V. Grimalsky\textsuperscript{1}, S. Koshevaya\textsuperscript{2}, A. Kotsarenko\textsuperscript{3}, V. Malnev\textsuperscript{4}, D. Juarez\textsuperscript{2}

\textsuperscript{1}National Institute for Astrophysics, Optics, and Electronics (INAOE), Z.P. 72000, Puebla, Mexico
\textsuperscript{2}Autonomous University of Morelos (UAEM), Faculty of Chemistry, CIICap, Z.P. 62210, Cuernavaca, Mor., Mexico
\textsuperscript{3}UNAM, Center of Geoscience, Juriquilla 1-742, ZP.76220, Que., Mexico
\textsuperscript{4}Kyiv National Shevchenko University, Ukraine

Passage of the acoustic waves caused by seismic and volcano activity through the lithosphere into the ionosphere

Received 24.03.95

During earthquakes and strong underground explosions the nonlinear passage of the acoustic waves takes place through the lithosphere into the ionosphere, which is analyzed in this paper. This nonlinear passage occurs due to nonlinear elastic modules of the lithosphere and hydrodynamic nonlinearity of the atmosphere. The waves are excited by underground sources under earthquakes. The acoustic wave propagation going almost vertically upwards causes a change of their spectrum. A wide spectrum of the acoustic waves fills the radio wave range, caused by fracturing of the rock in the surface, was observed by means of satellite measurements and radio telescope investigation of meteor bombing of the Moon. If the fracture occurs at deep depths, high frequencies due to nonlinear interaction transform into very low and extremely low frequency waves. Very low and extremely low elastic displacement waves achieve the Earth's surface and excite the response in seismograms. Acoustic waves move through the atmosphere into the ionosphere, and this causes changing their spectrum. More than 100 higher harmonics are excited. Excitation and passage of acoustic waves produce ELF and ULF waves in the atmosphere, as a result of the nonlinear transform and diffraction of waves.

I. INTRODUCTION

It is very important to investigate the mechanisms of the energy flows from the lithosphere into the atmosphere and the ionosphere caused by natural hazards (seismic and volcano activity, for example experiment MASSA [1—7]). All mechanisms possess different precursors [2] due to three basic channels of the lithosphere-ionosphere coupling, namely, electromagnetic, geochemical, and acoustic ones [3].

The acoustic channel of the lithosphere-ionosphere coupling seems to be quite effective. This takes place due to atmospheric acoustic waves excited by fluctuations of the terrestrial surface [6]. This channel manifests in different phenomena [7], like exciting the plasma waves and a periodic structure in the ionosphere, the increasing transparency for radio waves, linear and nonlinear generation of magnetic perturbations, oscillation of E-F-layers in the ionosphere caused by acoustic and acoustic-gravity waves, nonlinear change of the spectrum of waves in the atmosphere, ionosphere, and lithosphere [7—12]. The last-named case is analyzed below.

2. MODELLING OF PASSAGE OF ACOUSTIC WAVES IN THE LITHOSPHERE

The geometry of the model is shown in Fig. 1. The cylindrical surface is around the source of underground seismic explosion (or plate deformation). After that a seismic acoustic burst-like envelope, of a finite
transverse scale of the generated waves begins its passing through the Earth. It is possible to use the theory of elasticity with nonlinear modules, where damping the waves and their diffraction are taken into account. The elasticity theory for the case of the Earth’s crust as isotropic uniform medium results in the following equations for the mechanic displacement:

\[
\begin{aligned}
\frac{\partial^2 U}{\partial t^2} = & \nabla \left\{ (S^2_\rho - S^2_\eta) \cdot \nabla U \right\} + \\
& + S^2_\rho \Delta U + \Gamma(z) \frac{\partial}{\partial t} \Delta U + B(z) \nabla \left\{ (\nabla U)^2 \right\},
\end{aligned}
\]

where \(\Gamma(z)\) and \(B(z)\) describe the viscosity and nonlinearity coefficient of the elastic isotropic transversely uniform medium in the cylindrical geometry for the acoustic waves with longitudinal and transverse velocities \(S_\rho\) and \(S_\eta\), respectively. If it is possible to neglect the derivatives from \(S_\eta\) we take into account only a dependence of diffraction coefficient \(\Gamma(z)\) on the coordinate \(z\) in which the waves pass. The condition of neglecting the derivative of the sound velocity \(dS_\eta/dz\) in Eq. (1) is \(dS_\eta/dz \approx l_a^{-1} S_\eta \ll k S_\rho\), where \(l_a \approx 1\) km is the scale of the vertical variation of \(S_\eta\), \(k = \omega/S_\eta \approx 200\) km \(s^{-1}\) is the wave number of the ELF acoustic wave at the frequency \(\omega \approx 1000\) s \(^{-1}\). In another words, this condition is \(kl_a \gg 1\) and is here well satisfied.

