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Accuracy Analysis of Strapdown Inertial Navigation Systems

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This paper discusses correction methods of INS/GPS systems. Different schemes are considered: open loop correction scheme, scheme with estimated instrumental errors feedback, scheme with state space feedback and scheme with navigation parameter errors feedback INS correction is considered as the process of control on the base of combined method. Nonlinear equations of INS angular errors in quaternion form are obtained. Nonlinear and linear equations of navigation and orientation parameters errors are presented. Observability and detectability of linear equations of INS errors at different component combination of error vector are evaluated. Precision analysis of instrumental errors compensation is carried out. The results of testing INS/GPS systems on laboratory bench and car are presented.

1. INTRODUCTION

Integrated INS/GPS systems with microelectromechanical sensors (MEMS) are widely used for different moving plants because these inertial systems are small-sized and low-cost.

The development of integrated INS/GPS systems with MEMS gyros and accelerometers includes solving the problems to provide the required accuracy of navigation parameter calculation.

Correction of INS errors is realized using both internal loops without external information and adaptive optimal filtrating in close loop compensation scheme. In monograph [8], the informational equivalence of open loop and closed loop correction schemes under a certain ratio of coefficients in observation feedback was proved. The problem of correction becomes of great importance especially at development of INS, which is corrected using GPS receiver data, with low-cost miniature solid-state sensors, for example, MEMS — gyro and accelerometer sensors [10]. Development of low-cost integrated systems with this type of sensors, in our opinion, requires solving the following tasks.

1. Because of low precision of inertial sensors, accelerometers and gyros, which are determined by stability of scalar factor, stability of “zero” (bias) and parameters of random component of sensor error (noise), navigation parameters are formed (calculated) with low accuracy. The experience of practical use of the sensors demonstrates significant increase of fluctuating components of «zeros», such as Markovian processes with a correlation time from 10 to 50 s. These properties of sensor errors specify some additional requirements to compensation methods.

2. Inaccurate geometric placing sensors on axes of measurement trihedron because even proper orientation of the sensor element in the microchip is specified with an accuracy of 1°, and further placing the sensors on board, and board in the case results in additional errors. So, geometric calibration with high precision is required. But test bench for this type of calibration is expensive, and

total cost will increase. In our opinion, it is necessary to develop a new procedure on inexpensive test benches.

3. Temperature dependence of sensor parameters and significant deviations of temperature from pointed in specification narrows exploiting conditions of such a system and requires to carry out exact temperature calibration for not only systematic errors of sensors, but scale factor as well. This problem can be solved only by metrology. That means carrying out thorough temperature calibration.

4. Level of MEMS errors results in such navigation parameter errors when linear models can not be used. It is necessary to choose equations of INS errors thoroughly and validate all range of errors value.

5. Using the extended Kalman filter for evaluation of INS errors does not allow one to estimate all the errors, because precision of evaluation is decreased and calculable loading of processor is increased with extension of state vector. To achieve the necessary precision of error estimation, it is required conducting test motions of INS, not characteristic for the used object of navigation.

6. At disconnecting the GPS of the receiver at object shading or disappearing navigation satellites autonomous application of INS is not allowed more than a few minutes. The improvement of such performance indexes is possible due to the use of adequate mathematical models of sensor errors that requires the decision of identification of transformation processes of navigation information.

Basic methods of INS multidimensional correction of open loop type and closed loop type were offered and described in [8, 5]. Moreover, closed correction scheme is called compensation scheme or scheme of correction as controlled process. In this case control object is computational process of forming navigation parameters using measurements from sensors (controlled input actions), and control actions or signals are signals for compensation of sensor instrumental errors. Let us consider the compensation of uncontrolled disturbances (instrumental errors) as the purpose of control. Then control problem can be considered as the task of selective invariant control [9]. This approach is based on the use of the model of disturbances as in [8] where it is named wave approach. This approach is often applied to the tasks of INS errors, for example as in [4], using in the extended Kalman filter the model of sensor instrumental errors, although it does not correlate with invariant control. In this case the question is about the selective invariance of control or correction, and the precision of correction is determined by adequacy of the models used.

Other approach [5, 2] is based on the procedure of evaluation of the uncontrolled input signal of the dynamic system in the real time with the use of asymptotic observers of the state and disturbance. The disadvantage of this approach is high sensitivity of the system to information delay in the channels. Neglect of this fact results in instability of calculation process of navigation parameters. The approach under consideration is used at presence of INS hardware synchronization with external information sources.

It is possible to obtain high precision of integrated INS only at the comprehensive analysis of the closed computational process of navigation parameter calculation. Investigations into INS correction specify the actuality of improvement of compensation schemes of INS errors [10].

Let us consider the properties of correction schemes for type solutions of instrumental error compensation problem.

2. CORRECTION METHODS

2.1. Classification of correction methods. In recent years a number of INS correction methods using external measurements has been developed. Let us classify them according to the following features:

- according to integration level (tightly coupled / loosely coupled correction scheme, low level of integration / high level of integration);
- according to feedback (open-loop / closed correction scheme);
- according to functioning criterion (optimal / non-optimal).

