the input to low-pressure pumps needed for the LV POGO stability analysis of the Cyclone-4M LV based on the Nyquist criterion were determined.

1.3. Mathematical modeling of low-frequency dynamics of the LPRE feed system. The feed system lines for the LV first stage LRE RD-874 are made with branched thin-walled pipe-lines of relatively large length (Fig. 9). Developing a mathematical model of the low-frequency dynamics of the "LPRE — LV structure" closed system, the feed lines of the oxidizer and fuel were considered as systems of distributed parameters, taking into account the fluid compressibility and the ductility of the feed lines walls.

The mathematical model of the fluid dynamics in the feed lines of the main LRE RD-870 included the equations of unsteady fluid motion and the continuity equations [10]. Taking into account the acceleration of the LV structure during its flight, this mathematical model had the following form:

$$\begin{cases} \dfrac{\partial p}{\partial z} + \dfrac{1}{g \cdot F} \cdot \dfrac{\partial G}{\partial t} + \dfrac{k}{g \cdot F} \cdot G = \chi \ddot{q} \dfrac{\gamma_{lq}}{g} \cos\alpha, \\ \dfrac{\partial G}{\partial z} + \dfrac{g \cdot F}{c^2} \cdot \dfrac{\partial p}{\partial t} = 0, \end{cases} \quad (9)$$

where p, G are pressure and weight flow rate of the liquid, t is time, z is the axial (longitudinal) coordinate of the pipe-line axis, F is the area of the pipe-line cross section, k is a reduced factor of linear friction per unit length of the line length l, $\overline{G}$ is weight fluid flow rate at steady operation, g is a gravity acceleration, c is the sound speed in a liquid in a pipeline with elastic walls, q is the generalized coordinate in the equation of longitudinal oscillations of the studied longitudinal mode of the rocket structure, χ is a coefficient of the natural shape of the rocket structure longitudinal oscillations in the section of the pipe-line bracing, $\chi\ddot{q}$ is an acceleration of the pipe-line section along the longitudinal axis of the rocket, γ_{lq} is the liquid specific weight, α is the angle between the rocket longitudinal axis and the symmetry axis z of the feed line section.

In accordance with the approach [4], generalizing the impedance method [15] in the case of multidimensional and distributed external forces, a moving fluid in each section of the pipe-line was considered as an active quadripole.

For the rocket POGO stability analysis, the values of the resonant frequencies of fluid oscillations in the LRE feed lines (indicated as the oxidizer feed lines' low-frequency modes f_{ox1}, f_{ox2} and the fuel feed lines' low-frequency modes f_{f1}, f_{f2}) and the change in their values during the flight time according to the pressure and temperature change of the propellant at the engine inlet and the LRE operation mode in thrust (pressure in the combustion chamber) are of the primary interest. The dependences of these oscillation frequencies on the LV flight time are presented in Fig. 10. These dependences are obtained from (9) according to the results of calculations of the dynamic gains $W_{fl}^{ox}(j\omega)$ and $W_{fl}^{fuel}(j\omega)$ for the oxidizer and fuel feed lines of the LV first stage.

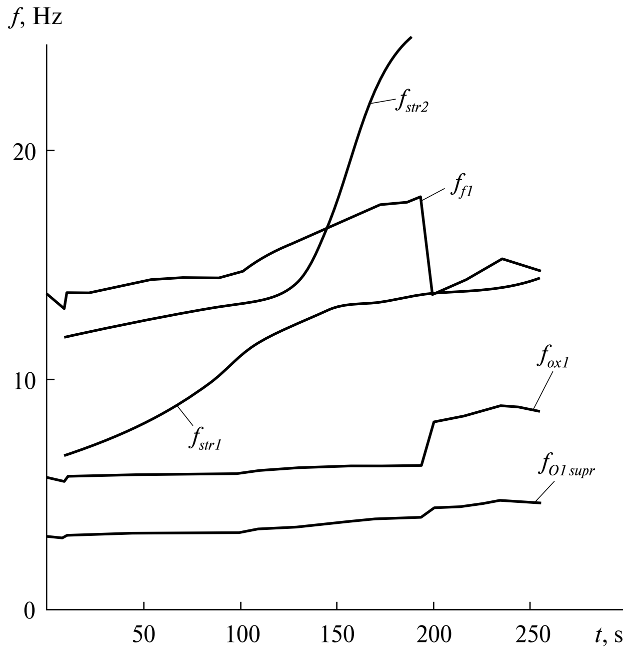

Fig. 10. Calculated dependences of the first and second modes of natural frequencies (f_{str1}, f_{str2}) of the Cyclone-4M LV structure longitudinal vibrations, the resonant frequencies (f_{ox1} and f_{o1supr}) of liquid oscillations in the first stage LV oxidizer feed system for the case without POGO suppressor in the feed line and for the case with POGO suppressor in the feed line, and resonant frequency in the fuel feed system f_{f1} on flight time t

1.4. Mathematical modeling of the Cyclone-4M LV structure low-frequency dynamics. For mathematical modeling of Cyclone-4M LV structure free oscillations, the one-dimensional oscillatory motion was considered only along the longitudinal axis. The estimated scheme of free longitudinal oscillation of the LV structure was built based on LV structural layout (see Fig. 1).