Rewrite Eq. (1) by the components. The projection on the axis \(Z\) is:

\[
\begin{aligned}
\frac{\partial^2 U_z}{\partial t^2} = & (S^2_\rho - S^2_\eta) \frac{\partial^2 U_z}{\partial z^2} + \frac{1}{\rho} \frac{\partial^2 (\rho U_z)}{\partial z \partial \rho} + \\
& + S^2_\rho \frac{\partial U_z}{\partial t} \frac{\partial^2 U_z}{\partial z \partial \rho} + \Gamma(z) \frac{\partial}{\partial t} \frac{\partial^2 U_z}{\partial z^2} + \\
& + B(z) \frac{\partial}{\partial z} \left( \frac{\partial U_z}{\partial z} \right)^2.
\end{aligned}
\]

It should be noted that the transverse profile is quite smooth. The transverse direction of the deformation is:

\[
\frac{\partial^2 U_z}{\partial t^2} = \frac{\partial^2 U_z}{\partial \rho^2} \frac{\partial^2 U_z}{\partial \rho \partial z} + S^2_\rho \frac{\partial^2 U_z}{\partial \rho \partial \rho}.
\]

The variables \(z, \rho\), and \(\eta = t - \int_{-\infty}^{z} \frac{dz'}{S_\eta(z')}\) are used. The method of slowly varying profile is applied so as this research is focused on the nonlinear evolution of a seismic acoustic burst-like envelope of a finite transverse scale in the cylindrical geometry.

For the \(\rho\)-component we obtain:

\[
\begin{aligned}
\frac{\partial^2 U_\rho}{\partial \rho^2} \left( 1 - \frac{S^2_\eta}{S^2_\rho} \right) \approx & - \frac{1}{\rho} \frac{\partial^2 U_\rho}{\partial \rho \partial \eta} - \\
& - \frac{\partial U_\rho}{\partial \eta} = - \frac{\partial U_\rho}{\partial \rho}.
\end{aligned}
\]

Therefore, it is possible to exclude the transverse component.

For the \(z\)-component we have:

\[
\begin{aligned}
\frac{\partial U_z}{\partial z} - \Gamma(z) \frac{\partial^2 U_z}{\partial \eta^2} + B(z) \left( \frac{\partial U_z}{\partial \eta} \right)^2 - \\
- \frac{S^2_\rho}{2 \Delta} \int_{-\infty}^{\eta} U_\rho(\eta')d\eta' = 0.
\end{aligned}
\]

This is the equation of Khokhlov – Zabolotskaya [13]. Below we use, in non-dimensional form, the Eq. (2) with change \(\Gamma(z) \rightarrow G(z)\) and \(B(z) \rightarrow N(z)\) and introducing the diffraction coefficient \(D\):

\[
\begin{aligned}
\frac{\partial U_z}{\partial z} - G(z) \frac{\partial^2 U_z}{\partial \eta^2} + N(z) \left( \frac{\partial U_z}{\partial \eta} \right)^2 - \\
- D \Delta \int_{-\infty}^{\eta} U_\rho(\eta')d\eta' = 0.
\end{aligned}
\]