2.2. Open loop correction with filtration (M1). This correction method is based on the evaluation of navigation parameter error vector and subtracting errors from the INS formed.

The condition for application of this correction scheme is saving smallness of navigation parameters errors in relation to supporting or basis point of linearization of navigation equations. With violating the condition of relative smallness of INS error estimation it is possible to change the supporting point of linearization and reduce estimation errors to zero, that is realized by connection of type Feedback with initial condition in the certain time moments. This type of correction is used for the high-precision systems with precise sensors, when the period of correction can last tens of minutes.

The equations for this correction scheme are described as:

$$\Delta \mathbf{y}(t) = \mathbf{y}_{GPS}(t) - \mathbf{y}_{INS}(t),$$

$$\Delta\hat{\mathbf{x}}(t) = \mathbf{A}\Delta\hat{\mathbf{x}}(t) + \mathbf{L}_{FK}(\Delta \mathbf{y}(t) - \mathbf{C}\Delta\hat{\mathbf{x}}(t),$$

$$\mathbf{x}_{out}(t) = \mathbf{x}_{INS}(t) - \Delta\hat{\mathbf{x}}(t),$$

where $\mathbf{L}_{FK}$ is matrix of feedback on an observation and $\mathbf{x}_{out}(t)$ is corrected vector of navigation parameters.

2.3. Correction scheme with feedback on estimation of sensor instrumental error (M2). For this scheme of correction, the requirements to the sensors can be significantly reduced, because the linearity of errors is valid for long time of the operation of the system. Adding direct connections as for the scheme M1, we have the scheme of the type M1 + M2.

The equations for this correction scheme are described as:

$$\Delta \mathbf{y}(t) = \mathbf{y}_{GPS}(t) - \mathbf{y}_{INS}(t),$$

$$\Delta\hat{\mathbf{x}}(t) = \mathbf{A}(t)\Delta\hat{\mathbf{x}}(t) + \mathbf{L}_{FK}(t)(\Delta \mathbf{y}(t) - \mathbf{C}\Delta\hat{\mathbf{x}}(t)),$$

$$\mathbf{u}_{\omega}(t) = -\Delta\hat{\boldsymbol{\omega}}(t),$$

$$\mathbf{u}_{w}(t) = -\Delta \mathbf{w}\hat{(t)},$$

$$\mathbf{u}(t) = [\mathbf{u}_{\boldsymbol{\omega}}^{T}(t)\ \mathbf{u}_{w}^{T}(t)\,]^{T},$$

$$\dot{\mathbf{x}}_{INS}(t) = \mathbf{f}_{INS}(\mathbf{x}_{INS}(t), \mathbf{u}(t), \mathbf{v}_{INS}(t)),$$

$$\mathbf{y}_{INS}(t) = \mathbf{h}_{INS}(\mathbf{x}(t), \mathbf{u}(t)),$$

$$\hat{\mathbf{x}}(t) = \mathbf{x}_{INS}(t) - \Delta \mathbf{x}_{FK}(t),$$

where $u_{\omega}(t)$ and $u_{w}(t)$ are the correction signals (control signals) on angular rate and accelerations sensors and $v_{INS}(t)$ denotes disturbances (sensor errors).

2.4. Correction scheme with feedback on estimation of navigation parameter error (feedback on the Kalman filter state vector), M3. This scheme of correction corresponds to the classic method of control using the state vector that can provide given dynamic properties of scheme by the matrix coefficient $\mathbf{K}_x$. The matrix coefficient can be calculated by solving the optimal control task using quadratic criterion (LQ) for the non-stationary system. However, the specificity of the matrix of the state allows one to get simplified solutions, for example as in [3]. The authors used the method of synthesis of stationary matrix coefficient by decomposition on autonomous channels. This method provides the required dynamics of adjustment on instrumental errors and, hence, ability of INS to operate.

The equations of this correction scheme are described as:

$$\Delta \mathbf{y}(t) = \mathbf{y}_{GPS}(t) - \mathbf{y}_{INS}(t),$$

$$\Delta\hat{\mathbf{x}}(t) = \mathbf{A}(t)\Delta\hat{\mathbf{x}}(t) + \mathbf{L}_{FK}(t)(\Delta \mathbf{y}(t) - \mathbf{C}\Delta\hat{\mathbf{x}}(t)),$$

$$\mathbf{u}(t) = \mathbf{K}_x\Delta\hat{\mathbf{x}}(t),$$

$$\dot{\mathbf{x}}_{INS}(t) = \mathbf{f}_{INS}(\mathbf{x}_{INS}(t), \mathbf{u}(t), \mathbf{v}_{INS}(t)),$$

$$\mathbf{y}_{INS}(t) = \mathbf{h}_{INS}(\mathbf{x}(t), \mathbf{u}(t)),$$

$$\hat{\mathbf{x}}(t) = \mathbf{x}_{INS}(t) - \Delta\hat{\mathbf{x}}(t).$$

2.5. Correction scheme with feedback on navigation parameter error (M4). The scheme of correction corresponds to the method of control with the use of output vector that can not always provide given dynamic properties of the scheme by the calculated matrix coefficient $\mathbf{K}_y$. The authors developed the procedure for synthesis of stationary matrix coefficient by decomposition on autonomous channels that provides the required dynamics of adjustment on the instrumental errors of sensors. The advantage of this correction scheme is defined by the fact that it does not use the multidimensional observer of the state and, hence, the INS software is simplified.