In the developed Cyclone-4M LV computation scheme, the LV structure is represented by an elastic thin-walled rod of the variable cross-section for a geometrically unchanged contour and free ends. There are elastically connected concentrated masses (oscillators) simulating oscillations of individual structural elements of the LV structure and liquid propellant in the tanks on the longitudinal axis of this rod in its various sections. The oscillations of each oscillator are a mechanical analog of the corresponding mode of fluid oscillations in the elastic tank or engine elastic oscillations (more precisely, the "elastic frame and LRE" subsystems) for rocket structure longitudinal oscillations. The LRE masses and liquids in the propellant tanks of the LV first stage were taken into account in the computation scheme as the masses of the LV individual structural elements.

The longitudinal displacements of the main first stage LRE relative to the plane of engine bracing to the LV structure were presented in the computation scheme as concentrated mass displacements on elasticity, simulating engine longitudinal oscillations for the fundamental frequency of the oscillation "elastic frame — LRE" subsystem. This elasticity was attached to the rod element. The axial coordinate of this rod element corresponds to the plane of engine bracing to the LV structure. The longitudinal oscillations of the LV second stage propulsion system were modeled in the same way.

The two lower (dominant) modes of the longitudinal oscillations of liquid in the LV first stage tanks (oxidizer and fuel) were taken into account in the computation scheme of the Cyclone-4M LV structure. The masses and frequencies values of the attached equivalent oscillators simulating the liquid longitudinal oscillations in the LV first stage tanks (with the 1st and 2nd modes frequencies) were calculated by modeling the free longitudinal oscillations of the liquid in the cylindrical tanks. The modeling was carried out for several levels of tank filling corresponding to different moments of the Cyclone-4M LV flight time during the operation of the LV first stage propulsion system. In this case, the liquid surface wave oscillations were not taken into account.

The calculated dependences of the frequencies f_{str1}, f_{str2} of the lower two modes of the LV structure longitudinal oscillations as a function of the active LV flight time during the operation of first stage main engines are shown in Fig. 10. Here, for comparison, the time dependences of the calculated frequencies (at the oxidizer line f_{ox1} and fuel line f_{f1}) of liquid oscillations in the feed system of the Cyclone-4M LV first stage propulsion system are shown.

The frequency responses of the Cyclone-4M LV structure were determined based on the equations of structure forced elastic longitudinal oscillations in deviations for generalized normal coordinates $q_i(t)$:

$$\delta\ddot{q}_i + \frac{\Delta_i\omega_i}{\pi}\delta\dot{q}_i + \omega_i^2\delta q_i = \frac{\beta_{D1i}\,\bar{R}}{m_{pi}\,\bar{p}_{cc}}\delta p_{cc}$$

$$(i = 1, \ldots, n), \qquad (10)$$

where q_i is the generalized coordinate of the i mode of LV structure elastic longitudinal oscillations, ω_i, Δ_i are angular (circular) frequency and decrement of the i mode of LV structure natural oscillations, β_{D1i} is the shape factor of the i mode of natural longitudinal oscillations of LV structure at the point of the first stage LRE mounting, m_{pi} is the i mode reduced mass of the LV structure longitudinal oscillations, R is the nominal value of the propulsion thrust of the LV first stage (it is supposed that the engine thrust amplitudes are directly proportional to the amplitudes of the pressure oscillations in the LRE combustion chamber [10]), n_T is the modes' number of LV structure natural longitudinal oscillations, taken into account in the mathematical model of the closed "LRE — LV structure" dynamic system, δ is a symbol indicating a small deviation of the parameter from its steady-state value.

The LV structure frequency responses were computed as the ratio of the complex amplitude of the LV structure generalized coordinate oscillations to the complex amplitude of the pressure oscillations in the LRE combustion chamber:

$$W_{str}(j\omega) = \frac{\delta q_i}{\delta p_{cc}}(j\omega) =$$

$$= \frac{\beta_{D1i}\,\bar{R}}{m_{pi}\,\bar{p}_{cc}\left(-\omega^2 + j\omega\cdot\dfrac{\Delta_i\omega_i}{\pi} + \omega_i^2\right)}, \qquad (11)$$

ω is the current value of the angular (circular) frequency changing from 0 to $\omega_{\max}$, j is the imaginary unit (square root of –1).

1.5. The results of the Cyclone-4M LV POGO stability analysis. The Cyclone-4M LV POGO stability analysis was developed based on a study of the closed "LRE — LV structure" dynamic system using the Nyquist stability criteria in the form given in [10]. The open system frequency responses $W_{ol}(j\omega)$ were calculated for the LV flight time interval during operation of the LV first stage LRE for various values of system parameters: spacecraft masses (from the range of its design values 500 kg up to 5000 kg), operational parameters of the first stage main LRE — pressures and temperatures of the propellants at the inlet to the main LRE RD-870 (for different combinations of their nominal, minimum and maximum values). The frequency response $W_{ol}(j\omega)$ of the open "LRE and LV structure" system was calculated as the product of the corresponding open subsystem frequency responses along the oxidizer and fuel lines:

$$W_{ol}(j\omega) = W_{ol}^{ox}(j\omega) + W_{ol}^{fuel}(j\omega), \qquad (12)$$

where $W_{ol}^{ox}(j\omega)$ and $W_{ol}^{fuel}(j\omega)$ are the frequency responses of the open system along the oxidizer and fuel lines.

The oxidizer line and the fuel line were investigated in the open "LRE — LV structure" system as dynamic subsystems consisting of three connected links: the LV structure, feed line, and engine.

