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GENERATION MECHANISM AND FEATURES OF PROPAGATION OF THE ULF PLANETARY-SCALE ELECTROMAGNETIC WAVY STRUCTURES IN THE IONOSPHERE

We give some results of a theoretical investigation of the dynamics of generation and propagation of planetary (with the wavelengths of 1000 km and more) ultra-low frequency (ULF) electromagnetic wave structures in the dissipative ionosphere. It is established that inhomogeneity (latitude variation) of the geomagnetic field and the Earth's rotation generates fast and slow planetary ULF electromagnetic waves. The waves propagate along the parallels to the east as well as to the west. In E-region the fast waves have phase velocities from 2 to 20 km/s and frequencies from 0.1 to 100 mHz; the slow waves propagate with local winds velocities and have frequencies 1—100 μHz. In F-region the fast ULF electromagnetic waves propagate with phase velocities from several ten to several hundred kilometres per second and their frequencies are in the range of 10 to 0.001 Hz. The slow mode is produced by the dynamo electric field, it represents the generalization of the ordinary Rossby type waves in the rotating ionosphere and is caused by the Hall effect in the E-layer. The fast disturbances are new modes, which are associated with oscillations of the ionospheric electrons frozen in the geomagnetic field and are connected with the large-scale internal vortical electric field generation in the ionosphere. The large-scale waves are weakly damped. The features and the parameters of the theoretically investigated electromagnetic wave structures agree with those of large-scale ULF midlatitude long-period oscillations and magnetoionospheric wave perturbations, observed experimentally in the ionosphere. It is established that because of relevance of the Coriolis and electromagnetic forces, generation of slow planetary electromagnetic waves at the fixed latitude in the ionosphere can give rise to the reverse of local wind structures and to the direction change of general ionospheric circulation. It is considered one more class of the waves, called as slow magnetohydrodynamic waves, on which inhomogeneity of the Coriolis and Ampere forces do not influence. These waves appear as an admixture of the slow Alfven and whistler type perturbations. The waves generate the geomagnetic field from several ten to several hundred nanotesla and more. Nonlinear interaction of the waves under consideration with the local ionospheric zonal shear winds is studied. It is established that planetary ULF electromagnetic waves, at their interaction with the local shear winds, can self-localize in the form of nonlinear solitary vortices moving along the latitude circles westward as well as eastward with velocity different from phase velocity of corresponding linear waves. The vortices are weakly damped and long-lived. They cause geomagnetic pulsations stronger than the linear waves. The vortex structures transfer trapped particles of medium and also energy and heat. That is why such nonlinear vortex structures can be structural elements of strong macroturbulence of the ionosphere.

1. INTRODUCTION

Increasing interest in large-scale planetary ultra-low frequency (ULF) wave perturbations is caused by the fact that ionospheric phenomena like superrotation of the Earth's atmosphere [45], ionospheric precursors of natural processes [24, 25], ionospheric response to the anthropogenic activity [42, 46] fall into the range of these waves. Large-scale wave structures play an important role in the processes of general energy balance and circulation of the atmosphere and ocean. It was supposed that in natural conditions planetary waves are generated in the tropo-stratosphere and reach the ionospheric altitudes. However, theoretical investigation of the wave processes, as the basis for energy transfer from the lower atmosphere to the upper one, shows that the system of the stable zonal winds screens (especially in summer) the upper atmosphere from the influence of large-scale planetary waves generated in the tropo-stratosphere [15, 18]. Conditions, most favourable to upward propagation of only very long planetary waves (with a wavenumber of 1 and 2), are created during equinoxes when the zonal winds change their direction [19]. Nevertheless, a great body of observational data has been stored up till now [7, 9, 13, 14, 37, 47—49, 56]. These data verify the permanent existence of ULF electromagnetic planetary-scale perturbations in E- and F-regions of the ionosphere. Among them, a special attention must be paid to large-scale zonal fast and slow wave perturbations propagating on a fixed latitude along the parallels around the Earth.

In midlatitude E-region of the ionosphere, the slow long-period planetary waves have phase velocity of order of local winds velocity 1—100 m/s, wavelength is equal to 1000 km and more, period varies from several days to tenth of a day, as it is obvious from observations carried out for many years [14, 37, 48, 56]; the fast waves propagate on the latitude circles along the Earth's surface with a velocity of order of 2 to 20 km/s, their periods vary from unit to several tenth of a minute and a few hours; their wavelength is of the order of 1000 km and more. They are revealed in the observations as midlatitude long-period oscillations (MLO) [7, 9, 47, 50]. Their phase velocities are different by one order in daily and nightly conditions in the ionosphere.

In F-region of the ionosphere in middle latitudes, the fast planetary electromagnetic wave perturbations are experimentally observed [9, 47, 49]. They propagate along the latitude circles with phase velocity from several ten to several hundred kilometres per second, with periods from a second to several minutes and with wavelengths of 1000 km and more. They are called as magnetoionospheric wave perturbations (MIWP) [47, 49]. Phase velocities of the fast MIWP have no important daily variations, but they depend on magnetospheric activity of the Sun.

Large phase velocities and their strong variations from day to night (in E-region) make it impossible to identify these perturbations with the ordinary magnetohydrodynamic (MHD) and gyrotropic waves. Amplitude of the geomagnetic pulsations in the waves mentioned above can vary from unit to several ten or several hundred nanotesla.

The ionospheric observations reveal one more class of the electromagnetic perturbations in E- and F-regions, called as the slow MHD waves [28, 50]. These waves (Alfven and whistler type) are insensitive to spatial inhomogeneities of the Coriolis and Ampere forces and are propagated in the ionospheric medium more slowly than the ordinary MHD waves.

In natural conditions, these perturbations are revealed as background oscillations. The forced oscillations of this kind, as can be seen from observations, are generated by impulse action on the ionosphere from above, during magnetic storms [24], or from below, as a result of earthquakes, volcanic eruption or artificial explosions [42, 46]. In the last case the perturbations are revealed as the solitary vortex structures.

It follows from the above discussion that the source of the background wave perturbations must exist in E- and F-regions of the ionosphere. There is a need to reveal the factors guaranteeing generation of such perturbations. Therefore, for adequate description and comprehension of the dynamic processes taking place in the ionospheric medium during formation and propagation of the waves, it is necessary to explore the nonlinear effects taking into account dispersion and dissipation of medium. Further, it is necessary to investigate the interaction of generated electromagnetic ULF waves with the medium, the possibility of generation of the exceeding winds and nonlinear solitary vortical structures on these modes.

This paper is devoted to an investigation of these phenomena.

2. MODEL OF THE MEDIUM AND BASIC EQUATIONS

Ionosphere represents partially ionized triple component plasma. To describe it, we take quasi-hydrodynamic equations which differ from hydrodynamic equations by the presence of «friction force» caused by collision of different particles [17, 28, 29]. Quasi-hydrodynamic equations describe the flows, electromagnetic currents and diffusive processes in the ionospheric plasma. However, the diffusive processes, compressibility and inhomogeneity of the atmosphere are of secondary importance for the large-scale ionospheric perturbations under consideration (wavelengths $\lambda \geq 1000$ km). Thus, we can substantially simplify these equations and obtain the following set of equations [17, 20, 28, 29, 31]:

$$\rho_n \frac{d\mathbf{V}_n}{dt} = \mathbf{F}_n - \rho_i \nu_{in}(\mathbf{V}_n - \mathbf{V}_i) - \rho_e \nu_{en}(\mathbf{V}_n - \mathbf{V}_e), \quad (1)$$

$$\rho_e \frac{d\mathbf{V}_e}{dt} = \mathbf{F}_e - \rho_e \nu_{en}(\mathbf{V}_e - \mathbf{V}_n) - \rho_e \nu_{ei}(\mathbf{V}_e - \mathbf{V}_i) - eN\mathbf{E} - eN\mathbf{V}_e \times \mathbf{B}, \quad (2)$$

$$\rho_i \frac{d\mathbf{V}_i}{dt} = \mathbf{F}_i - \rho_i \nu_{in}(\mathbf{V}_i - \mathbf{V}_n) - \rho_e \nu_{ei}(\mathbf{V}_i - \mathbf{V}_e) - eN\mathbf{E} - eN\mathbf{V}_i \times \mathbf{B}, \quad (3)$$

$$\nabla \cdot \mathbf{V}_n = 0, \ \nabla \cdot \mathbf{V}_e = 0, \ \nabla \cdot \mathbf{V}_i = 0, \quad (4)$$

where indices n, e, and i denote molecules (neutral particles), electrons, and ions; $d/dt = \partial/\partial t + (\mathbf{V}\nabla)$, $\mathbf{V}$ is hydrodynamic velocity of corresponding sorts of particles; $\rho_n = N_n M$, $\rho_e = Nm$, $\rho_i = NM$ are densities; m and M are masses of electrons and ions (molecules), respectively; N_n and N denote concentrations of the neutral particles and charged particles; ν_{ei}, ν_{en}, ν_{in} denote frequencies of collision of electrons with ions and molecules, of ions with molecules, respectively; $\mathbf{E}$ is the strength of the induced electric field; $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, $\mathbf{B}_0$ is the geomagnetic field, $\mathbf{b}$ denotes the perturbation of the geomagnetic field; $\mathbf{F}_n$, $\mathbf{F}_e$, $\mathbf{F}_i$

denote the nonelectromagnetic forces, containing gradients of impulse flux density tensor in general case; $\nabla \equiv (\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$ is the nabla operator.

The equations (1)—(4), state and thermal equations and Maxwell's equations form the close system for each component. For simplification of these equations, we take into account the results of experimental observations of the dynamical processes.

In the ionosphere at heights of 80 to 500 km ($\eta = N/N_n \sim 10^{-9}$—$10^{-4} \ll 1$) non-electromagnetic forces $\mathbf{F}_n$, $\mathbf{F}_e$, and $\mathbf{F}_i$ are proportional to the densities of medium components and, hence, $\eta \ll 1$, $|\mathbf{F}_i| \le |\mathbf{F}_e| \ll |\mathbf{F}_n|$. So, $\mathbf{F}_e$ and $\mathbf{F}_i$ cannot induce high currents. The inertia of electrons and ions can be neglected comparing with the inertia of neutral particles. Taking into account all these circumstances in Eqs (1)—(4), we derive the equation of ionospheric medium motion:

$$\rho_n \frac{d}{dt}\mathbf{V}_n = \mathbf{F}_n + \mathbf{j}\times\mathbf{B}, \qquad (5)$$

where $\mathbf{j} = eN(\mathbf{V}_i - \mathbf{V}_e)$ is the density of electric current.

Let us mention that at heights of E-region and higher the conductivity $\sigma_{||}$ along the geomagnetic force lines substantially exceeds the transversal $\sigma_\perp$ and Hall's σ_H conductivities, i. e., $\sigma_{||} \gg \sigma_\perp$, σ_H. Thus, the component of the electric field $E_{||}$ along the geomagnetic force lines usually is less than the component $E_\perp$ of the electric field across the geomagnetic field, i. e., $|E_{||}| \ll |E_\perp|$ [12]. Taking into account this circumstance and low-frequencity of considered perturbations ($\omega \ll \omega_{e,i}$), Eqs (2) and (3) may be rewritten as

$$-\frac{\nu_{en}}{\omega_e}(\mathbf{V}_e - \mathbf{V}_n) - \frac{\nu_{ei}}{\omega_e}(\mathbf{V}_e - \mathbf{V}_i) + \mathbf{V}_D\times\mathbf{n}_0 = \mathbf{V}_e\times\mathbf{n}_0, \qquad (6)$$

$$-\frac{\nu_{in}}{\omega_i}(\mathbf{V}_i - \mathbf{V}_n) - \frac{\nu_{ei}}{\omega_e}(\mathbf{V}_i - \mathbf{V}_e) + \mathbf{V}_i\times\mathbf{n}_0 = \mathbf{V}_D\times\mathbf{n}_0, \qquad (7)$$

where $\omega_e = eB_0/m$ and $\omega_i = eB_0/M$ denote cyclotron frequencies of electrons and ions, respectively; e is elementary charge; $\mathbf{V}_D = \mathbf{E}\times\mathbf{B}_0/\mathbf{B}_0^2$ is the electron drift velocity; $\mathbf{n}_0 = \mathbf{B}_0/B_0$ is a unit vector along the strength of the geomagnetic field. In the ionosphere, $\omega_e \approx 10^7$ s^{-1}, $\omega_i \approx 150$—300 s^{-1}, the collision frequency reaches its maximal value $\nu_{ei} \approx 10$ kHz, $\nu_{in} \approx 10$ kHz, $\nu_{en} \approx 100$ kHz at heights of 80 to 500 km in the lower layer of the ionosphere and quickly decreases in proportion to height. Thus, we can conclude that $\nu_{ei}/\omega_e \ll 1$, $\nu_{en}/\omega_e \ll 1$ in E- and F-layers of the ionosphere. This means that electron component of the ionospheric plasma is always magnetized in this region of the upper atmosphere. Taking into account these inequalities, Eqs (6) and (7) can be reduced to the following form:

$$\mathbf{V}_D\times\mathbf{n}_0 = \mathbf{V}_e\times\mathbf{n}_0 \Rightarrow \mathbf{V}_e = \mathbf{V}_D \Rightarrow E = -\mathbf{V}_e\times\mathbf{B}_0, \qquad (8)$$

$$\mathbf{V}_i = \mathbf{V}_n + \mathbf{j}\times\mathbf{B}_0/(\rho\nu_i),\ \nu_i = N\nu_{in}/N_n. \qquad (9)$$

Therefore, in E- and F-layers of the ionosphere, the electron component of the ionospheric plasma is always magnetized, moves with electron drift velocity ($\mathbf{V}_e = \mathbf{V}_D$) and the electrons are frozen into the geomagnetic field $\mathbf{B}_0$ ($\partial\mathbf{b}/\partial t = \nabla\times\mathbf{V}_e\times\mathbf{B}_0$). As for ion equation (9), for E-region of the ionosphere (altitudes are 80—150 km), we have $\omega_i/\nu_{in} \sim 0.01 \ll 1$, the second term in the right-hand side can be neglected in comparison with the first one and we have $\mathbf{V}_i = \mathbf{V}_n$. This means that in E-region of the ionosphere the neutral winds entrain ion components completely.