The equations for this correction scheme are described as:

$$u(t) = \mathbf{K}_y(\mathbf{y}_{GPS}(t) - \mathbf{y}_{INS}(t)),$$

$$\dot{\mathbf{x}}_{INS}(t) = \mathbf{f}_{INS}(\mathbf{x}_{INS}(t), \mathbf{u}(t), \mathbf{v}_{INS}(t)),$$

$$\mathbf{y}_{INS}(t) = \mathbf{h}_{INS}(\mathbf{x}(t), \mathbf{u}(t)).$$

3. INS ERROR EQUATIONS

3.1. The equations of orientation parameter errors (the Euler angles). Let us introduce the following designation:

- $\mathbf{\Lambda}$ is direction cosine matrix, which transforms vector from the body coordinate system to the inertial coordinate system, determined by quaternion $\boldsymbol{\lambda}$ and $\mathbf{\Lambda}(\boldsymbol{\lambda})$;
- $\Delta\mathbf{\Lambda}$ denotes the rotation matrix from calculated frame to navigation frame, then the relationship between the matrices is $\mathbf{\Lambda} = \Delta\mathbf{\Lambda}\tilde{\mathbf{\Lambda}}$;
- $\Delta\boldsymbol{\omega}_b$ is the vector of instrumental errors of gyros in the body frame.

The general equation of orientation parameter errors can be written as [7]:

$$\Delta\dot{\boldsymbol{\lambda}} = 0.5\Delta\boldsymbol{\lambda}\otimes\tilde{\boldsymbol{\lambda}}\otimes\Delta\boldsymbol{\omega}_b\otimes\tilde{\boldsymbol{\lambda}}^*, \tag{1}$$

where $\Delta\boldsymbol{\omega}_b = [0, \boldsymbol{\omega}_{bx}, \boldsymbol{\omega}_{by}, \boldsymbol{\omega}_{bz}]$ is quaternion of absolute angular rate in the body frame.

If the following representation is allowed:

$$\Delta\boldsymbol{\lambda} = [1, 0.5\alpha_x, 0.5\alpha_y, 0.5\alpha_z]^{\mathrm{T}},$$

$$\Delta\mathbf{\Lambda} = \mathbf{I} + \mathbf{\Phi},$$

$$\mathbf{\Phi} = \begin{bmatrix} 0 & -\alpha_z & \alpha_y \\ \alpha_z & 0 & -\alpha_x \\ -\alpha_y & \alpha_x & 0 \end{bmatrix},$$

the simplified equations of orientation parameter errors are used in the form:

$$\dot{\boldsymbol{\alpha}} = \mathbf{\Lambda}(\boldsymbol{\lambda})\Delta\boldsymbol{\omega}_b, \tag{2}$$

where vector $\boldsymbol{\alpha} = [\alpha_x, \alpha_y, \alpha_z]^{\mathrm{T}}$ is angular errors of vertical and course.

3.2. The equations of navigation parameter errors. If $\Delta\mathbf{V}$ is the vector of velocity errors in the inertial frame, and $\Delta\mathbf{R}$ is the vector of position errors, the complete equation of errors can be written as [7]:

$$\Delta\dot{\mathbf{V}} = -(\Delta\mathbf{\Lambda}(\Delta\boldsymbol{\lambda}) - \mathbf{I})\mathbf{\Lambda}(\tilde{\boldsymbol{\lambda}})\mathrm{w}_b + \Delta\mathbf{\Lambda}(\Delta\tilde{\boldsymbol{\lambda}})\Delta\mathrm{w}_b + \mathbf{g}(\Delta\mathbf{R}),$$

$$\Delta\dot{\mathbf{R}} = \Delta\mathbf{V}, \qquad \Delta R = |\Delta\mathbf{R}|, \tag{3}$$

where w_b is the vector of acceleration in the body frame and $\Delta\mathrm{w}_b$ is accelerometer bias.

The linear equations of errors can be represented in the form:

$$\Delta\dot{\mathbf{V}} = \mathbf{\Phi}(\alpha)\mathrm{w}_{nav} + \mathbf{\Lambda}(\boldsymbol{\lambda})\Delta\mathrm{w}_b + \Delta\mathbf{g}(\Delta\mathbf{R}),$$

$$\Delta\dot{\mathbf{R}} = \Delta\mathbf{V}, \qquad \Delta R = |\Delta\mathbf{R}|, \tag{4}$$

$$\Delta\mathbf{g}(\Delta\mathbf{R}) = [0, \omega_0^2\Delta R_y, 0]^{\mathrm{T}},$$

where w_{nav} is the vector of acceleration in the navigation frame and ω_0 denotes Shuler's frequency.