In summary

$$W_{ol}^{ox}(j\omega) = W_{str}(j\omega)\cdot W_{fl}^{ox}(j\omega)\cdot W_{en}^{ox}(j\omega), \quad (13)$$

$$W_{ol}^{fuel}(j\omega) = W_{str}(j\omega)\cdot W_{fl}^{fuel}(j\omega)\cdot W_{en}^{fuel}(j\omega), \quad (14)$$

where $W_{fl}^{ox}(j\omega)$ and $W_{fl}^{fuel}(j\omega)$ are the dynamic gains of the oxidizer and fuel feed lines by the external forces; $W_{en}^{ox}(j\omega)$ are the engine pressure dynamic gains along the oxidizer and fuel lines.

The POGO stability of the dynamic "LRE — LV structure" system was computed from the conditions of system amplitude stabilization [10]:

$$\text{mod } W_{ol}(j\omega) < 1, \; 0 < \omega < \omega_{\max}. \quad (15)$$

The open system frequency responses $W_{ol}(j\omega)$ were calculated separately for each mode of oscillations of the LV structure for a minimum value of LV structure oscillations decrement. When investigating the LV stability, the cases of separate and combined accounting of the LRE oxidizer and fuel part were analyzed in the mathematical model of the dynamic "LRE — LV structure" system.

Analysis of the computation results showed that for the studied "LRE — LV structure (I mode)" dynamic system, condition (15) don't satisfy. As it is shown in Fig. 11, 12, the $W_{ol}(j\omega_{str1})$ frequency response modules mod of the open "LRE — LV structure (I mode)" system in the range of the LV flight time (5 s, 70 s) significantly exceeds unity, reaching a maximum value of 3.4. In Fig. 11, sub numbers *1—3* show the results obtained for various combinations of propellant (LOX) pressure and temperature at the LRE inlet (*1* is minimum pressure and maximum temperature, *2* is nominal pressure and temperature, *3* is maximum pressure and minimum temperature).

The nonstability of the dynamic "LRE — LV structure (I mode)" system in the range (5 s, 70 s) of the LV flight time is, first of all, the result of the resonant dynamic interaction of the LV structure and the LRE oxidizer feed system. It happens for close values of the oscillations first mode frequency f_{ox1} of the liquid in the LRE oxidizer feed line and the 1st mode natural frequency f_{str1} of the LV structure longitudinal oscillations. The first mode frequency f_{ox1} of the fluid oscillations in the LRE oxidizer feed line in this interval of flight time varies in the range from 6.0 Hz to 8.0 Hz, and the frequency f_{str1} — from

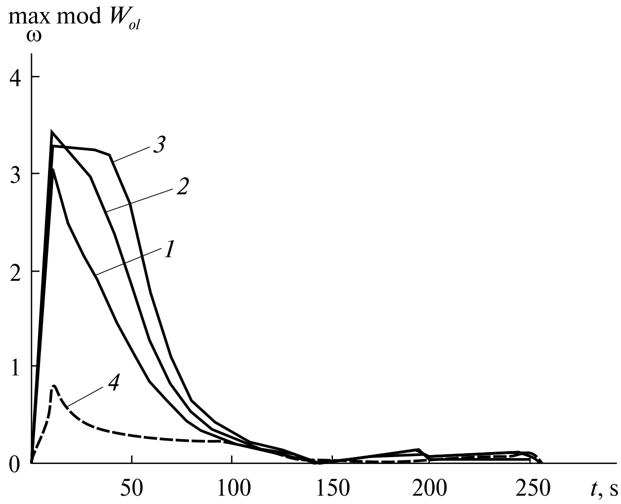

Fig. 11. Dependences of the maximum value of frequency response module of the open "LRE — LV structure (I mode)" system on the LV flight time t (*1* is a case of minimum pressure and maximum temperature, *2* is a case of nominal pressure and temperature, *3* is a case of maximum pressure and minimum temperature, *4* is a case of the LRE with the gas-generator cycle at nominal pressure and temperature)

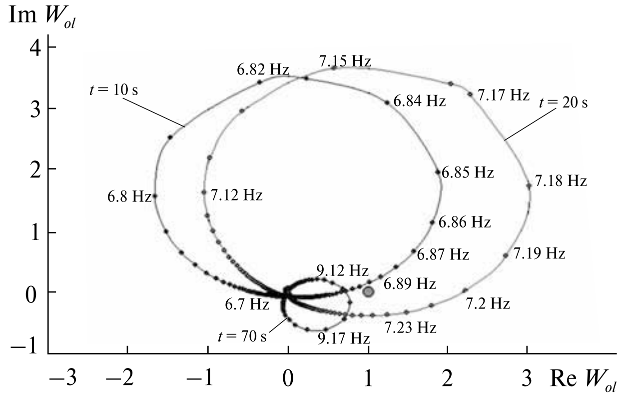

Fig. 12. Nyquist diagrams hodographs $W_{ol}(j\omega)$ of the open "LRE — LV structure (I mode)" system for t = 10 s; 20 s; 70 s of flight times

6.75 Hz to 9.1 Hz (see Fig. 10). At the same time, the dependency curves $f_{ox1}(t)$ and $f_{str1}(t)$ are close, but not intersecting.