During modelling, the fast Fourier transform with respect to \(\eta\) in Eq. (3) was applied. In this approximation we need to subdivide the equations into the LF and ELF parts. The classification of waves is taken as following: LF corresponding to wave frequencies about \(< 100\) kHz, ELF about \(< 300\) Hz and ULF about \(< 3\) Hz.
Figure 2. The nonlinear frequency conversion of AW (HF AW - ELF AW) under passing through the lithosphere. Part a) is initial distribution $|A_1(t, \rho)|^2$ of the first harmonic; part b) is a distribution of $|A_j(t, \rho)|^2$ for harmonics $j=1, 2, \ldots$; when the greatest ratio $|A_j/A_1|^2$ is observed; part c) is output distribution of the velocity of the ELF AW $v(t, \rho)$ on the Earth’s surface. Here, $\omega_0 = 5 \times 10^4 \text{ s}^{-1}$, $\omega_{\text{ELF}} = 1000 \text{ s}^{-1}$, $\rho_0 = 0.5 \text{ km}$. Norming for amplitudes of HF AW is $A_0 = 0.01 \text{ cm}^{-1}$, ratio of nonlinear and linear elastic modules is $c_{111}/c_{11} = 10$, a dissipation coefficient for the frequency $\omega_0 = 5 \times 10^4 \text{ s}^{-1}$ is taken to be 0.25 km$^{-1}$.

The equation for the LF part, i.e., a higher part of frequency spectrum, is:

$$\frac{\partial U}{\partial z} - G(z) \frac{\partial^2 U}{\partial \eta^2} + N(z) \left( \frac{\partial U}{\partial \eta} \right)^2 = 0.$$  

The equation for the ELF part is:

$$\frac{\partial U}{\partial z} - G(z) \frac{\partial^2 U}{\partial \eta^2} + N(z) \left( \frac{\partial U}{\partial \eta} \right)^2 - \frac{D \Delta}{\rho_0} \int_{-\infty}^{\eta} U(\eta')d\eta' = 0.$$

We take into account the diffraction only for the case of ELF part. The results of simulations are shown in Figs 2, 3. The source is located at a depth of 30 km. The results are presented for the uniform Earth’s crust, because the simulations with the scale of variation $l_0 \sim 1 \text{ km}$ of the sound velocity $S_0$ and of the viscosity $\Gamma$ gave qualitatively the same results on the transformation of the wave spectrum. The ratio of nonlinear and linear elastic modules is used as $c_{111}/c_{11} = 10$, a dissipation coefficient for the frequency $\omega_0 = 5 \times 10^4 \text{ s}^{-1}$ is taken as 0.25 km$^{-1}$ (it is proportional to the square of frequency, $\propto \omega^2$).

It is possible to estimate that LF waves possess high losses $G$, essential value of nonlinearity coefficient $N$, but diffraction is small in this frequency range. In the case of ELF part, essential nonlinear source due to LF part and an influence of diffraction take place.
Figure 3. The nonlinear frequency conversion of AW under passing through the lithosphere. Part a) is initial distribution $|A_1(t, \rho)|^2$ of the first harmonic; part b) is output distribution of the velocity of the ELF AW $v(t, \rho)$ at the Earth’s surface. Here, $a_0 = 5 \times 10^7\,\text{s}^{-1}$, $\omega_{\text{ELF}} = 1000\,\text{s}^{-1}$, $\rho_0 = 0.5\,\text{km}$. 600 times decrease of input intensity, as compared with Fig. 2.

Figs 2, 3 illustrate that only ELF waves reach the Earth’s surface without high losses. The LF part dissipates very fast, due to excitation of higher harmonics.

MODELLING OF ACOUSTIC WAVES PASSAGE INTO THE IONOSPHERE

The theory for different effects ensured by acoustic channel of coupling is based on the elasticity and hydrodynamic theory and Maxwell’s equations. We apply this theory to the nonlinear effects provided by seismic waves. The ELF atmospheric acoustic waves interact with each other in the atmosphere and ionosphere, producing ULF atmospheric acoustic waves, which reach the ionosphere altitudes, can be observed by satellites. During the earthquake the spectrum of the seismic waves is wide but only ELF, which changed due to nonlinear passing into ULF, reach the ionosphere. This process can be amplified due to the nonlinear interaction of ELF and their transformation into ULF acoustic atmospheric waves.