3.3. General equations of INS errors. Let us combine equations (1) with (3) and write nonlinear system:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{v}(t)), \ \mathbf{y}(t) = \mathbf{G}(\mathbf{x}(t), \mathbf{u}(t)), \tag{5}$$

where:

$\mathbf{x}(t) = [\Delta\boldsymbol{\lambda}(t), \Delta\mathbf{V}(t), \Delta\mathbf{R}(t), \mathbf{x}_\omega(t), \mathbf{x}_w(t)]^{\mathrm{T}}$ is the vector of INS errors;

$\mathbf{u}(t) = [\mathbf{u}_{\boldsymbol{\omega}}(t), \mathbf{u}_w(t), \mathbf{u}_v(t)]^{\mathrm{T}}$ is the vector of input;

$\mathbf{v}(t) = [\boldsymbol{\mu}_\omega(t), \boldsymbol{\mu}_w(t)]^{\mathrm{T}}$ is the vector of disturbance (sensor errors).

To investigate the observability and precision of instrumental error restoration, the equations of errors are used in linearized form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}_u(t)\mathbf{u}(t) + \mathbf{B}_v(t)\mathbf{v}(t),$$

$$\mathbf{A}(t) = \left.\frac{\partial\mathbf{F}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{v}(t))}{\partial\mathbf{x}}\right|_{\substack{x = x^0 \\ u = u^0 \\ v = v^0}},$$

$$\mathbf{B}_u(t) = \left.\frac{\partial\mathbf{F}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{v}(t))}{\partial\mathbf{u}}\right|_{\substack{x = x^0 \\ u = u^0 \\ v = v^0}}, \tag{6}$$

$$\mathbf{B}_v(t) = \left.\frac{\partial\mathbf{F}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{v}(t))}{\partial\mathbf{v}}\right|_{\substack{x = x^0 \\ u = u^0 \\ v = v^0}},$$

where x^0, u^0, v^0 denote working point of linearization.

For the Kalman filter realization, nonlinear system of equations (5) is used.

4. OBSERVABILITY AND DETECTABILITY OF INSTRUMENTAL ERRORS

Observability is fundamental concept of dynamic system identification theory, which is characterized by the ability to estimate state variable using system output measurements. In the case under consideration it is necessary to solve the problem on testability of sensor instrumental errors and navigation parameters using measurements of velocity and position (attitude).

Linear system (6) is considered as observable on finite time interval, if all the coordinates of state vector at initial moment of this time interval can be determined with the use of information on system input and measurements of system output at this time interval.

Criterion of Observability

In order to system (3) would be observable, it is necessary and sufficient that the matrix of observability $\mathbf{K}_O(\mathbf{A}, \mathbf{C}) = [\mathbf{C}^T, \mathbf{A}^T\mathbf{C}^T, ..., (\mathbf{A}^T)^{n-1}\mathbf{C}^T]$ *of dynamic system has* $\mathrm{rank}(\mathbf{K}_O) = n$ *equal to state vector dimension.*

But in practice the situation may take place when criterion of complete observability is not satisfied, while error evaluations are converged and correction is possible. This situation corresponds to the detectability of state vector.

If the system is unobservable and $\mathrm{rank}(\mathbf{K}_O) = \nu_2 < n$, using the Kalman structure transformation, system (6) can be expressed as:

$$\begin{bmatrix} \dot{\mathbf{x}}_1(t) \\ \dot{\mathbf{x}}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & 0 \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_v \\ \mathbf{B}_v \end{bmatrix} \mathbf{u}(t),$$
$$\mathbf{y}(t) = [\mathbf{C}_1, 0]\mathbf{x}(t) = \mathbf{C}_1\mathbf{x}_1(t), \qquad (7)$$

where

$$\mathbf{x}(t) = [\mathbf{x}_1(t), x_2(t)]^T,$$
$$n_1 = \nu_2, \qquad n_2 = n - \nu_2,$$
$$\mathrm{rank}[\mathbf{C}_1^T, \mathbf{A}_{11}^T\mathbf{C}_1^T, ..., (\mathbf{A}_{11}^T)^{\nu_2-1}C_1^T] = \nu_2 .$$

The pair $\{\mathbf{A}_{11}, \mathbf{C}_1\}$ *is observable. If at the same time* $\mathbf{A}_{22}$ *matrix is Gurwitz matrix, the system is called detectable. This means that the possibility exists of evaluating all the components of state vector.*

If the system is undetectable, principal possibility to calculate estimations of some INS errors is missed [6].