So, the dynamic interaction of the oxidizer feed line and the LV structure in the closed "LRE — LV structure (I mode)" system occurs in conditions of internal resonance. It leads to a critical increase (up to 3 or more units) of the frequency response module

$\mathrm{mod}\, W_{ol}(j\omega_{str1})$ of the open "LRE — LV structure (I mode)" system at the LV structure natural circular frequency ω_{str1}. Such a significant increase in $\max_{\omega} \mathrm{mod}\, W_{ol}(j\omega_{str1})$ is caused not only by the convergence of the first oscillation frequency of the liquid in the oxidizer feed line and the first mode natural frequency of the Cyclone-4M LV structure longitudinal oscillations. It is also caused by the very significant value of dynamic gain of the engine RD-870 for oxidizer reach staged combustion scheme at the frequency range from 6 Hz to 9 Hz (see Fig. 3, *b*). It leads to POGO instability in the initial interval of the Cyclone-4M LV flight time (5 s, 70 s).

It was found by calculation that the dynamics of the "turbopump — gas generator — gas path" circuit significantly affects the dynamic gain of the RD-870 engine, developed according to the with oxidizer-rich staged combustion cycle scheme, and the POGO stability of the investigated "LRE — LV structure" system. So, at the frequency range from 6 Hz to 9 Hz, the modulus of the dynamic gain $\mathrm{mod}\, W_{en}^{ox}(j\omega)$ of the RD-870 engine (Fig. 3, *b*, the curves *1*, *2*) significantly (at least in 5 times) exceeds the values of a similar dynamic characteristic computed without taking into account the feedback between the pressure at the oxidizer pump inlet and the pressure in the LRE gas generator, that is realized in the "turbopump — gas generator — gas path" circuit through the turbopump shaft speed (Fig. 3, *b*, the curve *3*). Time delays in the equations of the dynamics of LRE gas paths play an important role in the implementation of this feedback [8]. Note that in rocket engines without oxidizer-rich staged combustion cycle, this feedback in the above low-frequency range is practically absent, and the graph of the dynamic gain modulus of engines of such combustion cycle usually has a form similar to curve *3*.

Fig. 11 shows the results of computation of the modulus of the dynamic gain $\mathrm{mod}\, W_{ol}(j\omega_{str1})$ of the open-loop system taking into account the feedback between the pressure at the LRE oxidizer pump inlet and the pressure in the gas generator (curve *2*) and without taking it into account (curve *4*). A significant decrease in the values of the module dynamic gain $\mathrm{mod}\, W_{ol}(j\omega_{str1})$ of the RD-870 engine, calculated without taking into account this feedback, led to a significant decrease (to 0.84) in the frequency response module $\mathrm{mod}\, W_{ol}(j\omega_{str1})$ of the open-loop "LRE — LV structure (I mode)" dynamic system. It resulted in the stabilization of the system in the investigated closed-loop dynamic system "LRE — LV structure (I mode)" (see Fig. 11, curve *4*). This pattern of the POGO-instability development was discovered for the first time, and it can be noted as a characteristic feature of this dynamic phenomenon for rocket engines with an oxidizer-rich staged combustion cycle.

The computation results showed that the dynamic interaction of the LV structure and the LRE fuel feed line occurs for resonance conditions due to the convergence (coincidence) of the fluid oscillations first mode frequency in the LRE fuel line with the longitudinal oscillations LV structure first — third mode natural frequencies (in particular, at t = 130 s and t = 200 s). The dynamic "LRE — LV structure" system is stabilized for the LV second and third modes POGO oscillations and also has satisfactory stability margins. Moreover, the maximum values of the frequency response $\mathrm{mod}\, W_{ol}(j\omega_{str1})$ modulus of the open system are less than 0.66.

Finally, it was computed that the Cyclone-4M LV loses POGO stability at the structure oscillations first mode in the flight time interval (5 s, 70 s) during operation of the LV first stage main LRE. To ensure the POGO stability of the Cyclone-4M LV, it is necessary to correct the dynamic characteristics of the oxidizer feed line. It can be achieved by mounting POGO suppressors at the inlet to each main LRE. For this LV, mounting of POGO suppressors at the LRE fuel feed lines is not required.

2. THE CHOICE OF THE POGO SUPPRESSOR RATIONAL PARAMETERS FOR PROVIDING THE CYCLONE-4M LV POGO STABILITY

Native and foreign experience in providing the liquid rockets POGO stability has shown that the simplest and most effective way to LV POGO stability is changing the dynamic characteristics of the LRE feed lines by decreasing the liquid propellant oscillations resonant frequency in the LRE feed line [10, 12]. This is achieved by mounting special devices in the LRE feed line — POGO suppressors.

Decreasing the resonant frequency of liquid propellant oscillations in the LRE feed line is provided by the presence of elastic elements in the suppressors (for example, gas cavities or springs). Suppressors' elements contribute to the liquid propellant oscillation energy dissipation [10, 13]. Depending on the LRE schematic and the propellant physical properties, the POGO suppressor can be selected with or without separation of gas and liquid phase. Moreover, various separation elements (pistons, bellows, plastic, and rubber shells) can be used in the suppressor design for the separation of gas and liquid phase [2, 10, 12, 13, 19]. Choosing suppressor design for suppressing Cyclone-4M LV POGO vibrations, the suppressor without separation of gas and liquid phase was considered as one of the main technical solutions. This technical solution has shown its effectiveness in providing the POGO stability of the Zenit LV developed by Yuzhnoye State Design Office.

However, designing a POGO suppressor without a gas and liquid phases separator, it should be taken into account that the specified suppressor must be designed in such a way as to exclude any possibility of "transfer" of a certain volume of suppressor gas filling to the LRE feed line during transients in the LRE feed system. It may break normal engine operation. This requirement imposes some limitations on the geometry of such suppressor and its mounting in the LRE feed system. At the same time, the possibility of maintaining a constant liquid level using a special regulation system should be realized in the suppressor on the LV flight active part. It greatly complicates suppressor development as a part of the LRE.