For the acoustic channel of the lithosphere-ionosphere coupling, the above-mentioned circumstances are very important. A propagation of nonlinear acoustic waves in the atmosphere is described by the following set of hydrodynamic equations:

$$
\rho_0 \left( \frac{\partial V_z}{\partial t} + v_z \frac{\partial V_z}{\partial z} \right) + \rho' \frac{\partial V_z}{\partial t} = -\frac{\partial p'}{\partial z} + \frac{\partial}{\partial z} \left( \rho_0 \zeta(z) \frac{\partial V_z}{\partial z} \right) - \rho' g_z.
$$

Here, $C_i$ is the sound speed in the air, $v(z) = v(0) \exp(z/H)$ is the kinematic viscosity of the air ($\nu(0) = 0.14\,\text{cm}^2\,\text{s}^{-1}$), $\rho = \rho_0 + \rho'$, $p = p_0 + p'$ are its total density and pressure, in which $\rho_0$ and $p_0$ are stationary values of atmospheric density and pressure $\phi_0$, $p_0 \propto \exp(-z/H)$, $H = k_B T/m g$ is the effective atmosphere height, $k_B$ is the Boltzmann’s constant, $m$ is the neutral gas mass of the ionosphere and $g$ is gravitational acceleration, $\rho'$, $p'$ are their perturbed parts, and $v_x, v_y, v_z$ are the components of the air velocity.

We use the adiabatic equation for pressure with the adiabatic constant $\gamma$. It is assumed that nonlinearity is moderate and only quadratic nonlinear terms are preserved in Eqs (4). Note that all mechanisms of hydrodynamic nonlinearity are essential. Also we suppose that an acoustic wave moves preferentially vertically upwards, and inequalities $|V_{x,y}| \ll |V_z|$ are valid. In a linear approximation, when viscosity is neglected, from Eqs (4) we readily get for the acoustic
Figure 4. The nonlinear frequency conversion of AW, ELF AW into ULF AGW, under propagation through the atmosphere. Part a) is initial distribution $|A_1(t, \rho)|^2$ of the first harmonic; part b) is a distribution of $|A_i(t, \rho=0)|^2$ for harmonics ($i = 1, 2, ...$); part c) is distribution of the velocity of the ULF AGW $\nu(t, \rho)$ at the altitude $z = 50$ km; part d) is distribution of the velocity of the ULF AGW $\nu(t, \rho)$ at the altitude $z = 100$ km; part e) is distribution of the velocity of the ULF AGW $\nu(t, \rho)$ at the altitude $z = 150$ km; part f) is distribution of the velocity of the ULF AGW $\nu(t, \rho)$ at the altitude $z = 200$ km. Here, $\omega_{ELF} = 500$ s$^{-1}$, $\Omega = 2$ s$^{-1}$, $\rho_0 = 5$ km.
wave the following expression \( V(z, t) \sim \exp(z/2\eta) \cdot \exp(i(\Omega t - Kz)) \), where \( K \) is the acoustic wave number, and the energy flux remains constant, \( \rho_o(z) V(z, t) \cdot i = \text{const} \). Numerical simulation confirms this analysis. Because the wave acoustic dispersion is small (\( \Omega > 0.1 \text{ s}^{-1} \)), a lot of higher harmonics are excited under the nonlinear interaction.

Therefore, a slowly varying profile approach is suitable for the analysis of the nonlinear acoustic propagation in the atmosphere. It is possible to derive a single equation for the vertical component of velocity of the air \( V_z \). The density dependence on the single transverse coordinate is assumed only.

When introducing the dependent variable \( V = V_{\exp}(-z/2\eta) \), and \( z, \eta = t - z/C_s, \rho \), the resulting equation is (below, \( \rho, \rho_0 \) are used for notations of transverse coordinate only):

\[
\frac{\partial}{\partial \eta} \left[ \frac{\partial V}{\partial z} - \frac{v(0)e^{iH}}{2C_s^2} \frac{\partial^2 V}{\partial \eta^2} \right] = \frac{C_s}{2} \Delta \omega \cdot V. \tag{5}
\]

The Eq. (5) also is the Khokhlov — Zabolotskaya equation [13] for a medium with a density decreasing along \( z \)-axis. Here we consider the case when the envelope frequency is \( \Omega \sim 1 \text{ s}^{-1} > \Omega, \Omega = C_s/(2H) \). After changing the independent variable \( z, w = \exp[z/(2H)] \), it is possible to rewrite Eq. (5) to:

\[
\frac{\partial}{\partial \eta} \left[ \frac{\partial V}{\partial w} - \frac{v(0)Hw}{C_s^2} \frac{\partial^2 V}{\partial \eta^2} \right] = \frac{C_s}{w} \Delta \omega \cdot V,
\]

\[
V(w = 1, t, \rho) = V_{\rho_0} \sin(\omega_0 t) \exp[-(\rho/\rho_0)^{3}] \cdot \exp[-(\rho/\rho_0)^{3}].
\]

The input ELF wave is burst-like, namely, it is the wave packet with the ELF carrier frequency \( \omega_0 \); its envelope lies in ULF frequency region. The value of \( \rho_0 \) determines a transverse scale of the initial pulse.