4.1. Observability of instrumental errors in one channel of INS. The equations of INS error model for this case take the form:

$$\dot{\alpha}(t) = \Delta\omega,$$
$$\dot{V}(t) = \alpha(t)g + \Delta w(t), \qquad (8)$$
$$\dot{R}(t) = V(t),$$

where the state variables $\alpha(t)$, $V(t)$, $R(t)$, $\Delta\omega(t)$ and $\Delta w(t)$ are attitude error, velocity error, position error, gyro sensor error, and accelerometer error, respectively. Let us suppose that measurable coordinates of error vector are velocity error and position error. Then matrices of the system can be written as:

$$\mathbf{A}_0 = \begin{bmatrix} 0 & 0 & 0 \\ g & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$
$$\mathbf{B}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$
$$\mathbf{C}_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$\mathbf{D}_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

where g is gravity.

Using the R. Kalman criterion, determine the observability of system (8). The condition of observability is satisfied because $\mathrm{rank}(\mathbf{K}_O^0) = 3$.

Let us verify condition of observability for extended system with matrices $\{\tilde{\mathbf{A}}_0, \tilde{\mathbf{B}}_0, \tilde{\mathbf{C}}_0, \tilde{\mathbf{D}}_0\}$, which is obtained by adding to system (8) the models of INS instrumental errors of INS in the form:

$$\Delta\dot{\omega}_M(t) = \alpha_\omega\Delta\omega_M(t) + \xi_\omega,$$
$$\Delta\dot{w}_M(t) = \alpha_w\Delta w_M(t) + \xi_w,$$

where ξ_ω and ξ_w are “white noise” components.

Then condition of observability can be written as $\mathrm{rank}[\tilde{\mathbf{C}}_0^T, \tilde{\mathbf{A}}_0^T\tilde{\mathbf{C}}_0^T, ..., (\tilde{\mathbf{A}}_0^T)^4\tilde{\mathbf{C}}_0^T] = 4 < n$, which means that the components of state vector of extended system are unobservable. It should be mentioned that condition of observability of accelerometer errors and condition of observability of gyro sensor errors are satisfied separately.

Let us check condition of detectability of the extended system represented in the form similar to (7). Pair $\{\tilde{\mathbf{A}}_{011}, \tilde{\mathbf{C}}_{01}\}$ is completely observable because

$$\mathrm{rank}[\tilde{\mathbf{C}}_{01}^T, \tilde{\mathbf{A}}_{011}^T\tilde{\mathbf{C}}_{01}^T, ..., (\tilde{\mathbf{A}}_{011}^T)^3\tilde{\mathbf{C}}_{01}^T] = 4.$$

Matrix $\tilde{\mathbf{A}}_{022}$ is Gurwitz matrix, which allows one to conclude detectability of extended system and evaluation of all the components of space vector.

Transformed to the new basis, extended system can be written as:

$$\begin{aligned}\dot{\bar{x}}_1(t) &= \bar{x}_3(t),\\ \dot{\bar{x}}_2(t) &= \bar{x}_1(t),\\ \dot{\bar{x}}_3(t) &= \bar{x}_4(t),\\ \dot{\bar{x}}_4(t) &= \alpha_\omega \bar{x}_4(t),\\ &\ldots\ldots\ldots\ldots\\ \dot{\bar{x}}_5(t) &= \alpha_w \bar{x}_5(t).\end{aligned} \tag{9}$$

From this it is clear that the fifth equation extracts unobservable component

$$\bar{x}_5(t) = -\frac{1}{g}\alpha(t) - \frac{\alpha_w}{g}\Delta\omega(t) + \Delta w(t),$$

which includes accelerometer bias $\Delta w(t)$, while the other variables contains are included in block of observable variables.

4.2. Observability of INS instrumental errors in multidimensional case. *The first case.* If space vector of error equation (6) consists of INS parameter errors $x(t) = [\Delta\boldsymbol{\lambda}(t), \Delta V(t), \Delta R(t)]^{\mathrm{T}}$ and matrix of observations determines only measurements of position and velocity errors

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix},$$

the system is unobservable. Moreover, error of course angle α_y remains an unobservable variable. It is determined that this system is undetectable as well.

When this variable is considered as measurable (for example by external course system), the matrix of observation

$$\mathbf{C} = \begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}$$

is changed and system (6) becomes observable.

The second case. If space vector of error equation consists of INS parameter

$$\mathbf{x}(t) = [\Delta\boldsymbol{\lambda}(t), \Delta\mathbf{V}(t), \Delta\mathbf{R}(t), \mathbf{x}_\omega(t), \mathbf{x}_w(t)]^{\mathrm{T}}$$

and $\mathbf{A}_\omega$, $\mathbf{A}_w$ are nonzero matrices, the system is observable on observation of velocity and position errors.

The third case. Let us consider the state of the navigation system for which $\mathbf{A}_\omega$, $\mathbf{A}_w$ matrices are zero matrices and instrumental errors are constant. In this case the rank of constructed matrix of observability appeared to be smaller than the dimension of system space vector $\mathrm{rank}(\mathbf{K}_O^3) = 14$, which shows that the observability is missing completely from dynamic system. In that case let us verify detectability and determine unobservable components of navigation system error vector.