In a gas POGO suppressor with bellow separation of gas and liquid phases, this problem is naturally excluded. Such suppressor was used in the USSR on R-7 rocket, A30 (35) rocket [2]. In addition, as a result of theoretical investigations [19] of providing the Antares LV POGO stability performed by ITM NASU and GKAU, Yuzhnoye State Design Office, and Orbital Science Corporation, it was decided to mount the gas suppressor with a bellow separation of gas and liquid phases in the oxidizer feed system. The suppressor effectiveness was subsequently confirmed by the launches of this LV.

The choice of the conceptual scheme and rational parameters of the POGO suppressor as a tool of providing the Cyclone-4M LV POGO stability was made for both of these devices.

When developing a mathematical model of low-frequency dynamic processes in POGO suppressor, the following assumptions were made: the working medium in the suppressor gas cavity is an ideal gas; liquid oxygen located inside the bellows and communicating with the feed line does not contain gas-vapor inclusions (i. e., the internal cavity of the bellows is completely filled with liquid oxygen at the main engine start-up); heat exchange with the environment through the suppressor structure walls was not taken into account in the calculations.

Mathematical modeling of the POGO suppressor oscillation was performed in one-dimensional and three-dimensional form using computer finite element analysis [9].

To determine the compliance $\tilde{N}_d$ of the suppressor gas cavity, the state gas equation was used

$$C_d = \frac{\gamma_o \bar{V}_g}{\chi \bar{p}_g}, \qquad (16)$$

where $\bar{V}_g, \bar{p}_g$ are the volume and pressure in the suppressor gas cavity in steady state, χ is adiabatic index (factor), γ_0 is oxidizing specific weight.

Based on the developed one-dimensional and three-dimensional mathematical models of the POGO suppressor dynamics with a bellows separation of phases, the frequency responses necessary to perform an LV POGO stability analysis is determined. It is the POGO suppressor input hydraulic impedance. It can be represented in a simple form:

$$Z_d(j\omega) = \frac{\delta P_t}{\delta G_d}(j\omega) =$$

$$= R_d + j\omega J_d + \frac{k_{sf}}{j\omega\gamma_o F_{sf}^2} + \frac{1}{j\omega C_d}, \qquad (17)$$

where ω is the angular (circular) oscillations frequency, G_d is weight flowrate at the POGO suppressor inlet, R_d, J_d are linearized hydraulic and inertial resistance coefficients, F_{sf} is the effective bellow area. The symbol $\delta(*)$ in (17) marks the deviations of the parameter (*) from its steady-state value.

For the gas-liquid POGO suppressor with a free boundary between liquid and gas phase, the expression (17) for the input impedance of this type of sup-

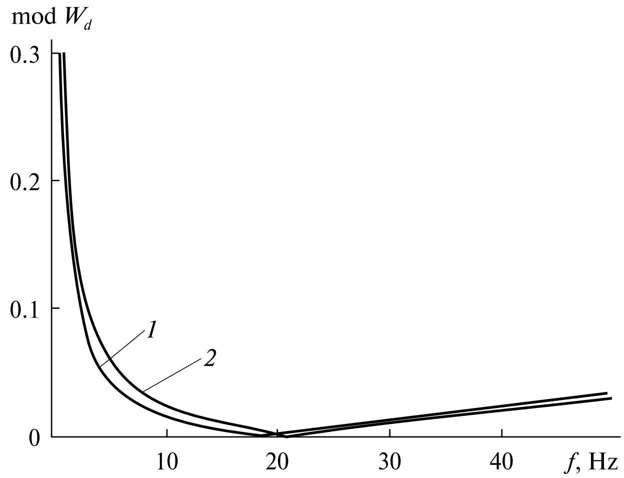

Fig. 13. Dependences of the POGO suppressor input impedance module for the case without bellows liquid and gas separation (curve *1*) and for the case with bellows liquid and gas separation (curve *2*) on the oscillation frequency *f*

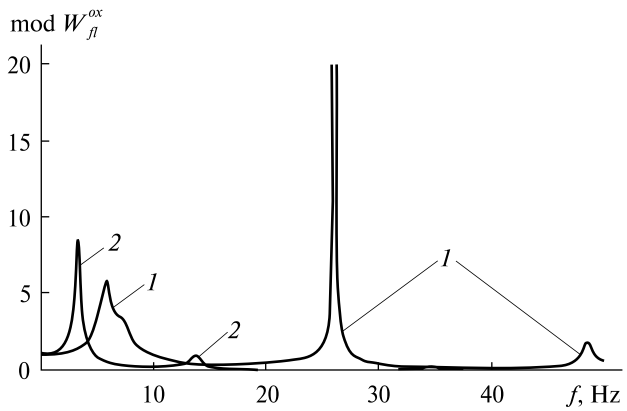

Fig. 14. Dependences of the dynamic gain module of the LRE oxidizer feed system on the oscillation frequency: *1* is a calculation case without POGO suppressor, *2* is a computation case with POGO suppressor for nominal pressure and nominal temperature from the LRE operating range at the LRE inlet

pressor has the form:

$$Z_d(j\omega) = \frac{\delta P_t}{\delta G_d}(j\omega) = R_d + j\omega J_d + \frac{1}{j\omega c_d}, \quad (18)$$

Fig. 13 shows the dependences of the input hydraulic impedance module $Z_d(j\omega)$ of the POGO suppressor without bellows separation of phases (curve *1*) and with bellows separation of phases (curve *2*) on the oscillation frequency *f*, calculated for POGO suppressors with a gas cavity volume $\bar{V}_g = 7.4 \times 10^{-3}$ m^3 at the suppressor working pressure of liquid oxygen $\bar{p}_g = 5$ bar.