Multiplying Eq. (8) by $\mathbf{B}_0$, we derive the important equality $\mathbf{E}\cdot\mathbf{B}_0 = 0$, $\Rightarrow \mathbf{E}\perp\mathbf{B}_0$, i. e., internal electric field generated in E- and F-layers of the ionosphere is always perpendicular to the geomagnetic field $\mathbf{B}_0$.

Using Maxwell's equations, we get the close system of equations (5), (8), and (9):

$$\frac{\partial}{\partial t}\mathbf{B} = -\nabla\times\mathbf{E}, \quad \mathbf{j} = \frac{1}{\mu_0}\nabla\times\mathbf{B}, \qquad (10)$$

where μ_0 is the permeability of free space. Excluding $\mathbf{E}$ and $\mathbf{j}$ with the use of Eq. (10) and taking into account that for considered wavy processes $\mathbf{F}_n/\rho = -\nabla P'/\rho + \mathbf{g} + \mathbf{V}\times 2\Omega_0$, dropping index n for velocity and density of the neutral particles, we obtain the system of nonlinear magneto-hydrodynamic equations for E- and F-layers of the ionosphere:

$$\frac{d}{dt}\mathbf{V} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \mathbf{V}\times 2\Omega_0 + \frac{1}{\mu_0\rho}\nabla\times\mathbf{B}\times\mathbf{B}, \qquad (11)$$

$$\frac{\partial}{\partial t}\mathbf{B} = \nabla\times\mathbf{V}_e\times\mathbf{B} = \nabla\times\mathbf{V}\times\mathbf{B} - \frac{\alpha}{\mu_0}\nabla\times\nabla\times\mathbf{B}\times\mathbf{B} + \frac{1}{\mu_0\rho\nu_i}\nabla\times\nabla\times\mathbf{B}\times\mathbf{B}\times\mathbf{B}, \qquad (12)$$

where Hall's parameter α in general case is $\alpha = 1/(\mathbf{B}_0\sigma_H)$, $\sigma_H = e^2N[\omega_e/(m(\omega_e^2 + \nu_e^2)) - \omega_i/(M(\omega_i^2 + \nu_{in}^2))]$ is Hall's conductivity; P is the perturbation of gas-kinematics pressure; $\nu_e = \nu_{ei} + \nu_{en}$; $\nu_i = \eta\nu_{in}$; $\mathbf{g}$ is the free fall acceleration; Ω_0 is angular velocity of the rotation of the Earth. For E-region of the ionosphere, we have $\omega_e \gg \nu_{en}$, $\omega_i \ll \nu_{in}$ and $\alpha = 1/(eN)$ (Hall's conductivity is disappeared higher than 150 km, $\sigma_H \to 0$).

It is obvious from equations (5) and (11) that the electromagnetic Ampere force $\mathbf{F}_A$, acting on a unit mass of medium, is determined by the expression:

$$\frac{\mathbf{F}_A}{\rho} = \frac{1}{\rho}\mathbf{j}\times\mathbf{B}_0 = \frac{1}{\mu_0\rho}\nabla\times\mathbf{b}\times\mathbf{B}_0 \approx$$

$$\approx \mathbf{V}\times 2\Omega_i - \mathbf{V}_D\times 2\Omega_i = \mathbf{u}\times 2\Omega_i, \quad (13)$$

where $2\Omega_i = \eta e\mathbf{B}_0/M = \eta\omega_i$; $\mathbf{u} = \mathbf{V} - \mathbf{V}_D$. From Eq. (13) it follows that the Ampere electromagnetic force $\mathbf{F}_A$ acting on an unit mass of the medium (or acceleration), $\mathbf{F}_A/\rho = \mathbf{u}\times 2\Omega_i$, has the same structure as the Coriolis acceleration $\mathbf{V}\times 2\Omega_0$. Therefore, the Ampere force must act on atmospheric-ionospheric medium similar to the Coriolis force. Similarity of the Ampere and Coriolis forces means that new modes of the large-scale electromagnetic oscillations must be generated due to inhomogeneity of the geomagnetic field $\mathbf{B}_0$ as well as Rossby-type usual planetary waves are generated due to inhomogeneity of angular velocity of the Earth's rotation Ω_0. In this case, as it will be shown below, the first term of the electromagnetic force $\mathbf{F}_A$ (13) caused by velocity of medium motion (dynamo field $\mathbf{E}_d = \mathbf{V}\times\mathbf{B}_0$) generates the slow Rossby-type electromagnetic waves; the second term of the electromagnetic force in (13) is appeared due to the vortex electric field $\mathbf{E}_V = \mathbf{V}_D\times\mathbf{B}_0$ and generates the fast electromagnetic waves.

Large-scale (planetary) waves are slightly damped due to turbulent and molecular viscosity and thermal conductivity, since the Reynolds number is large for such motions. Indeed, some estimations show that, for planetary-scale ($L \sim$ 1000—10000 km) perturbations in E-region of the ionosphere, magnetic Reynolds number ($R_m = \omega L^2/\nu_H \sim 1/\alpha$, where L and ω are characteristic linear scale and frequency of the perturbations, $\nu_H = 1/(\mu_0\sigma_H)$) reaches a rather small value ($R_m \sim 1$) [20, 28, 31]. Therefore, it is necessary to preserve Hall's term ($\propto \alpha$) in the induction equation (12), but the last one can be neglected due to the condition $\sigma_H \gg \sigma_\perp \approx \sigma_H\omega_i/\nu_{in}$ (where $\sigma_\perp$ is the transversal conductivity). Moreover, the large-scale ion motion velocity $\mathbf{V}_i = \mathbf{V}_n$ in E-region of the ionosphere, i. e., neutrals completely entrain ions. Correspondingly, the effect of ion drag on the large-scale motion in the ionospheric E-region can be neglected (i. e., the last term in (12)). In F-region of the ionosphere, where the Hall effect is not important, the last term in Eq. (12) also can be neglected for planetary-scale perturbations in the first approximation as far as Reynolds number $R_{m\perp} = \omega L^2/\nu_\perp$ ($\nu_\perp = 1/(\mu_0\sigma_\perp)$) is of the order of 100 [20, 28, 31]. Really, it can be concluded from observations that the planetary waves propagate over great distances in the ionosphere without substantial changes [9, 13, 14, 47, 48].

It is known that planetary Rossby waves are damped only due to the drag friction against the Earth's surface [23, 26]. Therefore, some authors suppose that it will be useful for qualitative understanding of the role of internal dissipation for large-scale flows to model the dissipative force in the form of the Reyleigh friction force, proportional to the velocity $\mathbf{F}_R = -\Lambda\mathbf{V}$, in the equation of motion [19, 23]. Here, Λ is the constant coefficient of surface friction of atmospheric layers, which reaches 10^{-5} s^{-1} at the ionospheric altitudes.

Hence, for E- and F-layers of the ionosphere, magnetohydrodynamic Eqs (11) and (12) may be written in the following form:

$$\frac{d}{dt}\mathbf{V} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \mathbf{V}\times 2\Omega_0 + \frac{1}{\mu_0\rho}\nabla\times\mathbf{b}\times\mathbf{B} - \Lambda\mathbf{V}, \quad (14)$$

$$\frac{d}{dt}\mathbf{b} = (\mathbf{B}\cdot\nabla)\mathbf{V} - (\mathbf{V}\cdot\nabla)\mathbf{B}_0 - \frac{\alpha}{\mu_0}\nabla\times\nabla\times\mathbf{b}\times\mathbf{B}, \quad (15)$$

$$\nabla\cdot\mathbf{V} = 0,\ \nabla\cdot\mathbf{b} = 0,\ \nabla\times\mathbf{B}_0 = 0,\ \nabla\cdot\mathbf{B}_0 = 0. \quad (16)$$

Here $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$, $\mathbf{B} = B_{0y}\mathbf{e}_y + B_{0z}\mathbf{e}_z$, $B_{0y} = -B_e\sin\theta'$, $B_{0z} = -2B_e\cos\theta'$, B_e = 32 μT is the value of geomagnetic field induction on the equator; $2\Omega_0 = 2\Omega_{0y}\mathbf{e}_y + 2\Omega_{0z}\mathbf{e}_z$, $2\Omega_{0y} = 2\Omega_0\sin\theta$, $2\Omega_{0z} = 2\Omega_0\cos\theta$, $\Omega_0 = 7.3\cdot 10^{-5}$ s^{-1}; $\theta = \pi/2 - \varphi'$, φ' is geomagnetic latitude; $\theta = \pi/2 - \varphi$, φ is geographical latitude; $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ denote unit vectors along x, y, z axes, respectively.

The close system of nonlinear Eqs (14) and (15) contains six scalar equations and gives the possibility to calculate six unknown quantities: V_x, V_y, V_z, b_x, b_y, b_z. On determining the values $\mathbf{V}$ and $\mathbf{b}$, pressure P will be determined from Eq. (11) in quadrature (as far as ρ = const); current density $\mathbf{j}$ and electric field are calculated from Maxwell's equations (10); electron velocity is determined from the expression $\mathbf{V}_e = \mathbf{V}_D$ and ion velocity is determined from the formula (9). Thus, the initial-boundary problem of large-scale dynamics of triple component plasma for E- and F-layers of the ionosphere is solved completely.

The planetary wavy perturbations discussed have wavelength of order of the Earth's radius R. Therefore, it is naturally to consider the creation of large-scale perturbations in the Earth's atmosphere in spherical coordinate system [4]. However, some mathematical difficulties arising from theoretical investigation of equations obtained oblige us to consider the problem in «standard» coordinate system [21, 23, 26, 40]. In this system, x-axis is directed to the east towards the parallels, y-axis is directed to the north along meridian, z-axis is directed vertically up (local Cartesian coordinate system). Length elements dx,

dy, dz are connected with the parameters of the spherical coordinate system λ, θ, r by the following approached formulas: $dx = R\sin\theta d\lambda$, $dy = -Rd\theta$, $dz = dr$. The velocities are: $V_x = V_\lambda$, $-V_y = V_\theta$, $V_z = V_r$. Here, λ is the longitude, R is the Earth's radius, r is the distance from the Earth's centre along its radius. This system is not equivalent to the ordinary Cartesian frame of reference as far as directions of the axes vary with the atmospheric particle motion from one point to the other one. However, for the large-scale processes, the terms related with spatial variations of coordinate axes, in equations of thermo-hydrodynamic atmosphere may be dropped in the first approximation [26, 40, 52]. Therefore, equation of motion in spherical coordinate system (taking into account relations between coordinates, mentioned above) has the same form as in the Cartesian frame of reference. This procedure simplifies the problem and investigation of dynamics of the large-scale processes in the atmosphere [21, 23, 26, 31, 40, 52] and therefore, it will be used also for magnetoactive ionospheric medium.

The method of «frozen-in» coefficients in dynamic equations will be also used below. This method is known as β-approximation (β-plane) [21, 23, 26, 40, 52] in spherical hydrodynamics and meteorology. In this approximation the parameters $\Omega_0(\theta)$, $\nabla\Omega_0(\theta)$, $\mathbf{B}_0(\theta')$, and $\nabla\mathbf{B}_0(\theta')$ are constant during integration of dynamical equations, in view of $\theta = \theta_0$, $\theta' = \theta'_0$. Medium motion is considered near θ_0 and θ'_0, i. e., average values of adjunction of the geographical φ_0 and the geomagnetic φ'_0 latitudes, respectively. In this case, dynamical equations transform into equations with constant coefficients, which may be investigated by plane wave method. Application of β-approximation (or β-plane) leads to simple results, which gives the possibility to reveal more important features of motion on a rotating sphere, which differs from motion on a rotating plane. Further we assume that geographical latitude φ coincides with geomagnetic latitude φ', i. e., $\theta = \theta'$, $\theta_0 = \theta'_0$.