Selecting S basis correspondingly, let us transform the system to the view similar to (6). Condition of observability of pair $\{\mathbf{A}_{11}, \mathbf{C}_1\}$ is satisfied because $\mathrm{rank}[\mathbf{C}_1^{\mathrm{T}}, \mathbf{A}_{11}^{\mathrm{T}}\mathbf{C}_1^{\mathrm{T}}, \ldots, (\mathbf{A}_{11}^{\mathrm{T}})^{13}\mathbf{C}_1^{\mathrm{T}}] = 14 = \nu_2$. Moreover, matrix $\mathbf{A}_{22}$ is Gurwitz one that allows one to conclude on detectability of the system, which corresponds to evaluation of all the components of system (6) state vector.

The transformed system (6) consists of two parts: the part of dimension with $\nu_2 = 14$ in which output $y(t)$ is observable and the unobservable part including the components of INS instrumental errors, Δw_y and Δw_z.

The results obtained demonstrate the following: when the parameters of both gyro and accelerometers error models are equal to zero, system (6) losses its observability. It should be noted that the availability at least, of rotation sensor or just accelerometers error model in the system provides the observability of the system. However, even if dynamic system losses its observability, the possibility exists of evaluating system space state, which means the detectability of the system.

5. ANALYSIS OF CORRECTION METHOD PRECISION

To analyse the precision of correction methods let us assume the following:

1. Unit of navigation parameters formation using inertial sensors is continuous and ideal system.
2. Unit of velocity vector and position measurements by correcting system is ideal.
3. It is analysed only influence of instrumental errors of inertial sensors on precision of navigation parameters in asymptotic mode (time tends to

infinity, $t \to \infty$).

Let us characterise the system precision by order of astaticism with respect to disturbance. The analysis of system astaticism can be performed through the structural method (using multidimensional transfer functions), at which the order of power of variable s is determined in numerator of transfer function, or by examining special matrices in state space.

5.1. Analysis of system astaticism in state space. Let us consider a controllable dynamic system (a closed system corrected for instrumental errors) in the form:

$$\Delta\dot{\mathbf{x}}(t) = \mathbf{A}_0\Delta\mathbf{x}(t) + \mathbf{B}_0\mathbf{v}(t), \qquad \Delta\mathbf{y}(t) = \mathbf{C}_0\Delta\mathbf{x}(t), \tag{10}$$

where $\mathbf{A}_0$ is the matrix of state in fixed time, $\mathbf{v}(t)$ denotes disturbance in the system (instrumental errors), $\Delta\mathbf{x}(t)$ is vector of INS errors.

If the component of the vector of navigation parameter error $\Delta\mathbf{x}_\nu(\infty)$ *does not depend on the* μ*th component of disturbance, the system has the first order of astaticism type* $(\nu, \mu)_1$ [1].

For the linear system in form (10) astaticism type $((\nu, \mu)_1$ is equivalent that the element of the μth row and νth column of matrix $A_0^{-1}\mathbf{B}_0$ is equal to zero. For astaticism of the second order type $(\nu, \mu)_2$, the corresponding element of $\mathbf{A}_0^{-2}\mathbf{B}_0$ matrix has to be equal to zero etc. If matrix $\mathbf{A}_0$ is singular, the astaticism order can be determined through structural method using matrix transfer function.

5.2. Analysis of correction scheme type M1. Let us introduce the extended state vector consisting of estimations of INS errors and estimations of error observer of state type Kalman filter $\Delta\mathbf{z} = [\Delta\mathbf{x}^{\mathrm{T}}, \Delta\hat{\mathbf{x}}^{\mathrm{T}}]^{\mathrm{T}}$. For this case matrices of state, control and observation are described in the form:

$$\mathbf{A}_z = \begin{bmatrix} \mathbf{A}_0 & 0 \\ 0 & \mathbf{A}_0 - L_{kf}\mathbf{C}_0 \end{bmatrix}, \qquad \mathbf{B}_z = \begin{bmatrix} \mathbf{B}_0 \\ 0 \end{bmatrix}, \quad \mathbf{C}_z = [I_{n\times n}, -I_{n\times n}]. \tag{11}$$

It is impossible to specify the order of system astaticism in state space, so we apply structural method and determine matrix transfer function in the form $W_z(s) = \mathbf{C}_z(s\mathbf{I} - \mathbf{A}_z)\mathbf{B}_z$ and matrix gain $\mathbf{K}_{sg} = \lim_{s\to 0} W_z(s)$.

The values of static gains of multidimensional transfer function show that coefficients of errors on coordinates are zero and the system is astatic. Respectively, the errors of velocity and errors of angles are limited, and the system remains static.