The analysis of the effectiveness of LV POGO stability providing for considered suppressor design versions showed the gas-liquid suppressor without bellows separation of phases is a bit more effective than the POGO suppressor with bellows separation of gas and liquid phases (for identical volumes of their gas cavities). At the same time, the bellows-type suppressor has an important advantage — it “prevents” gas leakage from the POGO suppressor to the main engine inlet during transients at the LV flight time. Finally, further investigations and assessment of the POGO suppressor rational design parameters were developed for a bellows-type POGO suppressor with gas-liquid separation.

The stability analysis of the dynamic “LRE with POGO suppressor — LV structure” system was made by varying the POGO suppressor design parameters — the gas cavity volume, the bellows stiffness, and the liquid cavity geometric dimensions.

The approach to determining the rational parameters of the POGO suppressor system to ensure the POGO stability of liquid LVs was developed. For the first time a rational choice of the design parameters of POGO suppressors was carried out from the conditions of a minimum of the module of the frequency response $W_{ol}(j\omega)$ for reliable amplitude stabilization of the open “LRE with POGO suppressor — LV structure” dynamic system, and not on the basis of the required removal of the first natural frequency of the oscillations of the LRE oxidizer feed system from the lowest natural oscillation frequency of the LV structure. Determining a rational variant of the suppressor parameters, the possibilities of its mounting were taken into account, limited by the geometry and features of the layout schematic of the Cyclone-4M LV first stage feed system. Finally, taking into account these limitations, it is most preferable to mount a POGO suppressor on almost horizontal branches of the oxidizer main pipeline leading to the engines.

It is shown the mounting of gas-liquid suppressors with a bellows separation of phases (for selected rational design parameters) in the branches of the oxidizer main pipeline leading to the RD-870 engines provides the POGO stability with satisfactory stability margins during Cyclone-4M LV flight time. In this case, the maximum modulus of the frequency responses of the open "LRE with POGO suppressor — LV structure (I mode)" system does not exceed 0.43, which is significantly less than the critical value (see (15)), i. e., the conditions for the amplitude stabilization of the investigated "LRE with POGO suppressor — LV structure (I mode)" closed system will be performed. The indicated dependence of the module $W_{ol}(j\omega_{K1})$ on the LV first stage flight time is shown in Fig. 15 (curve *1*).

The POGO stability of the Cyclone-4M LV is achieved as a result of a significant change in the LRE feed system dynamic gain $W_{fl}^{ox}(j\omega)$ (for the oxidizer pressure channel) by mounting the POGO suppressor with rational parameters (as it is shown in Fig. 14).

For the system without the POGO suppressor the maximum gain of the oxidizer feed line is realized at a frequency of 6 Hz (the curve *1* in Fig. 14), for the system with the POGO suppressor — at a frequency of 3.3 Hz (curves *2* correspond to a computation case with the POGO suppressor in the LRE feed at nominal pressure and nominal temperature of the liquid oxidizer at the LRE inlet from the operating range of these parameters). The change in the lowest oscillation frequency of the liquid oxidizer in the LRE feed line during the LV flight time for the selected variant of the POGO suppressor and the variant without mounting the POGO suppressor can be compared by the dependences, shown in Fig. 10 (the curves marked as f_{o1supr} and f_{ox1} respectively).

Furthermore, it was shown (see Fig. 15, curve *2*) in the case of mounting the POGO suppressor directly at the inlet to the engine low pressure pumps the LV POGO stability margins can be a bit increased.

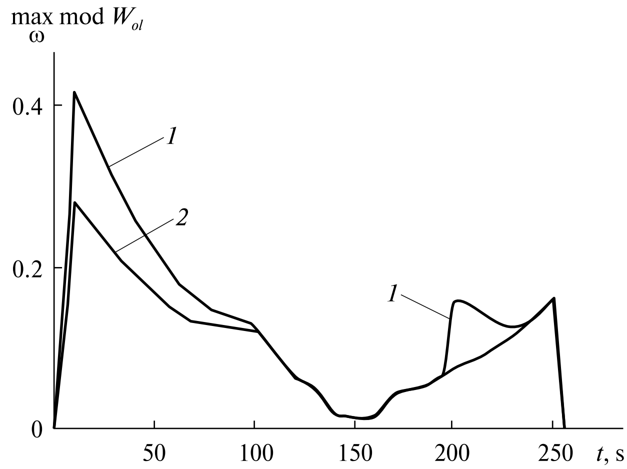

Fig. 15. Dependences of the maximum value of the frequency response module of the open "LRE and LV structure (I mode)" system on flight time *t* (*1* is a case of the POGO suppressor in the branches leading to the engines, *2* is a case of the POGO suppressor mounting at the inlet to the low-pressure pump)

CONCLUSION

The mathematical model of the POGO oscillations of the Cyclone-4M LV in the active flight during operation of the first stage propulsion system RD-874 (include four RD-870 engines designed according to the scheme with oxidizer-rich staged combustion cycle) is developed.