According to the experimental data on ionospheric E- and F-regions [13, 21, 23, 26, 31, 40, 43], the ratio of the characteristic vertical velocity V_V to the horizontal one V_h: $V_V/V_h \le L_V/L_h < 0.01$, where L_V and L_h are the characteristic vertical and horizontal size of disturbances, respectively. Thus, the large-scale motions in the ionosphere are mainly two-dimensional and quasi-horizontal. Thus, velocity vector has two nonzero components $\mathbf{V} = (V_x, V_y, 0)$. In these conditions, one can easily obtain two scalar equations for V_x and V_y from Eq. (14):

$$\left(\frac{\partial}{\partial t} + V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y}\right)V_x = -\frac{1}{\rho}\frac{\partial P}{\partial x} - \frac{B_{0z}}{\mu_0\rho}\left(\frac{\partial \mathbf{b}_z}{\partial x} - \frac{\partial b_x}{\partial z}\right) - \frac{B_{0y}}{\mu_0\rho}\left(\frac{\partial \mathbf{b}_y}{\partial x} - \frac{\partial \mathbf{b}_x}{\partial y}\right) + 2\Omega_z V_y - \Lambda V_x + \frac{b_z}{\mu_0\rho}\left(\frac{\partial \mathbf{b}_x}{\partial z} - \frac{\partial \mathbf{b}_z}{\partial x}\right) - \frac{b_y}{\mu_0\rho}\left(\frac{\partial b_y}{\partial x} - \frac{\partial \mathbf{b}_x}{\partial y}\right), \quad (17)$$

$$\left(\frac{\partial}{\partial t} + V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y}\right)V_y = -\frac{1}{\rho}\frac{\partial P}{\partial y} - \frac{B_{0z}}{\mu_0\rho}\left(\frac{\partial \mathbf{b}_z}{\partial y} - \frac{\partial b_y}{\partial z}\right) - 2\Omega_{0z}V_x - \Lambda V_y + \frac{b_x}{\mu_0\rho}\left(\frac{\partial \mathbf{b}_y}{\partial x} - \frac{\partial \mathbf{b}_x}{\partial y}\right) - \frac{b_z}{\mu_0\rho}\left(\frac{\partial \mathbf{b}_z}{\partial y} - \frac{\partial b_y}{\partial z}\right). \quad (18)$$

Analogously, from induction equation (15), one can derive three scalar equations for the components of perturbed magnetic field $\mathbf{b}$:

$$\left(\frac{\partial}{\partial t} + V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y}\right)b_x = \left(B_{0y}\frac{\partial}{\partial y} + B_{0z}\frac{\partial}{\partial z}\right)V_x - \frac{\alpha}{\mu_0}\left(B_{0y}\frac{\partial}{\partial y} + B_{0z}\frac{\partial}{\partial z}\right)\left(\frac{\partial b_z}{\partial y} - \frac{\partial \mathbf{b}_y}{\partial z}\right) + \left(b_x\frac{\partial}{\partial x} + \mathbf{b}_y\frac{\partial}{\partial y} + \mathbf{b}_z\frac{\partial}{\partial z}\right)V_x - \frac{\alpha}{\mu_0}\left(b_x\frac{\partial}{\partial x} + \mathbf{b}_y\frac{\partial}{\partial y} + \mathbf{b}_z\frac{\partial}{\partial z}\right)\left(\frac{\partial \mathbf{b}_z}{\partial y} - \frac{\partial \mathbf{b}_y}{\partial z}\right) + \frac{\alpha}{\mu_0}\left[\left(\frac{\partial b_z}{\partial y} - \frac{\partial b_y}{\partial z}\right)\frac{\partial}{\partial x} + \left(\frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x}\right)\frac{\partial}{\partial y} + \left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right)\frac{\partial}{\partial z}\right]b_x, \quad (19)$$

$$\left(\frac{\partial}{\partial t} + V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y}\right)b_y = \left(B_{0y}\frac{\partial}{\partial y} + B_{0z}\frac{\partial}{\partial z}\right)V_y - \beta_{B2}V_y + \left(b_x\frac{\partial}{\partial x} + b_y\frac{\partial}{\partial y} + b_z\frac{\partial}{\partial z}\right)V_y - \frac{\alpha}{\mu_0}\left[\left(B_{0y}\frac{\partial}{\partial y} + B_{0z}\frac{\partial}{\partial z}\right)\left(\frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x}\right) - \beta_{B2}\left(\frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x}\right) - \beta_{B1}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right)\right] + \frac{\alpha}{\mu_0}\left[\left(\frac{\partial b_z}{\partial y} - \frac{\partial b_y}{\partial z}\right)\frac{\partial}{\partial x} + \left(\frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x}\right)\frac{\partial}{\partial y} + \left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right)\frac{\partial}{\partial z}\right]b_y - \frac{\alpha}{\mu_0}\left(b_x\frac{\partial}{\partial x} + b_y\frac{\partial}{\partial y} + b_z\frac{\partial}{\partial z}\right)\left(\frac{\partial b_x}{\partial z} - \frac{\partial b_z}{\partial x}\right), \quad (20)$$

$$\left(\frac{\partial}{\partial t}+V_x\frac{\partial}{\partial x}+V_y\frac{\partial}{\partial y}\right)b_z=-\beta_{B1}V_y-$$
$$-\frac{\alpha}{\mu_0}\left(B_{0y}\frac{\partial}{\partial y}+B_{0z}\frac{\partial}{\partial z}\right)\left(\frac{\partial b_y}{\partial x}-\frac{\partial b_x}{\partial y}\right)+$$
$$+\frac{\alpha}{\mu_0}\left[\beta_{B1}\left(\frac{\partial b_x}{\partial z}-\frac{\partial b_z}{\partial x}\right)-\beta_{B2}\left(\frac{\partial b_y}{\partial x}-\frac{\partial b_x}{\partial y}\right)\right]-$$
$$-\frac{\alpha}{\mu_0}\left(b_x\frac{\partial}{\partial x}+b_y\frac{\partial}{\partial y}+b_z\frac{\partial}{\partial z}\right)\left(\frac{\partial b_y}{\partial x}-\frac{\partial b_x}{\partial y}\right)+$$
$$+\frac{\alpha}{\mu_0}\left[\left(\frac{\partial b_z}{\partial y}-\frac{\partial b_y}{\partial z}\right)\frac{\partial}{\partial x}+\left(\frac{\partial b_x}{\partial z}-\frac{\partial b_z}{\partial x}\right)\frac{\partial}{\partial y}+\right.$$
$$\left.+\left(\frac{\partial b_y}{\partial x}-\frac{\partial b_x}{\partial y}\right)\frac{\partial}{\partial z}\right]b_z. \qquad (21)$$

Here the parameters $\beta_{B1,2}=\partial B_{0z,y}/\partial y$ characterize the spatial (latitudinal) inhomogeneity of the geomagnetic field.

The system of equations (17)—(21), in corresponding initial and boundary conditions, describes nonlinear evolution of the spatial three-dimensional large-scale electromagnetic perturbations in noncompresible ionospheric E- and F-regions.

Further, only for simplicity, we shall neglect the dependence of the perturbed (wavy) quantities on the coordinate axis z. Moreover, for considered large-scale zonal wavy perturbations (as it will be shown below) dependence on axis z is not substantial and, consequently, it can be assumed that $\partial/\partial z\approx 0$. In general case $\partial/\partial z$ can be conserved, but in the dynamical equations (17)—(21) it can be integrated according to vertical conductive layer (according to axis z) in corresponding boundary conditions on the Earth's surface and in magnetosphere. Herewith, submitting the tensor components of conductivity, integrated with respect to height, the results will be identical to that for $\partial/\partial z\approx 0$, and only some constants will be changed by the order of unit.

Then, substituting z component of vector potential A of the magnetic field by formula $(\nabla\times\mathbf{b})_z=-\nabla_\perp^2 A$, and by the condition of noncompressibility of medium (4), we represent the velocity component in the form $V_x=-\partial\Psi/\partial y$, $V_y=\partial\Psi/\partial x$, where Ψ is the stream function. Then, operating on (17) by operator $\partial/\partial y$, on (18) by operator $\partial/\partial x$ and subtracting the second equation from the first one, we obtain:

$$\left(\frac{\partial}{\partial t}+\Lambda\right)\nabla_\perp^2\Psi+\beta\frac{\partial\Psi}{\partial x}-$$
$$-\frac{1}{\mu_0\rho}\left[\beta_{B1}\frac{\partial b_z}{\partial x}-\left(\beta_{B2}+B_{0y}\frac{\partial}{\partial y}\right)\nabla_\perp^2 A\right]=$$
$$=J(\nabla_\perp^2\Psi,\Psi)+\frac{1}{\mu_0\rho}J(A,\nabla_\perp^2 A), \qquad (22)$$

where the parameter β characterizes the value of gradient of the angular velocity of the Earth's rotation $\beta=\partial 2\Omega_{0z}/\partial y$ or Rossby parameter.

Analogously, from equations (19)—(21) we get:

$$\left(\frac{\partial}{\partial t}-C_{B1}\frac{\partial}{\partial x}\right)\nabla_\perp^2 A+B_{0y}\frac{\partial}{\partial y}\nabla_\perp^2\Phi-$$
$$-\beta_{B2}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\Phi=J(\nabla_\perp^2 A,\Phi)+J(A,\nabla_\perp^2\Phi)+$$
$$+2J\left(\frac{\partial A}{\partial x},\frac{\partial\Phi}{\partial x}\right)+2J\left(\frac{\partial A}{\partial y},\frac{\partial\Phi}{\partial y}\right), \qquad (23)$$

$$\left(\frac{\partial}{\partial t}+C_{B1}\frac{\partial}{\partial x}\right)b_z+\beta_{B1}\frac{\partial\Psi}{\partial x}-\frac{\alpha}{\mu_0}\left(\beta_{B2}+B_{0y}\frac{\partial}{\partial y}\right)\nabla_\perp^2 A=$$
$$=J(b_z,\Psi)-\frac{\alpha}{\mu_0}J(A,\nabla_\perp^2 A). \qquad (24)$$

Here, we used the abbreviations: $C_{B1,2}=\alpha\beta_{B1,2}/\mu_0$, $\nabla_\perp^2=\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2\Phi\equiv\Psi+\alpha b_z/\mu_0andJ(a,b)$ $=\partial a/\partial x\cdot\partial b/\partial y-\partial a/\partial y\cdot\partial b/\partial x$.

From the set of Eqs (22)—(24) we can determine the temporal evolution of energy E:

$$\frac{\partial E}{\partial t}=\frac{\partial}{\partial t}\left\{\frac{1}{2}\int[\rho(\nabla_\perp\Psi)^2+\mu_0^{-1}((\nabla_\perp A)^2+b_z^2)]\,dxdy\right\}=$$
$$=-\rho\Lambda\int(\nabla_\perp\Psi)^2dxdy, \qquad (25)$$

and the potential enstrophy Q of wave perturbations:

$$\frac{\partial Q}{\partial t}=\frac{\partial}{\partial t}\left\{\frac{1}{2}\int[\rho(\Delta_\perp\Psi)^2+\mu_0^{-1}((\Delta_\perp A)^2+(\nabla_\perp b_z)^2)]\,dxdy\right\}=$$
$$=-\rho\Lambda\int(\Delta_\perp\Psi)^2dxdy. \qquad (26)$$

Energy E and entropy Q of the waves are conserved in the nondissipative case ($\Lambda=0$).

The close system of nonlinear equations in partial derivations (22)—(26) describes the nonlinear dynamics of the planetary low-frequency electromagnetic wave perturbations in the ionospheric medium.

3. IONOSPHERIC PLANETARY ELECTROMAGNETIC LINEAR WAVY PERTURBATIONS IN THE FRAMEWORK OF β-PLANE

Let us begin an analysis of the system of dynamic equations (22)—(24) for the planetary-scale small-amplitude perturbations, for which these equations can be linearized.

As it was mentioned above, we consider a motion in neighbourhood of a fixed latitude $\varphi=\varphi_0(\theta=\theta_0)$. Then, all the coefficients in equations (22)—(24) became constant and the solutions can be sought in the form of the plane waves: $\exp\{i(k_x x+k_y y-\omega t)\}$, where $\mathbf{k} = = (k_x, k_y, 0)$ is wavy vector, ω is frequency of the perturbations. In these conditions, we get the following dispersion equation from Eqs (22)—(24):

$$\rho\alpha^2 k_\perp^2\left(\omega + \frac{k_x}{k_\perp^2}\beta + i\Lambda\right)[\omega^2 - (k_x^2 C_B^2 + \omega_H^2 - k_y^2 C_{B2}^2 + 2ik_y^2 C_{B2}^2 k_y R\mathrm{tg}\theta_0)] - \mu_0(\omega + k_x C_{B1})(k_x^2 C_B^2 + \omega_H^2 - k_y^2 C_{B2}^2 + 2ik_y^2 C_{B2}^2 k_y R\mathrm{tg}\theta_0) = 0, \quad (27)$$

which has the third order with respect to frequency ω. Here $k_\perp^2 = k_x^2 + k_y^2 = k^2$, $C_B^2 = C_{B1}^2 + C_{B2}^2$ and ω_H is frequence of the helicons (whistlers), i. e.,

$$\omega_H = \frac{\alpha k}{\mu_0}(\mathbf{k}\cdot\mathbf{B}_0), \quad (28)$$

herewith, the parameters characterizing wavy perturbations in (27) are determined by the expression:

$$\beta = \frac{\partial 2\Omega_{0z}}{\partial y} = -\frac{\partial 2\Omega_{0z}}{R\partial\theta} = \frac{2\Omega_0}{R}\sin\theta_0 > 0,$$

$$C_{B1} = \frac{\alpha}{\mu_0}\beta_{B1} = -\frac{2\alpha B_e}{\mu_0 R}\sin\theta_0 < 0,$$

$$C_{B2} = \frac{\alpha}{\mu_0}\beta_{B2} = \frac{\alpha B_e}{\mu_0 R}\cos\theta_0 > 0; \quad (29)$$

they do not depend on the spatial coordinates and are constants.

Cubic equation (27) has three classes of eigen solutions. Let us carry out an analysis of these solutions for different layers of the ionosphere.

3.1. Low-Frequency Long-Scale Electromagnetic Waves in E-Layer

3.1.a. Slow MHD Waves

Let us begin investigation of the roots of equation (27) for quasi-horizontal waves, characterized by periods from several ten minutes to two hours, with the wavelength from several hundred to several thousand kilometres and propagating in the ionosphere with a velocity of 1—2 km/s. Contrary to the classical MHD waves, these waves entrain into collective motion not only ionized but also neutral components of the ionospheric plasma. This process leads to decrease of the phase velocity of the waves by the factor $\eta = N/N_n$ (η is plasma ionization rate). That is why they are called as slow MHD waves [28, 50].

The slow MHD waves are insensitive to the Coriolis force as well as to the latitudinal inhomogeneity of the geomagnetic field. Therefore, in this limiting case dispersion relation (27) is reduced to

$$(\omega + i\Lambda)(\omega^2 - \omega_H^2) - \omega_{AM}^2\omega = 0, \quad (30)$$

where ω_{AM}^2 is the square of the slow modified Alfven frequency,

$$\omega_{AM} = \frac{(\mathbf{k}\cdot\mathbf{B}_0)}{(\mu_0\rho)^{1/2}} = k_y V_{AM}\sin\theta_0, \quad V_{AM} = \frac{B_e}{(\mu_0\rho)^{1/2}}. \quad (31)$$

Representing perturbation frequency ω by the sum of real ω_0 (the eigen frequency) and imaginary γ (decrement) parts:

$$\omega = \omega_0 + i\gamma \ (|\gamma| \ll \omega_0),$$

we obtain from (30):

$$\omega_0(\omega_0^2 - \omega_H^2 - \omega_{AM}^2) = 0,$$

$$\gamma(3\omega_0^2 - \omega_H^2 - \omega_{AM}^2) = -\Lambda(\omega_0^2 - \omega_H^2). \quad (32)$$

From the first equation (32) we have following three eigen frequencies for considered perturbations:

$$(\omega_0)_{1,2} = \pm(\omega_{AM}^2 + \omega_H^2)^{1/2}, \quad (\omega_0)_3 = 0. \quad (33)$$

Trivial solution $(\omega_0)_3$ (corresponding to the stationary state of medium $\partial/\partial t = 0$) also has a physical meaning (we shall consider this in the end of the next section).