After an excitation at the Earth’s surface, ELF wave is subject to nonlinearity that leads to a generation of higher harmonics and also to down-conversion, namely, increasing ULF components. More than 100 harmonics are excited. The growth of higher harmonics leads to the creation of a saw-tooth like structure. This structure dissipates due to viscosity. The ULF part of the spectrum is not subject to dissipation, it increases due to this nonlinear interaction. When the transverse scale \( \rho_0 \sim 1-10 \text{ km} \), diffusion is not essential for ELF wave but it can decrease the peak amplitudes of the ULF part. The results of numerical simulations (Fig. 4) demonstrate that there is an effective nonlinear interaction of ELF and ULF atmospheric acoustic waves and a good nonlinear excitation of ULF atmosphere-ionosphere waves caused by ELF seismic acoustic bursts on the Earth’s surface.

The nonlinear interaction of atmospheric acoustic waves demonstrates the importance of the acoustic channel via the example of the nonlinear mechanism of the energy flux propagation from the lithosphere into the ionosphere. The efficiency of the nonlinear acoustic transformation is quite high. For an initial ELF wave amplitude \( V(z = 0) = V_0 = 2.5 \text{ cm/s} \), one can obtain that the corresponding ULF wave amplitude is \( V_z = V(0) \exp(z/(2H)) = 300 \text{ cm/s at the altitude z = 130 km} \).

CONCLUSIONS

The modelling of acoustic wave passages from the lithosphere into the ionosphere, excited by fracturing, is studied. The simulation of the nonlinear passage of the acoustic waves caused by underground fracturing through the lithosphere, atmosphere, and ionosphere shows that there is an effective nonlinear interaction of the burst in the lithosphere. First of all, a strong excitation of LF and ELF acoustic waves occurs. Second, a good nonlinear excitation of ULF waves (< 3 Hz) takes place in the atmosphere. The diffraction decreases the value of the pulse of ULF, but losses of these waves are small. The LF waves are excited in the lithosphere, but these waves possess high losses and they are so small on the Earth’s surface that they have not been detected by a seismograph. In fact, a transformation of the seismic burst after passing through lithosphere into ELF acoustic waves takes place. In the ionosphere, ELF waves excite ULF waves. The nonlinear interaction of acoustic bursts demonstrates an importance of acoustic channel using the example of the nonlinear mechanism of the energy flow from the lithosphere into the ionosphere.

5. Molchanov O. A., Hayakawa M., Rafalsky V. A. Penetration characteristics of electromagnetic emissions from an underground seismic source into the atmosphere, ionosphere, and magnetosphere // J. Geophys. Res.—1995.—100A, N 2.—
Під час землетрусів та потужних підземних вибухів відбувається нелінійне проходження акустичних звуків через гідрофіру, аналіз якого проведено в даній статті. Широкий спектр акустичних хвиль, що бує викликаний розріджуванням гірських порід та простираних аж до діапазону радіохвиль, спостерігається на сучасних широтах та у роботі телескопічних досліджень землетрусів. Нелінійні процеси хвиль від підземних джерел забезпечено еластичною нелінійністю гідрофіру та гідродинамічними нелінійностями атмосфери.

Якщо розріджування відбувається на великих глибинах, нелінійна вагомість високочастотних коливань здебільшого над та в низькочастотні хвилі пружності, які досить повністю змінюються в гідрофіру, спостерігається незначний спектр радіохвиль. Попереднє вертикальне насищення акустичних хвиль в атмосфері викликає послідовні зміни в його спектрі, збуджуючи понад 100 гармонік. Генерація ВЧ і УЧ хвиль в атмосфері відбувається в наслідок нелінійної трансформації та дифракції акустичних хвиль, що поширюються та збуджуються в ній.