As a result of a theoretical analysis of the POGO stability of the Cyclone-4M LV using the Nyquist criterion, it was found the "LRE — LV structure" dynamic system is unstable with respect to the first mode of the LV structure longitudinal vibrations at the initial flight time interval (5 s, 70 s) due to the dynamic interaction of the oxidizer feed line and the LV structure under conditions of internal resonance. This resonance is caused not only by the convergence of the first oscillation frequency of the liquid in the feed line of the oxidizer and the natural frequency of the first mode of the longitudinal vibrations of the Cyclone-4M LV structure but also by a significant increase in the frequency range from 6 Hz to 9 Hz of the dynamic gain of the RD-870 engine along the LRE oxidizer pressure channel. It leads to POGO instability of the Cyclone-4M LV in the initial interval of the flight time. This pattern of the POGO-instability development was discovered for the first time, and it can be noted as a characteristic feature of this dynamic phenomenon for rocket engines with an oxidizer-rich staged combustion cycle.

The approach to determining the parameters of the POGO suppressor system to ensure the POGO stability of liquid LVs was developed. For the first time a rational choice of the design parameters of POGO suppressors was carried out from the conditions of the amplitude stabilization of the "LRE with POGO suppressors — LV structure" open dynamic system, and not on the basis of the required removal of the first natural frequency of the oscillations of the LRE feed system from the lowest natural oscillation frequency of the LV structure.

It is shown that the installation of POGO suppressors designed with a bellows separation of gas-liquid media in the feed branches from the oxidizer main pipeline to the engines, ensures the POGO stability of the Cyclone-4M LV with satisfactory stability margins during all flight not only with the nominal values of the operational parameters of the rocket engine but also with various combinations of limiting values of pressure and temperature at the inlet to the engines. At the same time, for all the options for choosing the values of the operational parameters of the liquid propellant rocket engine, the conditions for the amplitude stabilization of the studied open "liquid propellant rocket engine with POGO suppressor — LV structure" dynamic system are satisfied.

REFERENCES

1. About G., Hauguel N., Hrisafovic N., Lemoine J. C. (1983). La prevention des instabilites POGO Sur Ariane 1. *Acta Astronautica*, **10**, No. 4, 179—188.
2. Balakirev Yu. G. (2014). Solving the problem of POGO oscillations in flight of Soviet liquid rockets: achievements and failures. *Cosmonautics and Rocket Engineering*, Part 1, No. 6 (79), 185—191 [in Russian].
3. Dotson K. (2003). Mitigating Pogo on Liquid-Fueled Rockets. Crosslink. *Aerospace Corporation magazine of advances in aerospace technology,* 26—29.
4. Fomenko P. V. (1989). Method of transferring boundary conditions for determining the transfer functions of hydraulic systems from distributed external effects. *Applied problems of hydrodynamics and heat and mass transfer in power plants.* Kyiv: Science Dumka, 134—140 [in Russian].
5. Gemranova E. A., Kolbassenkov A. I., Koshelev I. M., Levochkin P. S., Martirosov D. S. (2013). Ways to suppress low-frequency oscillations in the LRE on deep throttling modes. *NPO Energomash named after academician V. P. Glushko*, No. 30, 104—110.
6. Gladky V. F. (1969). *Dynamics of the launch vehicle structure*. Moscow: Nauka, 496 p. [in Russian].
7. Glikman B. F. (1989). *Automatic regulation of liquid rocket engines.* Moscow: Mechanical Engineering, 296 p. [in Russian].
8. Khoriak N. V., Nikolayev O. D. (2007). Decomposition and stability analysis of the dynamic system "feed lines — mid-range rocket engine with oxidizer-rich staged combustion". *Technical mechanics*, No. 1, 28—42 [in Russian].
9. Kohnke P. (2001). *Ansys, Inc. Theory Manual 001369.* Twelfth Edition. Canonsburg: SAS IP, Inc. 1266 p.
10. Natanzon M. S. (1977). *POGO self-oscillations of a liquid rocket.* Moscow: Mechanical Engineering, 208 p. [in Russian].
11. Official site of the Yuzhnoye State Design Office. URL: https://www.yuzhnoye.com/home/ (Last accessed: 10.10.2019).
12. Oppenheim B. W., Rubin S. (1993). Advanced Pogo Stability Analysis for Liquid Rockets. *J. Spacecraft and Rockets,* **30**, No. 3, 360—383.
13. Preventing POGO on Titan IVB (2003). Crosslink. *The Aerospace Corporation magazine of advances in aerospace technology. Summer*, 3—12.
14. Pylypenko O. V., Prokopchuk A. A., Dolgopolov S. I., Khoriak N. V., Nikolayev O. D., Pisarenko V. Yu., Kovalenko V. N. (2017). Mathematical modeling and stability analysis of low-frequency processes in main LRE with oxidizer-rich staged combustion. *Bulletin of engine building,* No. 2, 34—42 [in Russian]. URL: http://nbuv.gov.ua/UJRN/vidv_2017_2_8 (Last accessed: 10.10.2019).
15. Pylypenko V. V., Zadontsev V. A., Natanzon M. S. (1977). *Cavitation self-excited oscillations and dynamics of hydraulic systems.* Moscow: Mashinostroenie Publishing Company, 351 p. [in Russian].
16. Pylypenko V. V., Dovgotko N. I., Pylypenko O. V. (2011). Studies in the dynamics of liquid rocket propulsion systems and the longitudinal stability of liquid launch vehicles. *Technical Mechanics,* No. 4, 16—29 [in Russian].
17. Pylypenko V. V., Dovgotko N. I., Dolgopolov S. I., Nikolayev O. D., Serenko V. A., Khoriak N. V. (1999). Theoretical evaluation of the amplitudes of POGO vibrations in liquid propellant launch vehicles. *Kosm. nauka tehnol.*, **5**(1), 90—96 [in Russian]. doi.org/10.15407/knit1999.01.90.
18. Pylypenko V. V., Dolgopolov S. I. (1998). Experimental and calculated determination of the coefficients of the equation of dynamics of cavitation cavities in inducer and centrifugal pumps of various sizes. *Technical Mechanics*, No. 8, 50—56 [in Russian].
19. Ransom D. L. (2016). Probabilistic Design Analysis of Bellows Type Pogo Accumulator, AIAA 2016-0682.
20. Rubin S. (1972). Analysis of POGO Stability. The Aerospace Corporation, El Segundo, California, USA. — 23 International Astronautical Congress. — Vienna, Austria, Oct. 8—15. P. 19.