Correspondingly, from the second equation (32), decrement can be determined:

$$\gamma = -\frac{\omega_{AM}^2}{2\omega_0^2}\Lambda. \quad (34)$$

In the long wavelength limit, when $\rho_i/\rho \gg k^2c^2/\omega_p^2$ (here c is the light speed, $\omega_p = (Ne^2/\varepsilon_0 M)^{1/2}$ is the ion plasma frequency, $\varepsilon_0 = 1/\mu_0 c^2$ is the permittivity of free space), the waves propagate along the meridians to the north as well as to the south as slow modified Alfven waves:

$$\omega_0 = \pm\omega_{AM} = \pm\frac{k_y B_e}{(\mu_0 M N_n)^{1/2}}\sin\theta_0, \quad \gamma = -\frac{\Lambda}{2}, \quad (35)$$

and exhibiting a weak damping due to Rayleigh friction.

In the short wavelength limit, when $\rho_i/\rho \ll k^2c^2/\omega_p^2$, it follows from equations (32) and (33) that in E-layer of the ionosphere the helicons (whistlers) are propagated, i. e.,

$$\omega_0 = \pm\omega_H = \pm\frac{kk_y B_{0y}}{eN\mu_0}, \quad \gamma = -\frac{\rho_i\omega_p^2}{\rho k^2c^2}\frac{\Lambda}{2}, \quad (36)$$

which are damped very weakly.

The existence of slow MHD waves with speeds of the order of 1—2 km/s in E-region [50] can not be explained in terms of ordinary atmospheric gravity waves (AGW), since their typical speeds at ionospheric altitudes do not exceed 700 m/s. Although these speeds are larger than those for AGW's in the ionosphere, they are still small for ordinary MHD type waves. The physical reason for such properties of the MHD perturbations in the ionosphere is that the plasma in E-region is not completely frozen into the magnetic field. Due to the fact that the ions are completely dragged by the neutral particles ($\mathbf{V} = \mathbf{V}_i$), any perturbation arising in the ionized component immediately exchanges energy with the neutral component and thus it starts to propagate with the Alfven speed, that is loaded by the heavy and dense neutral population, i. e., with the speed $B_{0y}/(\mu_0\rho)^{1/2}$. The value of that decelerated Alfven speed is apparently much smaller than that for the plasma (ion) component, which is $B_{0y}/(\mu_0\rho_i)^{1/2}$. In the E-layer $\rho_i/\rho \approx \approx N/N_n \approx 10^{-8}$—$10^{-9}$ and owing to this fact the loading effect is quite substantial.

As can be seen from (35), the phase velocity of the modified Alfven wave is $V_{ph} = \omega_0/k_y = B_{0y}/(\mu_0\rho)^{1/2}$. For typical values of the neutral particle number density in the ionospheric E-layer, $N_n = 10^{18}$—10^{19} m^{-3}, and $B_e \approx 30$ μT at middle latitudes, we obtain $V_{ph} \approx \approx 1$ km/s. The corresponding wavelength of the wave $\lambda = V_{ph}T$ (where T is the period of oscillations) has the characteristic value $\lambda \sim 1000$ km. Thus, the slow MHD waves with a phase velocity of 1 to 2 km/s [28, 50] can be identified with the slow modified Alfven waves, which smoothly are converted into helicons governed by the neutral component of E-layer. According to (35), they propagate northwards or southward along the meridian and are induced by the y-component of the geomagnetic field.

As it follows from (27), a weak dispersion of the modified Alfven waves is caused by the spatial inhomogeneity of the Coriolis force, i. e., by the β-effect:

$$\omega_0 = \pm\omega_{AM} - \frac{k_x}{k_\perp^2}\frac{\beta}{2}. \quad (37)$$

A value of the geomagnetic pulsations generated by the waves under consideration can be estimated with the help of the equation (15) and we can obtain the formula $b_y \approx B_{0y}V/V_{ph}$, where V is characteristic amplitude of local wind velocity in E-region of the ionosphere and V_{ph} is phase velocity of the waves. For typical values of amplitude of the wind velocity V in E-region of the ionosphere, $V \sim 5$ m/s, and $V_{ph} \sim 1$ km/s, $B_0 \sim 30$ μT, we have $b_y \approx 150$ nT.

3.1.b. Fast and Slow Planetary Electromagnetic Modes

For planetary-scale waves, we can not neglect the latitude variations of angular velocity of the Earth's rotation $\Omega_0(\theta)$ and the geomagnetic field $\mathbf{B}_0(\theta)$. In the present paper we use the term «planetary waves» in reference to the class of large-scale perturbations (with wavelengths of the order of the Earth's radius and more) having periods of order of a few seconds, a few hours and more, propagating along the latitude circles (along x-axis), i. e., they are zonal waves. These zonal waves are horizontal-transversal, i. e., these particles oscillate along y-axis, but the waves propagate along x-axis. Exactly such waves are observed in above-mentioned experimental works [7, 9, 14, 37, 47—49, 56]. Naturally, for such waves the solution of initial dynamic equations (22)—(24) can be sought in the form $\exp\{i[k_x x - \omega t]\}$, at which $(\mathbf{k}\cdot\mathbf{B}_0) \equiv 0$ and according to (31) and (28) the slow MHD waves, helicon and Alfven type waves will be filtered out. This is equal to the assumption $k_y R \operatorname{tg}\theta_0 \ll 1$ or $k_y \to 0$ in general dispersion equation (27). The last condition means that the oscillations do not propagate along y-axis, i. e., the waves do not propagate along y-axis and in this direction only the particles will oscillate.

Thus, for planetary zonal wave perturbations, the dispersion equation (27) reduces to the following form:

$$\omega\,[\omega(\omega - \omega_R) - \omega'^2_B] + i\Lambda(\omega^2 - k_x^2C_B^2) + \omega_B^2\omega'_R = 0, \quad (38)$$

where $\omega_R = -\beta/k_x$ is frequency of the Rossby waves, $\omega'_R = -\beta'/k_x$, $\beta' = \beta + eN\beta_{B1}/\rho$ is magnetic analog of the Rossby parameter, $\omega_B^2 = k_x^2C_B^2$, $\omega_B^2 = k_x^2C_B^2$, $\omega'^2_B = = (k_x^2 + k_0^2)C_B^2 = K^2C_B^2$, $k_0^2 = \mu_0/(\alpha^2\rho) = N\omega_p^2/(N_n c^2)$.

Similar to the previous section, decomposing the frequency into its real and imaginary parts, we find from (38):

$$\omega_0\,[\omega_0(\omega_0 - \omega_R) - \omega'^2_B] = -\,\omega_B^2\omega'_R,$$

$$\gamma = -\frac{\omega_0^2 - \omega_B^2}{3\omega_0^2 - 2\omega_0\omega_R - \omega'^2_B}\Lambda. \quad (39)$$

In the high-frequency band when the conditions $\omega_0 \sim \omega_B \sim \omega'_B \gg \omega_R$, ω'_R are satisfied (or in the short wavelength limit when $Kk_x \gg \beta'/C_B$ and $k_x^2 \gg \beta/C_B$), from (39) one finds dispersions for the fast planetary electromagnetic modes:

$$\omega_0^f = \pm\omega'_B = \pm(k_x^2 + k_0^2)^{1/2}C_B =$$

$$= \pm\frac{B_e}{eN\mu_0}\frac{(1 + 3\sin^2\theta_0)^{1/2}}{R}(k_x^2 + k_0^2)^{1/2},$$

$$\gamma^f = -\frac{k_0^2}{k_x^2 + k_0^2}\frac{\Lambda}{2}. \quad (40)$$

The fast mode (40) is an additional new mode of own oscillations of E-region of the ionosphere. Wave has the electromagnetic origin and can exist only in the presence of the latitudinal gradient of the equilibrium magnetic field, which is inevitably inherent in a dipole type magnetic configuration such as that of the Earth's ionosphere-magnetosphere. The linear fast waves (40) can propagate along the latitude circles to the west as well as to the east. Herewith, the waves propagate virtually without damping ($\gamma \ll \Lambda$, as well as for characteristic parameters in E-region we have:

$k_x \gg k_0 \sim 10^{-6}$ m^{-1}). Numerical calculations of the parameters of planetary waves (40) were carried out using models of the ionosphere and the neutral atmosphere [27] for low and high solar activity. Numerical calculations show that at $\theta_0 = 45°$ in the interval of heights from 90 to 150 km phase velocity of waves $C_B = \omega'_B/k_x = (k_x^2 + k_0^2)^{1/2} B_e \sqrt{1 + 3\sin^2\theta_0} \times (eN\mu_0 k_x R)^{-1}$ vary from 4 to 1.4 km/s at night and from 400 to 800 m/s by day. Periods $T_B = \lambda_B/C_B$, where wavelength $\lambda_B = 2\pi/k_x$ at $\lambda_B = 2000$ km are in the range of 1.5 to 6 hours by day and from 4 to 12 minutes at night. Perturbation of the geomagnetic field of these waves $b_B = B_e\sqrt{1 + 3\sin^2\theta_0}\,\xi_e/R$ (where ξ_e is the electron displacement) is 8 and 80 nT at $\xi_e = 0.1$ km and $\xi_e = 1$ km. The influence of exosphere temperature on C_B and T_B is insignificant but is important for the magnetic field perturbations. The C_B and T_B values are substantially different in daytime and at night as far as electron concentration in E-region of the ionosphere varies by one order of magnitude during a day. From the equation for C_B and T_B one can see that, for measured values of C_B and T_B, height profiles of the electron concentrations in Hall's layer of ionosphere may be built exactly.

Parameters of C_B waves correlate well with parameters of MLO, which were observed experimentally [7, 9, 47, 49] at middle latitudes of E-region of the ionosphere and were extracted as middle-latitude long-period oscillations (MLO). But it is evident from Eq. (40) that there are not any restrictions for the existence of these perturbations at both high and low latitudes. They are revealed especially by world-wide network of ionospheric and magnetospheric observatories during earthquakes, magnetic storms and artificial explosions [8, 24, 25].

In the low-frequency band $\omega_0 \le \omega_R \sim \omega'_R \ll \omega_B$, ω'_B, or in the long wavelength limit when $k_x^2 \ll \beta/C_B$ and $Kk_x \ll \beta'/C_B$, dispersion equation (39) has the solution in the form of frequency of the slow (Rossby type) modes:

$$\omega_0^s = -\frac{k_x}{k_x^2 + k_0^2}\beta', \qquad \gamma^s = -\frac{k_x^2}{k_x^2 + k_0^2}\Lambda, \qquad (41)$$

which is damping substantially ($\gamma^s \approx -\Lambda$ as well as $k_x \gg k_0$); but for more large-scale waves the damping can be weak.

Some calculations show that the phase velocities of these Rossby type waves (41) $C'_R = \omega_0^s/k_x = -\beta'\lambda^2/[4\pi^2(1 + k_0^2/k_x^2)]$ are in the range of −2 to +80 m/s in daytime, at heights of 90 to 150 km, $T_{exos} = 600$ K and $\lambda = 2\pi/k_x = 2000$ km. For $\lambda = 20000$ km phase velocities vary from −41 m/s to +1.8 km/s in daytime and from −41 to −11 m/s at night. Velocities change from −3 m/s to +60 m/s in daytime and from −2 m/s to −1.3 m/s at night, $T_{exos} = 2600$ K and $\lambda = 2000$ km. In this case sign «−» points to the direction of phase velocity from the east to the west and sign «+» from the west to the east. Calculations show that $\beta' = (\Omega_0 - N\omega'_i/N_n)2\sin\theta_0/R$ (here $\omega'_i = eB_e/M$) tends to zero and $C'_R = 0$ in daytime at a height of 115 km. The parameter β' tends to zero also at a height of 150 km of nightly ionosphere. Hence, ordinary slow planetary Rossby waves, moving from the west to the east direction in daytime, prevail in the lower E-region at heights of 90 to 115 km; but higher than the critical height they are prevailed by the planetary waves having electromagnetic nature and moving from the west to the east directions which, at increase of the height, can not be called as slow waves as well as the velocities of these waves at a height of 150 km in the daytime can reach a value of 1 km/s and more at low solar activity ($T_{exos} = 600$ K). The Hall region is completely occupied by the slow Rossby waves at nightly ionosphere. Hence, magnetic control of planetary waves in the ionosphere depends on critical altitude where the condition $\beta' = 0$ is fulfilled. Experimentally, these altitudes may be revealed from detection of planetary waves jointly at both ionospheric and magnetospheric observatories. It can be seen from calculations that periods $T'_R = 2\pi/\omega_0^s$ are in the range from 14 days to 8 hours at heights from 90 to 150 km, $T_{exos} = 600$ K, $\lambda = 2000$ km. T'_R vary from 14 days to 2 hours at $T_{exos} = 2600$ K. Dependence of C'_R on the exosphere temperature is conditioned by the fact that β' parameter of the electromagnetic planetary waves includes the ionization level η, which leads us to the conclusion on increasing of phase velocities C'_R with height increasing. Perturbation of the geomagnetic field can be determined from the Maxwell equations $b_R \approx |\mu_0 eNC'_R\xi|$ and it reaches a few tens of nanotesla (here ξ is the ion (neutral) displacement). The solution (41) practically coincides with the frequency of so-called magnetohydrodynamic gradient (MHG) modes revealed in the investigations [30, 53].