5.3. Analysis of correction scheme type M2. For this case the matrices of state, control and observation are described in the form:

$$\mathbf{A}_z = \begin{bmatrix} \mathbf{A}_0 & -\mathbf{B}_0\mathbf{K}_{err} \\ \mathbf{C}_0\mathbf{L}_{kf} & \mathbf{A}_{0e} - L_{kf}\mathbf{C}_0 - \mathbf{B}_{0e}\mathbf{K}_{err} \end{bmatrix},$$

$$\mathbf{B}_{zu} = \begin{bmatrix} \mathbf{B}_0 \\ \mathbf{B}_{0e} \end{bmatrix}, \qquad \mathbf{B}_{zv} = \begin{bmatrix} \mathbf{B}_0 \\ 0 \end{bmatrix},$$

$$\mathbf{C}_z = \begin{bmatrix} \mathbf{I}_{n\times n} & 0 \\ 0 & \mathbf{I}_{ne\times ne} \end{bmatrix},$$

where

$$\mathbf{K}_{err} = \begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}$$

is matrix of feedback.

It is impossible to specify the order of system astaticism in state space, so we apply the structural method and determine matrix transfer function in the form $W_z(s) = \mathbf{C}_z(s\mathbf{I} - \mathbf{A}_z)\mathbf{B}_z$ and matrix gain $\mathbf{K}_{sg} = \lim_{s\to 0} W_z(s)$.

The values of static gains of multidimensional transfer function show that the coefficients of errors on angles are nonzero, hence the system is static. Respectively, the errors of velocity and errors of coordinates are unlimited. Similarly, from accelerometers errors we obtain unlimited coefficients, and system is even not static.

Let us analyse scheme M2 + M1, the matrix of observation will be changed:

$$\mathbf{C}_z = \begin{bmatrix} \mathbf{I}_{n\times n} & -\mathbf{I}_{ne\times ne} \\ 0 & \mathbf{I}_{ne\times ne} \end{bmatrix}.$$

The values of static gains of multidimensional transfer function show that the coefficients of errors on angles are zero and system is astatic. Respectively, the errors of velocity and errors of coordinates are limited, and system remains static. From accelerometers errors we obtain limited coefficients, and system become static.

5.4. Analysis of correction scheme type M4. For this case the matrices of state, control and observation are described in the form:

$$\mathbf{A}_z = [\mathbf{A}_0 - \mathbf{B}_0\mathbf{K}_y],$$

$$\mathbf{B}_{zu} = [\mathbf{B}_0], \quad \mathbf{B}_{zv} = [\mathbf{B}_0], \quad \mathbf{C}_z = \mathbf{I},$$

where

$$\mathbf{K}_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K \\ 0 & K & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K & 0 & 0 & K & 0 & 0 \\ 0 & 0 & 0 & K & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K & 0 & 0 & K & 0 \\ 0 & 0 & 0 & 0 & 0 & K & 0 & 0 & 0 \end{bmatrix}.$$

Using matrix $\mathbf{A}_z^{-1}\mathbf{B}_{zv}$ gives the possibility to determine the order of astaticism of scheme in state space. The elements of the given matrix show that the system is static as a whole, and is astatic with respect to some navigation parameters. If we supplement the system with the new variable, integral from error of course, the coefficients of staticism will be changed.

In this case the system remains astatic with respect to the angular variables from errors of gyros, and static with respect to the angular variables from errors of accelerometers.

6. EXPERIMENTAL RESULTS

Testing of the INS was carried out. The INS is built on the basis of microelectromechanical sensors (the accelerometer ADXL311 and gyros ADXR150) and the microcontroller of C8051F133 type. The INS correction was conducted with the use of GPS receiver of EM-406 type (GlobalSat, Taiwan) which provides a positioning accuracy of 10 m (RMS) and a speed accuracy of 0.1 m/s from datasheet. The system was located on the land vehicle driven on a given trajectory. The results of testing are shown in Fig. 1 and Fig. 2.

The conclusions made on the basis of the testing are the following:

1. Noticeable increase of the error of yaw occurs at turning when an intensive change of yaw takes place. This error results in increasing the error in projection velocity, which produces increasing the estimate of gyro drift (Fig. 2, part 1—3). The effect is caused by low accuracy of determination of velocity and course by increment of coordinate.
2. Noise components of inertial sensors were within the limits specified in technical data.
3. Values of gyro drifts were evaluated within the limits specified in technical data, but the errors of drift evaluation significantly depended on error of course GPS. The best result was obtained through the method of correction type M1+M2.
4. The coincidence of INS and GPS navigation parameters did not exceed 20 m, which is sufficient for INS with microelectromechanical sensors.