21. Rubin S. (1970). Prevention of coupled structure-propulsion instability (POGO). National Aeronautical and Space Administration, USA. NASA SP—8055. 48 p.
22. Shevyakov A. A., Kalnin V. M., Naumenkova N. V., Dyatlov V. G. (1978). *The theory of automatic control of rocket engines*. Moscow: Mechanical Engineering, 287 p. [in Russian].
23. Sterett I. B., Riley G. F. (1970). Saturn V/Apollo vehicles POGO stability problems and solutions. *AIAA Paper*, No. 1236, 12.
24. Swanson L. A., Giel T. V. (2009). Design Analysis of the Ares I Pogo Accumulator 45th AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit, 02 August 2009 — 05 August 2009, Denver, Colorado. doi.org/10.2514/6.2009-4950.
25. Wang O., Tan S., Wu Z., Yang Yu., Yu Z. (2015). Improved modelling method of Pogo analysis and simulation for liquid rockets. *Acta Astronautica*, **107**, 262—273.

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ЗАБЕЗПЕЧЕННЯ ПОЗДОВЖНЬОЇ СТІЙКОСТІ РАКЕТИ-НОСІЯ «ЦИКЛОН-4М»

Низькочастотні поздовжні коливання рідинних ракет-носіїв (POGO) — явище, властиве майже всім рідинним ракетам. Ці коливання можуть призвести до різних аварійних ситуацій: пошкодження конструкції ракети і системи подачі рідкого палива, неприпустимих порушень в роботі системи керування ракетою. Використання рідинних ракетних двигунів з допалюванням окислювального генераторного газу як маршових двигунів першого ступеня ракет-носіїв може внести ряд особливостей в аналіз поздовжньої стійкості РН. Насамперед, в цьому випадку поздовжні коливання ракети можуть бути спричинені низькочастотною нестійкістю рідинної двигунної установки, пов'язаною з динамічними процесами в контурі «газогенератор — газовід — турбонасосний агрегат».

Для прогнозування поздовжньої стійкості проєктованої в даний час двоступеневої ракети-носія «Циклон-4М» було розроблено математичну модель низькочастотної динаміки системи «рідинна ракетна двигунна установка — корпус ракети». Ця модель описує взаємодію пружних поздовжніх коливань конструкції РН і низькочастотних процесів в її маршовій двигунній установці. Розроблена модель містить математичне описання низькочастотної динаміки маршової двигунної установки RD-874 (до її складу входять чотири двигуни RD-870 з допалюванням окислювального генераторного газу), живильних магістралей та корпусу ракети-носія. В результаті теоретичного аналізу повздовжньої стійкості ракети-носія, виконаного на основі розробленої математичної моделі з використанням критерію Найквіста, було виявлено, що на початковому інтервалі часу польоту ракети (5 с, 70 с) динамічна система «рідинна ракетна двигунна установка — корпус РН» є нестійкою по відношенню до I поздовжньої моди корпусу ракети. Визначено, що нестійкість досліджуваної системи обумовлено не лише зближенням частоти I тону вільних поздовжніх коливань корпусу ракети «Циклон-4М» з першою частотою коливань рідини в лінії живлення окислювачем, але й значним зростанням динамічного коефіцієнта підсилення двигуна RD 870 у частотному діапазоні від 6 до 9 Гц. Сукупна дія цих двох чинників призвела до втрати поздовжньої стійкості ракети-носія «Циклон-4М» у зазначеному інтервалі часу польоту. Таку схему розвитку POGO-нестійкості рідинних ракет було виявлено вперше, і її можна відзначити як характерну особливість явища POGO для ракет, в яких використовуються рідинні ракетні двигуни з допалюванням окислювального генераторного газу.

Для забезпечення поздовжньої стійкості ракети-носія «Циклон-4М» запропоновано встановити в живильні магістралі окислювача демпфери повздовжніх коливань. Розроблено математичну модель низькочастотної динаміки газорідинного демпфера з сильфонним поділом середовищ і визначено його раціональні параметри. Запропоновано підхід до визначення параметрів системи демпфування POGO-коливань, згідно з яким раціональний вибір конструктивних параметрів демпфера повздовжніх коливань здійснюється виходячи з умови амплітудної стабілізації динамічної системи «двигунна установка з демпфером подовжніх коливань — корпус ракети-носія».

Ключові слова: поздовжня стійкість ракети-носія, кавітаційні явища в насосах, демпфер поздовжніх коливань, сильфонний поділ середовищ, рідинний ракетний двигун, допалювання окислювального генераторного газу, критерій стійкості Найквіста.