In the critical layers where $\beta' = 0$, the reverse of the local winds' directions takes place (i. e., the winds change their directions) and the superrotation of the Earth's upper atmosphere can arise [45]. Phase velocity of the wave (a wind) propagation, as it was mentioned above, is of order of $C'_R \approx -41...+100$ m/s and it covers the values of superrotation velocity of the upper atmosphere (of order of 55 m/s) observed from satellites [45]. Thus, the slow planetary waves C'_R can be a reason of the superrotation of the Earth's upper atmosphere at different latitudes.

Parameters of C'_R-waves correlate well with observable parameters of planetary electromagnetic waves in E-region of the ionosphere at the middle latitudes during any season of a year [13, 14, 37, 48, 56].

3.2. Planetary-Scale Electromagnetic Waves in F-Layer

It was already mentioned in the section 2 that in F-region of the ionosphere the Hall effect, causing an additional electromagnetic gyroscopic action on medium, is lacked ($\alpha \approx 0$). Here, transversal conductivity prevails, which causes negligibly weak damping of the considered perturbations (see sec. 2). In this case, general dispersion equation (27) reduces to the following form:

$$\omega[\omega^2 - \omega(\omega_R - i\Lambda) - \omega_n^2 + \omega_{n2}^2 - \omega_{AM}^2 - 2i\omega_{n2}^2 k_y R \mathrm{tg}\theta_0] = 0, \qquad (42)$$

where

$$\omega_n = \frac{1}{(\mu_0\rho)^{1/2}}\frac{k_x}{k_\perp}(\beta_{B1}^2 + \beta_{B2}^2)^{1/2} = \frac{B_e}{(\mu_0\rho)^{1/2}}\frac{k_x}{k_\perp}\frac{(1+3\sin^2\theta_0)^{1/2}}{R}, \qquad (43)$$

$$\omega_{n2} = \frac{1}{(\mu_0\rho)^{1/2}}\frac{k_y}{k_\perp}\beta_{B2} = \frac{B_e}{(\mu_0\rho)^{1/2}}\frac{k_y}{k_\perp}\frac{\cos\theta_0}{R}.$$

3.2.a. Slow Alfven Waves

Taking into account (as in the previous section) that inhomogeneities of the angular velocity of the Earth's rotation and the geomagnetic field practically do not affect on the dynamics of the slow MHD waves, so for eigen frequency ω_0 and for decrement of damping γ of the slow waves we get the following solution from the equation (42):

$$(\omega_0)_{1,2} = \pm\omega_{AM} = \pm\sqrt{\eta}\,\omega_A = \pm k_y V_{AM}\sin\theta_0, \quad \gamma = -\frac{\Lambda}{2}, \qquad (44)$$

$$(\omega_0)_3 = 0. \qquad (45)$$

Here, non-dimensional parameter $\eta = N/N_n$ denotes the degree of plasma ionization and $\omega_A = (\mathbf{k}\cdot\mathbf{B}_0)/\sqrt{\mu_0 NM}$ is the ordinary Alfven frequency. Dispersion equation (44) has two roots for positive and negative propagation directions. Group velocity of these perturbations is directed along the force lines of the geomagnetic field $\mathbf{B}_0$. The electromagnetic perturbations with the frequency (41) are modified, or slow Alfven waves. Modified Alfven waves are slow waves as far as parameter η varies in the range of 10^{-7} to 10^{-3} for F-region of the ionosphere at heights from 200 to 500 km. They can propagate in F-region of the ionosphere along the meridian to the north as well as to the south and are weakly damped by Raileigh friction. For typical value of particle density in F-region of the ionosphere, $\rho \sim 5\cdot10^{-11}$ kg/m^3, it follows from (44) that phase velocity of the slow Alfven waves reaches the value $V_{ph} = \omega_0/k_y \approx 4$ km/s. The wavelength is of the order of $\lambda = V_{ph}T \sim 1000$—10000 km, frequency occupies the range from 0.01 to 0.001 Hz. The waves generate geomagnetic pulsations of the order of $b \approx B_0V/V_{ph} \sim 1\ \mu$T. Consequently, in F-region of the ionosphere the observed large-scale electromagnetic perturbations, propagating along the meridians to the north or to the south with a velocity more than 2.5 km/s [22, 50], can be identified with the slow Alfven waves. The features of the solution (45) will be discussed in the end of the section.

3.2.b. Fast Planetary Electromagnetic Modes

Similarly to the item 3.1.b., from Eq. (42) we obtain the following dispersion equation describing frequency characteristics of the zonal perturbations propagated along the latitude circles (along x-axis directing towards parallel) for large-scale processes in F-region of the ionosphere, when latitudinal inhomogeneity of geomagnetic field $\mathbf{B}_0$ is not negligible:

$$\omega_0[\omega_0^2 - \omega_0\omega_R - \omega_n^2] = 0, \gamma = -\frac{\omega_0}{2\omega_0 - \omega_R}\Lambda. \qquad (46)$$

Taking into account that the penetration of the ordinary Rossby waves to the heights of F-region of the ionosphere is difficult [15, 18] and with $\omega_R \ll \omega_n$, from (46) we obtain following three solutions:

$$(\omega_0)_{1,2} = \pm\omega_n = \pm\sqrt{\eta}\frac{B_e}{\sqrt{\mu_0 MN}}\frac{(1+3\sin^2\theta_0)^{1/2}}{R}, \quad \gamma_n = -\frac{\Lambda}{2}, \qquad (47)$$

$$(\omega_0)_3 = 0.$$

The solution (47) characterizes only the oscillating regime of the zonal perturbations and represents standing waves around the parallels as it takes place for the Langmuir waves in completely ionized plasma [33]. As the height increases, the neutral concentration in F-region of the ionosphere and higher varies in a wide range (by several orders); the eigenfrequency of the zonal oscillations ω_n (47) caused by permanently acting factor, which is global for the ionosphere (latitudinal inhomogeneity of the geomagnetic field), varies in a wide range of frequencies as well. At the magnetic equator ($\theta = 90°$), the frequency ω_n will be twice as large as that at the magnetic pole ($\theta = 0$).

The large-scale electromagnetic perturbation with frequency (47) is a new eigen mode for F-region of the ionosphere.

In the presence of the planetary Rossby waves in F-region of the ionosphere (to which a favourable condition takes place during equinoxes) [15, 18, 19], it follows from equation (46) that the considered perturbations already propagate along the latitude circles, i. e., they became waves and acquire the dispersion:

$$V_{\rm ph}^2 = \frac{C_{\rm n}^2}{1 - \omega_{\rm R}/\omega},$$

where $C_{\rm n} = \omega_{\rm n}/k_x$, $V_{\rm ph} = \omega/k_x$ is the phase velocity of the wave propagation. These waves also can propagate in F-region of the ionosphere and also cause an interaction with a zonal wind V_0. Indeed, taking into account the zonal winds $\omega_0 = k_x V_0 + \omega_{\rm n}$, we obtain for phase velocity: $V_{\rm ph} = \omega/k_x = V_0 + C_{\rm n}$. The group velocity of the wave $C_{\rm gr} = \partial\omega/\partial k$ will coincide with zonal wind velocity $C_{\rm gr} = V_0$ in F-region of the ionosphere. Thus, measuring the group velocity of these electromagnetic perturbations on the basis of the data derived at the ionospheric and magnetospheric observatories gives the possibility to determine zonal wind velocity in F-region of the ionosphere, for measurements of which an effective experimental method has yet to be developed.

For F-region of the ionosphere, these waves were detected experimentally in [6, 9, 47, 49] as magneto-ionospheric wave perturbations (MIWP).

It can be seen from the calculations that the phase velocity of these waves $C_{\rm n} = \omega_{\rm n}/k_x$ is in the range of 20 to 1400 km/s at heights of 200 to 500 km (wavelength is $\lambda_{\rm n}$ = 2000 km, θ = 45°, exosphere temperature is $T_{\rm exos}$ = 600 K) and 10—50 km/s at $T_{\rm exos}$ = 2600 K. Period of these waves $T_{\rm n} = 2\pi/\omega_{\rm n}$ does not depend on the wavelength and is in the range of 3—105 s at $T_{\rm exos}$ = 600 K and of 40—210 s at $T_{\rm exos}$ = = 2600 K. Magnetic pulsations induced by these waves are of the same order of magnitude as $C_{\rm B}$-waves, $b_{\rm n}$ = = $b_{\rm B}$. The strong dependence of the parameters $C_{\rm n}$ and $T_{\rm n}$ on the exosphere temperature can be explained by «swelling» of atmosphere and lifting of heavy particles from lower layers of ionosphere. General increasing of phase velocities $C_{\rm n}$ with height and latitude θ increasing is a result of equation (47) for $\omega_{\rm n}$. The waves are weakly damped with the decrement $|\gamma|$ = = $0.5\Lambda \approx 10^{-6}$ s^{-1}. The periods, phase velocities, and amplitudes of geomagnetic pulsations for $C_{\rm n}$-waves in the middle-latitude ionosphere are in good agreement with observational data on both middle-latitude and large-scale electromagnetic perturbations generated in F-region of the ionosphere during powerful earthquakes and magnetic storms [24, 25]. This is a new fast mode of own oscillations of F-region of the ionosphere.

3.3. Stationary State of the Considered Medium

Let us mention that in the general case from the initial system of equations (14) and (15) it follows the dispersion equation of the sixth order according to the frequency ω. From them in E-region of the ionosphere there exist four types of eigen oscillations: the slow Alfven waves (35) and the atmospheric whistles (the helicons) (36); the planetary fast electromagnetic waves (40) and the slow Rossby type electromagnetic waves (41). In F-region there also exist four types of the eigen oscillations: two modified slow Alfven waves (propagating along the meridians to the north and to the south) (44) and two fast large-scale electromagnetic waves (47). Two frequencies (out of six frequencies mentioned above) are equal to zero ($\partial/\partial t$ = 0, $\omega_{5,6}$ = 0) and they also have a physical meaning corresponding to the hydrodynamic and electromagnetic equilibrium in the main state of the ionosphere.

Indeed, in order to determine equilibrium (stationary) state in initial system of equations (14) and (15), it is necessary to assume $\partial/\partial t$ = 0. Since in equilibrium state the geomagnetic field $\mathbf{B} = \mathbf{B}_0$ (i. e., $\mathbf{b}$ = 0) has nonvorticity (dipole) character $\nabla\times\mathbf{B}_0 = 0$, the electromagnetic Ampere force becomes mainly equal to zero $\mathbf{F}_{\rm A} = \nabla\times\mathbf{B}_0\times\mathbf{B}_0/(4\pi\rho) = 0$. Then, from the equations (14) and (15) (at $\partial/\partial t$ = 0) we obtain:

$$\mathbf{V}_0\times 2\Omega_0 = -\frac{1}{\rho}\nabla P_0 + \mathbf{g} - \Lambda\mathbf{V}_0, \qquad (48)$$

$$\nabla\times(V_0\times\mathbf{B}_0) = -\nabla\times\mathbf{E} = 0. \qquad (49)$$

Writing equation (48) with respect to components, we get the condition of quasistaticity $\partial P_0/\partial z = -\rho g$ and gradientity of the wind

$$V_{0x} = -(\Lambda\partial P_0/\partial x + \Omega_0\partial P_0/\partial y)/[\rho(\Lambda^2 + 4\Omega_0^2)],$$

$$V_{0y} = -(\Lambda\partial P_0/\partial y - \Omega_0\partial P_0/\partial x)/[\rho(\Lambda^2 + 4\Omega_0^2)],$$

which in those regions of the ionosphere where the Raileigh friction can be neglected (Λ = 0) transits into condition of quasigeostrophicity $\mathbf{V}_0 = \mathbf{V}_g$ = = $[\mathbf{e}_z\nabla P_0]/(2\omega_{0z}\rho)$ of the atmosphere. Here, P_0 is equilibrium pressure, $\mathbf{g}$ is gravity acceleration, $\mathbf{V}_g$ is geostrophic wind velocity. Correspondingly, from equation (49) it follows that in equilibrium state the electric field of polarization $\mathbf{E}$ has the electrostatic character ($\nabla\times\mathbf{E}$ = 0) and it will be generated in the upper atmosphere by the wind $\mathbf{V}_0$. Indeed, integrating

(49), we get $\mathbf{E} = -\nabla\varphi = -\mathbf{V}_0\times\mathbf{B}_0$. Thus, zero frequencies also have some physical meanings and represent the known stationary solutions of the equations of magnetohydrodynamics of the ionosphere and correspond to quasistatic, quasigeostrophic, and electromagnetic equilibrium.

4. NONLINEAR INTERACTION OF PLANETARY ELECTROMAGNETIC WAVES WITH SHEAR ZONAL WINDS IN THE IONOSPHERE

It is shown in the previous section that large-scale modified slow MHD, planetary fast and slow electromagnetic linear waves propagate in E- and F-regions (direction of propagation depends on the frequency). Now we consider the influence of the nonlinear effects on the dynamics of linear eigenwaves of the ionospheric medium, which are discussed in the previous sections. Experimental data and observations show [16, 39, 41] that the nonlinear solitary vortex structures may exist in the different layers of the Earth's atmosphere. These structures carry away the trapped circulating particles. The ratio of the particle rotational velocity U_C and the vortex displacement velocity U satisfies the following condition:

$$U_C/U \geq 1.$$

Let us define T and L as the characteristic temporal and spatial scales. According to the formula $V \propto \partial\Psi/\partial y$, we have the expressions: $U_C \propto \Psi/L$, $U \propto L/T$. On introducing (22), we get: $J(\Psi, \Delta\Psi)/\partial\Delta\Psi/\partial t \propto \Psi T/L^2 \propto U_C/U$. Hence, nonlinearity is very important for the wave processes satisfying the condition $U_C \geq U$. Thus, these estimations show that the nonlinear effects can play a substantial role in the case of the large-scale low-frequency electromagnetic waves studied in the linear stage in the previous sections. The inequality $U_C \geq U$ coincides with antitwisting condition, only at which the corresponding system of the nonlinear dynamic equations can have a solitary vortex solution [55].