CONCLUSIONS

In the majority of the correction schemes applied the accuracy of INS is determined by the property of staticism from instrumental errors and just in some cases the property of astaticism exists. To increase the precision of correction schemes it is

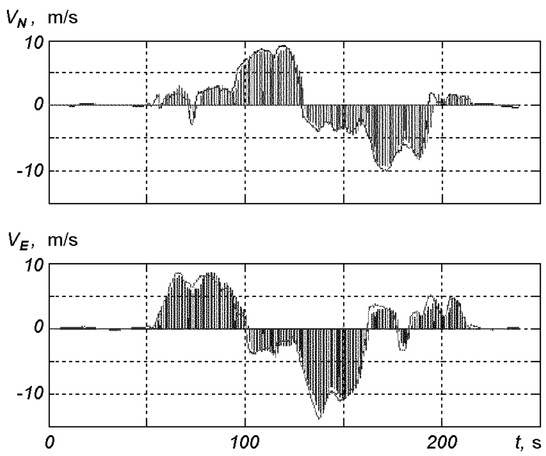

Fig. 1. Velocity of INS/GPS (north and east components)

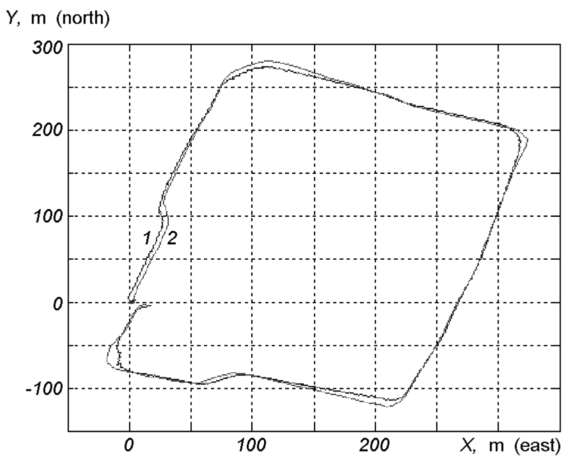

Fig. 2. Test trajectory: 1 — GPS, 2 — INS

necessary to increase the order of astaticism, to extend vector of control by integral variables and to apply combination. To provide selective invariance of instrumental errors it is sufficient to include models of errors into general model of INS. For low cost INS, we conclude that sufficient accuracy of the system is reached with the use of correction scheme with navigation parameter errors feedback, which does not require high specification to CPU. We consider that perspective direction of further research is the examination of the precision of two sample discrete system and the development of some methods for the enhancement of astaticism in discrete systems.

REFERENCES

1. Babich O. A. Information Processing in Navigation Complexes. — Moscow: Mashinostroenie, 1991.—512 p.
2. Branets V. N., Shmyglevskiy I. P. Introduction into Inertial Navigation Systems Theory. — Moscow: Nauka, 1992.— 280 p.
3. Chernodarov A. V., Patrikeev A. P., Gladkin C. M., et al. Performance updating of integrated navigation systems on the base of closely coupled damping schemes of sensor errors // Proceedings of XIIIen St. Petersburg International Conference on Integrated Navigating Systems. — St. Petersburg, 2005.—P. 111—120.
4. Dmitriev S. P., Stepanov O. A., Koshaev D. A. Investigation of data interconnecting at inertial satellite systems development // Integrated Inertial Satellite Systems. — St. Petersburg, 2004.—P. 43—59.
5. Kuzovkov N. T., Salychev O. S. Inertial navigation and optimal filtration. — Moscow: Mashinostroenie, 1982.— 216 p.
6. Kortunov V. I., Proskura G. A. Observability and detectability of strapdown INS instrumental errors // Aerospace technique and technology.—2006.—N 3 (29).—P. 31—38.
7. Kortunov V. I., Proskura G. A. Error equations of strapdown INS and observability analysis // Data reduction systems.—2006.—N 9 (58).—P. 112—115.
8. Parusnikov N. A., Morozov V. N., Borzov V. I. Correction problem in inertial navigation. — Moscow: Moscow State University, 1982.—176 p.
9. Panov A. P. Mathematical fundamentals of inertial navigation theory. — Kyiv: Naukova dumka, 1995.—278 p.
10. Stepanov O. A. Development features and future prospects of navigation inertial satellite systems // Integrated Inertial Satellite Systems. — St. Petersburg, 2004.—P. 25—43.

АНАЛІЗ ТОЧНОСТІ БЕЗПЛАТФОРМНИХ ІНЕРЦІАЛЬНИХ НАВІГАЦІЙНИХ СИСТЕМ

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Наведено методи корекції в інтегрованих ІНС/GPS-системах. Розглянуто схеми корекції розімкненого й замкнутого типу зі зворотними зв'язками за оцінками інструментальних помилок, зі зворотними зв'язками за оцінками вектора стану помилок та зі зворотними зв'язками за помилками навігаційних параметрів. Корекція ІНС розглядається як процес управління обчисленням навігаційних параметрів і вирішується задача селективного інваріантного управління. Отримано лінійні рівняння помилок ІНС у кватерніонній формі. Представлено лінійні рівняння параметрів орієнтації та навігації. Оцінено спостережуваність і відновлюваність лінійних рівнянь помилок ІНС для різних комбінацій вектора помилок. Проведено аналіз точності компенсації інструментальних помилок для приведених схем корекції. Наведено результати тестування ІНС/GPS-системи на автомобілі.