From the general theory of nonlinear waves it is known [38, 54] that if in the system the nonlinear effects are sufficient, superposition principle can not be used and solution can not be given in a form of a plane wave. Nonlinearity destroys the wave profile and a waveform differs from sinusoid. If in the nonlinear system dispersion is lacked (or β-effect, inhomogeneity of the equilibrium parameters of the medium), then all the small-amplitude waves with the different wave numbers κ are propagated with equal velocities and can interact with each other for a long time. Thus, a small nonlinearity also leads to the accumulation of distortions. Such nonlinearity distortion, as a rule, leads to an increase of the wave front curvature and its upset or to a shock-wave formation. In the presence even a small dispersion (or β-effect, and so on) the phase velocities of the waves with different k are not equal, the wave packet has a tendency to a spreading and, because of this, at not so large wave amplitude the dispersion can compete with nonlinearity. Consequently, the waves can be decayed into different nonlinear wave structures before its upset and the shock-wave will not be formed. In the real atmosphere, the shock-wave actually will not be formed itself (spontaneously, without external influence). First of all, this shows that in the atmospheric-ionospheric medium the dispersion effect (or equivalent to it β-effect, inhomogeneity of the equilibrium parameters of the medium, and so on) is strongly expressed and successfully competes with the nonlinear distortion. If nonlinear increase of the wave front curvature is compensated by their dispersion spreading, there can be existed the stationary waves, i. e., the solitary waves propagating without changing their shapes.

Results of ground-based and satellite observations bring out clearly that at different layers of the ionosphere there permanently exist zonal winds (flows) having inhomogeneous velocity along the meridians [20, 23, 28, 31]. Because of this, we take further into account interaction of the considered waves with an inhomogeneous (shear) wind (a flow). It will be shown that when there is a velocity inhomogeneity in the zonal flow, the wave perturbations can acquire an additional dispersion at interaction with it and the nonlinear effects will be appeared in their dynamics. Thus, the ionospheric medium creates a condition which is favourable to formation of the nonlinear stationary solitary wave structures.

Thus, we will seek the solution of the full system of nonlinear dynamic equations (22)—(24) (in non-dissipative stage, $\Lambda = 0$) in the form $\Psi = \Psi(\eta, y)$, $A = A(\eta, y)$, $b_z = b_z(\eta, y)$, where $\eta = x - Ut$, i. e., in the form of stationary solitary waves propagating along x-axis (along the parallels) with velocity U = const without changing its shape. To avoid confusion, let us notice that by a nonlinear solitary wave is here meant a perturbation localized even if for one coordinate. We will consider also that waves are propagated on background of the mean horizontal wind with zonal shear of velocity $\overline{V}(y)$. Stream function Ψ in this case is equal to

$$\Psi = \psi - \int_{-\infty}^{y} \overline{V}(y)dy, \tag{50}$$

where ψ is the deviation of stream function from average value.

To simplify further investigation, we will seek a class of solutions for which $\Psi = -\alpha b_z/\mu_0$ and $\nabla_\perp^2 A = \partial b_y/\partial x - \partial b_x/\partial y = f(y)$, where f is an arbitrary function of its argument; further it will be assumed decreasing fastly at infinity. Then, substituting the expression (50) into system (22)—(24), on simple transformations we obtain:

$$J\left(\psi - \int_{-\infty}^{y} \overline{V}(y)dy + Uy, \nabla_\perp^2\psi - \frac{\partial \overline{V}}{\partial y} + \left(\beta' + \frac{\mu_0}{\alpha^2\rho}U\right)y\right) = 0, \quad (51)$$

where $J(a, b) = \partial a/\partial x \cdot \partial b/\partial y - \partial a/\partial y \cdot \partial b/\partial x$ is the Jacobian.

Let us mention that the linear electromagnetic waves investigated in the section 3 are basically zonal, i. e., for them the direction along a parallel (on the axis x) is primary. Therefore, for perturbations of such polarization, it is more adequate to consider nonlinear wave structures which are longer on the axis x, i. e., the scale on the axis x is much greater than the scale on the axis y.

Let us consider in more detail the construction of long-wave (on the axis x), solitary spatially two-dimensional solution of equation (51). We will show that in this case equation (51) can be reduced to stationary equation of Korteveg — de Vries (KDV) for the zonal component of motion [11, 36, 44]. In this section the results of these works will be generalized on the basis of the general equation of the planetary electromagnetic waves — vortices (51) describing propagation of stationary nonlinear wave structures.

Let us pass to non-dimensional variations in the equation (51): $\psi = \psi' VL$, $\overline{V} = \overline{V}'\omega_0 L$, $U = U'\omega_0 L$, $\beta' = (\beta')'\omega_0 L$, $\eta = \eta' L/\delta = \eta' L_x$, $y = y'L$, where L and L_x are spatial scales of a vortex according meridian and zonal directions, V is characteristic velocity of medium particles, and ω_0 is eigenfrequency of one of the considered linear electromagnetic waves, we get:

$$\varepsilon\delta^2 J\left(\psi', \frac{\partial^2\psi'}{\partial\eta'^2}\right) + \varepsilon J\left(\psi', \frac{\partial^2\psi'}{\partial y'^2}\right) + \left(\alpha_0 U' + (\beta')' - \frac{\partial^2\overline{V}'}{\partial y'^2}\right)\frac{\partial\psi'}{\partial\eta'} + \delta^2(\overline{V}' - U')\frac{\partial^3\psi'}{\partial\eta'^3} + (\overline{V}' - U')\frac{\partial^3\psi'}{\partial y'^2\partial\eta'} = 0, \quad (52)$$

where $\varepsilon = V/(\omega_0 L) \ll 1$, $\delta = L/L_x < 1$, $\alpha_0 = \mu_0 L^2/\alpha^2\rho = N\omega_p^2 L^2/N_n c^2$ (further we will not use the primes on the non-dimensional values). We are interested in the waves, for which nonlinearity and dispersion are of the same order. Assuming that $\delta^2 = \varepsilon$ in (52), we get:

$$(\overline{V} - U)\frac{\partial^3\psi}{\partial y^2\partial\eta} + \left(\alpha_0 U + \beta' - \frac{\partial^2\overline{V}}{\partial y^2}\right)\frac{\partial\psi}{\partial\eta} + \varepsilon\left[(\overline{V} - U)\frac{\partial^3\psi}{\partial\eta^3} + J\left(\psi, \frac{\partial^2\psi}{\partial y^2}\right)\right] + \varepsilon^2 J\left(\psi, \frac{\partial^2\psi}{\partial\eta^2}\right) = 0. \quad (53)$$

We will seek the solution in the form:

$$\psi = \psi_0 + \varepsilon\psi_1 + \varepsilon^2\psi_2 + ...,$$

$$U = U_0 + \varepsilon U_1 + \varepsilon^2 U_2 + ... \quad (54)$$

Substituting (54) into (53) and equalizing to zero the components of the same order of smallness, we get a linear equation in zero approximation:

$$\left(\alpha_0 U_0 + \beta' - \frac{\partial^2\overline{V}}{\partial y^2}\right)\frac{\partial\psi_0}{\partial\eta} + (\overline{V} - U_0)\frac{\partial^3\psi_0}{\partial y^2\partial\eta} = 0, \quad (55)$$

with boundary conditions $\psi_0(\eta, 0) = \psi_0(\eta, 1) = 0$ corresponding to the flows bounded along the meridians (let us mention that in the case of infinite region this condition leads to equality of the stream function to zero at infinity: $\psi \to 0$ at $y \to \pm\infty$). Separating the variations $\psi_0 = F(\eta)\cdot\Phi(y)$, we obtain standard problem of Sturm — Liouville:

$$\frac{\partial^2\Phi}{\partial y^2} + G(y)\Phi = 0, \ \Phi(y_1) = \Phi(y_2) = 0, \quad (56)$$

$$G(y) = \frac{1}{\overline{V} - U_0}\left(\alpha_0 U_0 + \beta' - \frac{\partial^2\overline{V}}{\partial y^2}\right),$$

where y_1 and y_2 are coordinates of the zonal flow edges.

For definition, let us consider the case where a zonal wind has a weak shear and its velocity harmonically varies in the meridian direction:

$$\overline{V} = V_0[1 + a_0\sin(\kappa_0 y)], \ a_0 \ll 1. \quad (57)$$

Then, the general solution (56) takes the form:

$$\Phi = \Phi_0\sin(n\pi y), \ U_0 = \frac{m^2(V_0 - \beta'/m^2)}{m^2 + \alpha_0}, \quad (58)$$

where Φ_0 is an arbitrary constant amplitude, $m = n\pi$.

The next approximation includes the effects of dispersion and nonlinearity:

$$\frac{\partial^3\psi_1}{\partial y^2\partial\eta} + G(y)\frac{\partial\psi_1}{\partial\eta} = -\frac{\partial^3\psi_0}{\partial\eta^3} - \frac{\alpha_0 U_1}{\overline{V} - U_0}\frac{\partial\psi_0}{\partial\eta} + \frac{1}{\overline{V} - U_0}\frac{\partial\psi_0}{\partial y}\frac{\partial^3\psi_0}{\partial y^2\partial\eta} - \frac{1}{\overline{V} - U_0}\frac{\partial\psi_0}{\partial\eta}\frac{\partial^3\psi_0}{\partial y^3} + \frac{U_1}{\overline{V} - U_0}\frac{\partial^3\psi_0}{\partial y^2\partial\eta}. \quad (59)$$

Here ψ_1 satisfies the same boundary conditions as ψ_0. Assuming $\psi_0 = F(\eta)\cdot\Phi(y)$, multiplying (59) by Φ and integrating transverse to the flows, we get:

$$\frac{\partial}{\partial\eta}\int_{y_1}^{y_2} \psi_1\left(\frac{\partial^2\Phi}{\partial y^2} + G(y)\Phi\right)dy = = a_1U_1\frac{\partial F}{\partial\eta} + a_2F\frac{\partial F}{\partial\eta} + a_3\frac{\partial^3 F}{\partial\eta^3}. \quad (60)$$

It is clear that, according to (56), the left-hand side of equation (60) is equal to zero. Consequently, the function $F(\eta)$ must satisfy the stationary KDV equation:

$$\frac{\partial F}{\partial\eta} + \frac{a_2}{a_1U_1}F\frac{\partial F}{\partial\eta} + \frac{a_3}{a_1U_1}\frac{\partial^3F}{\partial\eta^3} = 0, \quad (61)$$

where

$$a_1 = -\int_{y_1}^{y_2}(\alpha_0 + G(y))\frac{\Phi^2}{\overline{V} - U_0}dy,$$

$$a_2 = \int_{y_1}^{y_2}\frac{\Phi^3}{\overline{V} - U_0}\frac{\partial G}{\partial y}dy, \quad a_3 = -\int_{y_1}^{y_2}\Phi^2 dy.$$

In accordance with integration constants, the solution of KDV equation represents either the solitary wave (the vortex) or the periodic (cnoidal) wave. We are interested in solutions of the soliton form:

$$F(\eta) = \mathrm{sgn}(a_2a_3)A\mathrm{sech}^2(\kappa\eta), \quad (62)$$

where A is an arbitrary constant amplitude of soliton, $\kappa^{-1} = d$ is characteristic spatial width of soliton.

Substituting (62) into (61), we get the following expressions for U_1 and κ:

$$U_1 = -\,\mathrm{sgn}(a_2a_3)\frac{a_2}{3a_1}A,$$

$$\kappa^2 = \frac{1}{12}\left|\frac{a_2}{a_3}\right|A. \quad (63)$$

Let us mention that expression (58) fixes the velocity U_0 for every eigen function Φ and that amplitude A is a unique arbitrary constant. For the existence of a solution of KDV equation, it is necessary that the value $\overline{V} - U_0$ should not become zero in the interval $y_1 < y < y_2$. This condition and the expression $\varepsilon = \delta^2$ are unique restrictions on amplitude of the nonlinear vortex structures.

Correspondingly, the stream function can be determined by integration of expression (50) taking into account (57) and (58) at smallness of the parameter of velocity shear ($a_0 \ll 1$):

$$\psi = -V_0y + \psi_0^0\sin(n\pi y)\mathrm{sech}^2(\kappa\eta). \quad (64)$$

Here, $\psi_0^0 = \Phi_0A\mathrm{sgn}(a_2a_3)$ and in accordance with (63) we determine the characteristic scale of the nonlinear vortex:

$$\kappa^{-1} = d = = \left|\frac{\psi_0^0V_0a_0\kappa_0^3n\pi(7n^2\pi^2 + \kappa_0^2)[1-(-1)^n\cos\kappa_0]}{8(V_0 - U_0)^2(n^2\pi^2 - \kappa_0^2)(9n^2\pi^2 - \kappa_0^2)}\right|^{-1/2}. \quad (65)$$

It is obvious that the characteristic size of the vortex is inversely proportional to the amplitude of the structure (ψ_0^0), as it must be for the nonlinear soliton structures [54], it depends on the amplitude of the wind velocity V_0, and on the velocity U_0 of the vortex motion as well.

Analogously, for full velocity of movement of the nonlinear vortical structures we get:

$$U = \frac{m^2}{m^2 + \alpha_0}\left(V_0 - \frac{\beta'}{m^2}\right) + + \frac{\psi_0^0V_0}{2}\{a_0\kappa_0^3n\pi(7n^2\pi^2 + \kappa_0^2)(4n^2\pi^2 - \kappa_0^2)\times \times[1 - (-1)^n\cos\kappa_0]/[(n^2\pi^2 - \kappa_0^2)(9n^2\pi^2 - \kappa_0^2)\times \times[(\alpha_0U_0 + \beta')(4n^2\pi^2 - \kappa_0^2) + + 4a_0V_0\kappa_0n^2\pi^2(1 - \cos\kappa_0)]]\}. \quad (66)$$

It is obvious that velocity of movement of the nonlinear vortical structures along the parallel depends on the wind velocity V_0, inhomogeneities of the geomagnetic field and angular velocity of the Earth's rotation (β'), and also on amplitude of the structures (A, ψ_0^0), as it is peculiar for nonlinear solitary formation [38, 54]. The expression (66) shows that velocity of movement of the vortex can become zero, i. e., standing structures can be formed. Moving vortices can exist on the background of both western ($V_0 > 0$) and eastern ($V_0 < 0$) winds.

For these wave perturbations, velocity vortex is distinct from zero, $\nabla\times\mathbf{V} \approx \nabla_\perp^2\Psi\mathbf{e}_z \neq 0$. Consequently, they are vortical formations carrying trapped-rotating particles.

According to the formula (64), the current lines, the level lines and spatial relief of the vortex structures in the moving coordinate system (η, y) can be built. For definiteness, as linear scale in the zonal direction, the Earth's radius is chosen, $L_x \sim 1000$ km; $a_0 \sim \varepsilon \sim \delta^2 \sim 0.01$; accordingly, the scale of length to the meridian direction is $L \approx 0.1L_x$. The time scale is determined by the characteristic period of the wave perturbations. For characteristic value of the zonal wind amplitude, the value $V_0 = 100$ km/s is chosen. Fig. 1 shows the stream lines for the characteristic values of the parameters of medium and motion: $V_0 = 0.1$, $\Psi_0^0 = = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.9$, $n = 1$, with formation of the anticyclone vortex structure

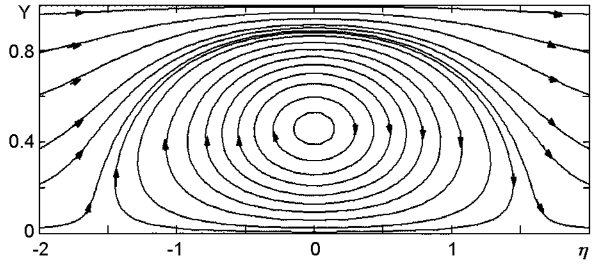

Fig. 1. The isolines of non-dimensional stream function $\Psi = -V_0 y + \Psi_0^0 \mathrm{sech}^2(\kappa\eta)$ = const (the current lines) in the system of coordinates η, y, moving together with the wave structure: $V_0 = 0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.9$, $n = 1$ (anticyclone)

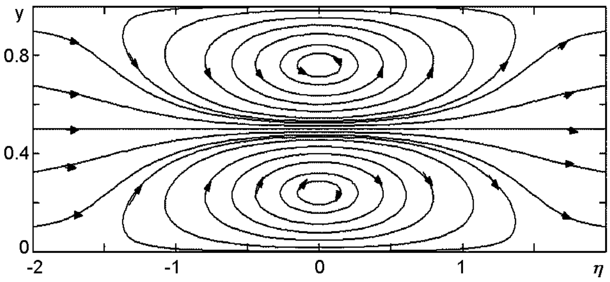

Fig. 3. The current lines in the moving system of coordinates (η, y): $V_0 = 0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.9$, $n = 2$ (cyclone-jet-anticyclone)

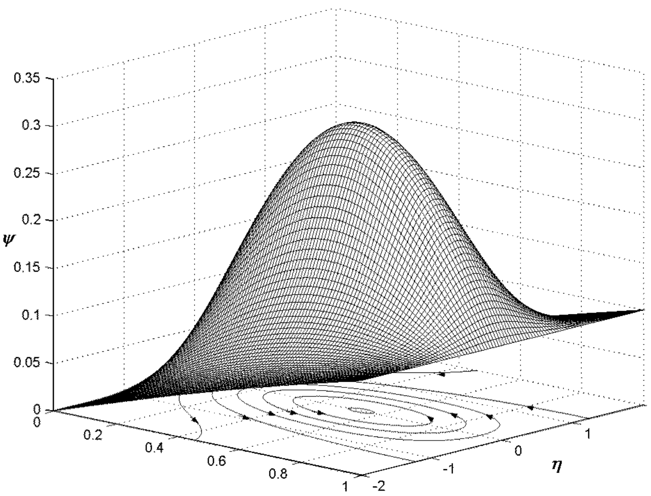

Fig. 2. The level lines of the stream function and the relief of two-dimensional vortex solution in the moving system of coordinates at $V_0 = -0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.5$, $n = 1$ (cyclone)

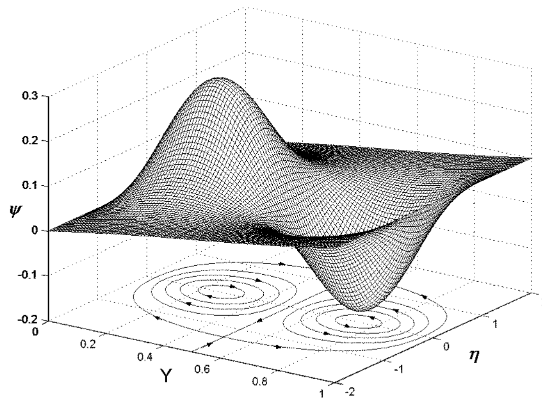

Fig. 4. The level lines of the stream function and the relief of two-dimensional vortex solution in the moving system of coordinates at $V_0 = -0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.6$, $n = 2$ (cyclone-anticyclone)

near the southern boundary of the zonal wind flow in the ionosphere. At $V_0 < 0$ it generates the cyclone vortex (see Fig. 2 where $V_0 = -0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.5$, $n = 1$) near the northern boundary of the wind flow. It is obvious from Figs 3 and 4 that at $n = 2$ in the wind flow of the ionosphere the connected vortex structure will be formed, consisting of cyclone and anticyclone of equal intensity. At $V_0 > 0$ (see Fig. 3 where $V_0 = 0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.9$, $n = 2$) cyclone and anticyclone have the joint zone of maximal velocities (the jet flows). In the case of $V_0 < 0$ (see Fig. 4 where $V_0 = -0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.6$, $n = 2$) the interstructural jet flow is absent and the nonlinear structure represents two-dimensional dipole vortex solution of cyclone-anticyclone character of type [34]. For $n > 2$ the transversal vortex chains will be generated, which consists of n alternating cyclone and anticyclone structures and locating across to the zonal wind (flow) (see Fig. 5).

It must be mentioned that equation (51) has also the exact solution in the form of solitary two connected dipole vortices of cyclonic-anticyclonic type of equal intensity [2, 34] (see Fig. 4); in the form of a regular vortical chain, «cat eyes» [23, 41], and also in the form of the asymmetric solitary vortices at which the stream function depends only on radial coordinate [35]. But for the waves under investigation, possessing zonal asymmetry, as it was already mentioned, more probable is generation of nonlinear vortical structures of considered type (64).

In the dissipative ionosphere ($\Lambda \neq 0$), the vortex is not a stationary wave and, to study the dynamics of the nonlinear structures, it is necessary to use the appropriate equations of transfer. In this case the integral properties of structures, namely, energy E and enstrophy Q (25) and (26) are not conserved and vary with respect to time due to dissipation. In accordance to [3], solutions (64)—(66) can be placed

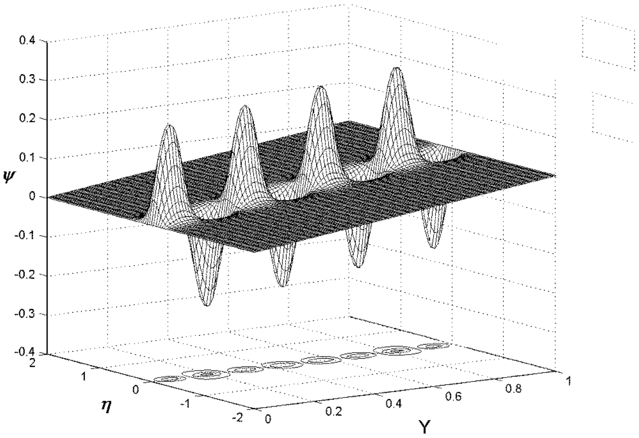

Fig. 5. The level lines of the stream function and the relief of two-dimensional vortex solution in the moving system of coordinates at $V_0 = -0.1$, $\Psi_0^0 = 0.25$, $a_0 = 0.01$, $\alpha_0 = 0.2$, $\beta' = 0.01$, $\kappa_0 = 0.6$, $n = 8$ (transversal vortex chain)

in (25) and (26) within the parameters ψ_0^0, U and κ, varying slowly with respect to time within the limits of the weak dissipation. In order to analyse the evolution of energy and enstrophy (25) and (26) in the dissipative medium, we estimate the order of integrals:

$$\int(\nabla_\perp\psi)^2 dxdy \sim d^{-2}\int\psi^2 dxdy,$$

$$\int(\nabla_\perp^2\psi)^2 dxdy \sim d^{-2}\int(\nabla_\perp\psi)^2 dxdy,$$

where d is the characteristic spatial scale of the vortices. If we consider the small-scale vortical structures $d \ll k_0^{-1}$, the energy and enstrophy have the same order of magnitude as the dissipative term, and Eqs (25) and (26) may be rewritten as

$$\partial E/\partial t \approx -2\Lambda E, \quad \partial Q/\partial t \approx -2\Lambda Q. \quad (67)$$

This means that the energy and the enstrophy of these vortices damped exponentially. For the large-scale vortices with $d > k_0^{-1}$, dissipative term in transfer equations (25) and (26) is less than energy and enstrophy, so the relaxation of the vortices proceeds more slowly.

Thus, the large-scale electromagnetic nonlinear vortical structures are long-lived in the ionosphere and, therefore, they can make an important contribution to transfer processes of substance, heat, energy, and formation of strong turbulent state of the medium [1].

5. DISCUSSION OF THE RESULTS AND CONCLUSION

It is shown on the basis of our investigation that in dynamo-region (E-layer) of the ionosphere, with the S_q currents, Hall conductivity, and wind system, which are inherent in it, a wide class of large-scale, low-frequency electromagnetic wave structures can be generated. First of all, slow MHD waves must be mentioned, on which dynamics of the inhomogeneity of the Coriolis and Ampere forces, does basically not influence. Besides, it is necessary to point out slow Alfven waves which, in distinction to completely ionized plasma, are decelerated in the ionosphere because of involving neutral components of the medium in fluctuations. And they are connected to the effect of full entrainment of ion components by neutrals in E-region (see the section 2), resulting in transferring perturbations of charged particles to neutral ones. In short-wave limit this wave continuously passes to the helicons. In E-region the slow Alfven waves have typical periods of 0.5 to 2 hours, wavelengths of the order of 1000 km and phase velocities of the order from 1 to 2 km/s. The waves generate geomagnetic pulsations about 150 nT. The resulted characteristics of the waves well correlate with properties of large-scale, low-frequency slow electromagnetic waves, revealed experimentally [28, 50].

With the decrease of frequencies of the waves the effect of inhomogeneity of the Coriolis and Ampere forces becomes essential. This effect leads to the generation of the planetary, ULF electromagnetic fast and slow modes.

The generation of the slow electromagnetic linear waves, in the ionospheric E-region by the gradient of both geomagnetic field and angular velocities of the Earth's rotation is shown. They propagate in E-region along the latitudinal circles westward and eastward against background of mean zonal wind and are the waves of the Rossby type. The frequency of the slow waves vary in the range of 1 to 100 μHz; period of these waves vary in the range from 2 hours to 14 days; wavelength is about 1000 km and longer, the phase velocity has the same order as the local winds, namely, from a few metres per second to one hundred metres per second ($C'_R \approx V \approx 1$—100 m/s). The slow waves experience a strong attenuation by Rayleigh friction between the layers of the local atmosphere and the damping factor is $|\gamma^S| = \Lambda \sim 10^{-5}$ s^{-1}. Though, the attenuation would be weaker for longer large-scale waves with a wavelength of about 10000 km and a timescale of a week or longer. The linear slow waves perturb the magnetic field which has the order of $b_R = |\mu_0 e N C'_R \xi|$ (ξ is transversal shift of the charged particles). For the value of the phase velocity $C'_R = 50$ m/s and $\xi = 1$ km, we have $b_R \approx 1$ nT. Perturbed magnetic field strength increases up to 20 nT, if transversal displacement of the system $\xi = 10$

km and the phase velocity $C'_R \sim 100$ m/s. Thus, the linear slow electromagnetic waves in the dynamo-region are accompanied by the noticeable micro-pulses of the geomagnetic field and have the same order as the micro-pulses caused by S_q currents in the same region. The slow wavy structures (41) can cause observed superrotation (SR) of the upper atmosphere at different layers. These waves were observed in some experiments [13, 14, 37, 48, 56].

The generation of linear fast planetary electromagnetic waves in the ionospheric E-region by the gradient of geomagnetic field and the Hall effect is established. These waves propagate along the latitude against the background of the zonal-mean flow westward and eastward at a speed of a few kilometres per second ($C_B \approx 2$—20 km/s) in the dynamo-region. The waves have a frequency of the order of 0.1 to 100 mHz; periods are in the range from 4 minutes to 6 hours; wavelength is about 1000 km and longer. They attenuate weakly and $|\gamma^f| \sim 0.08\Lambda \sim 10^{-6}$ s^{-1}. The essential micro-pulses of the geomagnetic field caused by the fast waves equal $b_B \approx |2eNC_B\lambda^f| \sim 1$ μT. They could be assumed as a new mode of the own oscillations in E-region of ionosphere. Frequencies and phase speeds of fast waves depend on density of the charged particles. Therefore, the phase velocities of fast disturbances in E-region of the ionosphere differ almost by one order of magnitude for daytime and nighttime conditions. High phase velocities as well as their strong change for day and night precludes the identification of these disturbances with MHD waves. The fast waves are caused by oscillations of the electrons completely frozen in the geomagnetic field (see Sect. 2, formulas (8)—(10)), $\partial\mathbf{b}/\partial t = \nabla\times\mathbf{V}_e\times\mathbf{B}_0$, for motionless ions and electrons, $|\mathbf{V}_e| \gg |\mathbf{V}_i| = |\mathbf{V}|$. As, thus $\mathbf{V}_e = \mathbf{V}_D = -\mathbf{j}/eN = -\nabla\times\mathbf{b}/(eN\mu_0)$, for linear waves induction equation (equation of frozen-in) closes on itself and gets the form: $\partial\mathbf{b}/\partial t = \nabla\times\mathbf{B}_0\times\nabla\times\mathbf{b}/(eN\mu_0)$. The Fourier transformation of this equation recovers the dispersion relation (40).

We investigated dynamics of the slow Alfven waves in F-region of the ionosphere. It is shown that they can be propagated along the meridian to the north or to the south with phase velocity of order from 2 to 5 km/s. The waves have lengths of order from 1000 to 10000 km and periods in the range from 3 min to 1.5 h. They damp weakly enough with decrement $|\gamma| \approx 10^{-6}$ s^{-1} and generate magnetic pulsations of about 1 μT. Some experimental observations of similar waves are reported in [22, 28, 50].

It is established that, in the ionospheric F-region, inhomogeneity of the geomagnetic field generates fast planetary electromagnetic waves, propagating along the latitude circles to the east or to the west with phase velocity $V_{ph} = C_n = 20$—1400 km/s. Frequency of the waves is within limits from 0.001 to 10 Hz and the waves are weakly damped with decrement $|\gamma_n| \approx 10^{-6}$ s^{-1}. The period of perturbations varies in the range from 1 to 110 s. Amplitude of geomagnetic micropulsations generated by these waves is about $b_n \approx b_B \approx 1$ μT. The C_n waves are new modes of eigen oscillations of F-region of the ionosphere. These waves as magnetospheric wave perturbations (MIWP) have been detected in experiments [6, 9, 48, 49]. Measurements of group velocity of these electromagnetic perturbations on the basis of observational data from ionospheric and magnetospheric observatories give the possibility of definition of the zonal wind velocity in F-region of the ionosphere, for the determination of which there not exist yet a direct effective experimental method.

Two eigen-frequencies $\omega = 0$ also have a physical meaning and correspond to hydrodynamic and electromagnetic equilibrium state of the ionospheric medium in a background state where geostrophycal wind velocity coincides with electric drift velocity.

In Table, spectra of investigated eigen planetary electromagnetic oscillations of the ionosphere are shown together with decrements of damping and the condition of their existence.

Of special note is the fact that, up to now, in physics of the ionosphere it was supposed that in the ionosphere it can exist basically only the dynamo electric field $\mathbf{E}_d = \mathbf{V}\times\mathbf{B}_0$ caused by a local wind. Our results show that the fast planetary electromagnetic waves under investigation (C_B and C_n waves) can generate intensive large-scale internal vortical electric fields in E-and F-regions of the ionosphere. It is possible to determine size of the field $\mathbf{E}_v$ on the basis of the equation (15):

$$(\nabla\times\mathbf{E}_v)_z \sim \beta'\left(1 - V^f_{ph}/V^s_{ph}\right)V_y, \tag{68}$$

where V^f_{ph} is phase velocity of the fast planetary waves (C_B or C_n waves) and V^s_{ph} is phase velocity of the slow planetary waves C'_R. Taking into account that in E-region of the ionosphere $V^f_{ph} = C_B$, $V^s_{ph} = C'_R$, $\beta' \sim \beta_{B1} \sim B_{0z}/R$ and setting characteristic size of the perturbations along the axis x, $L = \lambda/2\pi$, for the ratio of the vortex $E_{v,y}$ as well as considering that dynamo fields is $E_{d,x} = -V_yB_{0z}$, we obtain from (68):

$$\frac{|E_{v,y}|}{|E_{d,x}|} \approx \frac{L}{R}\frac{|V^f_{ph}|}{|V^s_{ph}|} = \frac{\lambda}{2\pi R}\frac{|C_B|}{|C'_R|}, \tag{69}$$

When $C_B = 20$ km/s and $\lambda = 6000$ km, $|E_{v,y}|/|E_{d,x}| \approx 30$. The values of the dynamo field $E_{d,x} \approx V_yB_{0z}$ and internal vortical electric field $E_{v,y} \approx$

Spectrum of the Large-scale Ultra-Low-Frequency Electromagnetic Waves in the Ionosphere

E-region			F-region		
Frequencies	Conditions of the existence	Decrements of damping	Frequencies	Conditions of the existence	Decrements of damping
		Slow MHD waves ($\lambda \le 1000$ km)			
1. Slow Alfven waves $\omega_{1,2}=\pm\frac{k_yB_e\sin\theta}{(\mu_0MN_n)^{1/2}}$ and $\omega'_{1,2}=\omega_{1,2}-\frac{k_x}{k^2}\frac{\beta}{2}$	$\rho_i/\rho \gg k^2c^2/\omega_p^2$ ($\beta, \beta_{B1,2}\to 0$)	$\gamma=-\frac{\Lambda}{2}$	1. Slow Alfven waves $\omega_{1,2}=\pm\frac{k_yB_e\sin\theta}{(\mu_0MN_n)^{1/2}}$	$\rho_i/\rho \gg k^2c^2/\omega_p^2$ ($\alpha, \beta, \beta_{B1,2}\to 0$)	$\gamma=-\frac{\Lambda}{2}$
2. Slow helicons (whistlers) $\omega_{3,4}=\pm\frac{kk_yB_{0y}}{eN\mu_0}$	$\rho_i/\rho \ll k^2c^2/\omega_p^2$ ($\beta, \beta_{B1,2}\to 0$)	$\gamma=-\frac{\rho_i\omega_p^2}{\rho k^2c^2}\frac{\Lambda}{2}$			
		Planetary hydromagnetic gradient waves ($\lambda > 1000$ km)			
3. Fast waves $\omega_{1,2}=\pm\frac{B_e}{eN\mu_0}\frac{(1+3\sin^2\theta)^{1/2}}{R}k_x$	$k_x^2 \gg \beta'/C_B,\ \beta/C_B$	$\gamma=-\frac{k_0^2}{k_x^2}\frac{\Lambda}{2}$	2. Fast waves $\omega_{1,2}=\pm\omega_n=\pm\frac{B_e}{(\mu_0MN_n)^{1/2}}\times\frac{(1+3\sin^2\theta)^{1/2}}{R}$ and	$\alpha, k_y, \omega_R\to 0$	$\gamma=-\frac{\Lambda}{2}$
4. Slow waves $\omega_{3,4}=-\beta'/k_x$	$k_x^2 \ll \beta'/C_B,\ \beta/C_B$	$\gamma=-\Lambda$	$\omega'_{1,2}=\pm\frac{\omega_n}{(1-\omega_R/\omega_n)^{1/2}}$	$\alpha, k_y\to 0, \omega_R\neq 0$	

$V_yB_{0z}\lambda C_B/(2\pi RC'_R)$ may also be estimated. For the characteristic values of the E-region parameters, namely, $V_y \sim 50$ m/s, $B_{0z}\sim 64$ μT, $\lambda \sim 10000$ km, $C_B\sim 20$ km/s, $C'_R\sim 50$ m/s, we get $E_{d,x}\sim 1.6$ mV/m and correspondingly, for strength of planetary internal vortical electric field we obtain $E_{v,y}\sim 50$ mV/m (which coincides with the values measured experimentally [10, 51]. Thus, the value of the long-scale internal vortical electric field may be several times greater than the dynamo-field generated in the same ionospheric layer by local wind motion.

Existence of the large-scale fast waves C_B (in E-region), C_n (in F-region) and slow Rossby-type planetary waves C'_R (in both E- and F-regions) are caused by constantly affecting factor, fundamental for ionosphere, namely, spatial inhomogeneity of the geomagnetic field $\mathbf{B}_0$. The slow waves are generated by the dynamo field of polarization ($\mathbf{E}_d = \mathbf{V}\times\mathbf{B}_0$) and the fast waves are caused by vortex electric field $\mathbf{E}_V = \mathbf{V}_D\times\mathbf{B}_0$.

Dynamics of the slow planetary electromagnetic waves in the ionosphere are studied experimentally, more or less. Experimental investigation of features of the fast large-scale electromagnetic waves must be realized. Formulas (41) and (47) show that fast electromagnetic large-scale ($L\sim 1000$—10000 km) atmospheric waves both in E- and F-regions of the ionosphere have general-planetary character and occupy latitudes from the pole ($\theta = 0$) to the equator ($\theta=\pi/2$).

The fast electromagnetic atmospheric waves at ionospheric altitudes can be experimentally revealed and detected using their specific features:

1) a wide range of phase velocity dependence on the latitude (phase velocities of the waves are increased from the pole to the equator; they are doubled at the equator).

2) a high variation (by magnitude) of electron concentration N substantially increases phase velocity of $C_B=\omega_B/k_x$ — waves in E-region of the ionosphere in nightly conditions (from a few hundred metres per second in daytime to a few ten kilometres per second at night).

3) application of the well-known profiles $N(h)$ allows us to calculate uniquely height distribution of the C_B-waves in E-region of the ionosphere and, conversely, from the height distribution of $C_B(h)$-waves we can plot the dependence of concentration $N(h)$ on altitude.

4) altitude variation of the neutral component concentration $N_n(h)$ leads to strong increase of the phase velocity of C_n-waves (phase velocity of C_n-waves at heights of 200 to 500 km is increased from a few kilometres per second up to 1000 kilometres per second) in F-region of the ionosphere.

5) response of C_B- and C_n-waves to earthquakes, magnetic storms, artificial explosions and magnetic activity of the Sun.

6) detection of electromagnetic and large-scale (1000—10000 km) character of both C_B- and C_n-waves by world-wide network of ionospheric and magnetospheric observatories.

The self-localization of the planetary electromagnetic waves in the non-dissipative ionosphere is proved on the basis of the analytical solution of the nonlinear dynamic equations (22)—(24). As follows from (64), the solution has the asymptote Ψ, $b_z \sim \exp(-\kappa|x|)$ at $|x| \to \infty$, so the wave is localized along the Earth's surface (η, y).

The generated nonlinear vortex structures represent a monopolistic (solitary) cyclone (Fig. 1) and/or anticyclone (Fig. 2) or a cyclone — anticyclone pair, connected in a certain manner (Fig. 3) and/or a pure dipole cyclone — anticyclone structure of equal intensity (Fig. 4), rotating in the opposite direction and moving along the latitudinal circles (along the parallels) against the background of the mean zonal wind.

The nonlinear large-scale vortices generate stronger pulses of the geomagnetic field than the corresponding linear waves. Thus, the fast vortices generate the magnetic field $b_v^f \approx 10\,\mu$T, and the slow vortices form the magnetic field $b_v^s \approx 100$ nT. The formation of such intensive perturbations can be related to specific properties of the low-frequency planetary structures under consideration. Indeed, they trap environmental particles, and charged particles in E- and F-regions of the ionosphere are completely or partially frozen into the geomagnetic field. That is why the formation of the structures is indicative of the significant densification of the magnetic force lines and, respectively, the intensification of the disturbances of the geomagnetic field in their location. Since the number of captured particles is of the order of the number of passed-by (transient) ones, the perturbation of the magnetic field in the stronger faster vortices would be the same order as of the background field. On the Earth's surface, located R_0 (~ 100—300 km) below the region of the wave structure under consideration, the level of the geomagnetic pulses would be less by the factor $\exp(-R_0/\lambda_0$, here λ_0 is the characteristic length of the electromagnetic perturbations. Since $\lambda_0 \sim (10—100)R_0 \gg R_0$, the magnetic effect on the Earth's surface is less then in E- and F-regions, but in spite of this they are easily detected as well.

The motion of medium particles in the nonlinear vortex structures (64) under study is characterized by nonzero vorticity $\nabla\times\mathbf{V} \neq 0$, i. e., the particle rotate in vortices. The characteristic velocity of this rotation U_c is of the same order as the vortex velocity U, $U_c \geq U$. In this case the vortex contains the group of trapped particles (the number of these particles is approximately the same as the number of transit particles); rotating, these particles move simultaneously with the vortex structure. Therefore, being long-lived objects, nonlinear planetary-scale electromagnetic vortex structures may play an important role in transporting matter, heat, and energy, and also in driving the macroturbulence of the ionosphere [1]. In particular, the vortex structures that play the role of «turbulent agents» can be treated as elements of the horizontal macroscopic turbulent exchanges in global circulation processes in the ionospheric E- and F-layers. The coefficient of the horizontal turbulent exchange can be estimated from the Obukhov formula [26, 43]: $K_T \approx 0.01d^{4/3}$ m^2/s. Thus, for vortices with dimensions of about $d \sim 1000$ km at latitudes of about $\varphi = 50°—55°$, we obtain $K_T \approx 3\cdot10^6$ m^2/s. This estimate (which can be regarded as an upper one) shows that, in the global exchange processes between high and low latitudes, the meridional heat transport from north to south in the ionospheric E- and F-layers should be of macroturbulence nature (recall that, in the ionosphere, the polar regions are warmer than the equatorial region).

The frequencies of the waves under investigation vary in the band $\omega \sim 10—10^{-6}$ Hz and occupy both infrasound and ULF bands. Wavelength is $\lambda \sim 1000—10000$ km, period of oscillation is T about of 0.1 s to 14 days. The electromagnetic perturbations from this band are biological active [32]. Namely, they can play an important role as a trigger mechanism of the pathological complications in people having the tendency to hypertensional and other diseases. Thus, these waves deserve great attention, as they are a significant source of the electromagnetic pollution of environment.

The fast and slow electromagnetic planetary waves are own degree of freedom of the E and F-regions of the ionosphere. Thus, first of all, the impact on the ionosphere from the top or the bottom (magnetic storms, earthquakes, artificial explosions and so on) induces (or intensifies) the wave structures of these modes [5]. At the certain strength of the source, the nonlinear solitary vortices would be generated [2], which is proved by the observations [16, 39, 46].

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МЕХАНІЗМ ГЕНЕРАЦІЇ ТА ХАРАКТЕРИСТИКИ ПОШИРЕННЯ СТРУКТУР УЛЬТРАНИЗЬКОЧАСТОТНИХ ІОНОСФЕРНИХ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ ПЛАНЕТАРНОГО МАСШТАБУ

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Наведено результати теоретичного дослідження генерації та поширення електромагнітних УНЧ-хвиль планетарного масштабу (λ > 1000 км) у дисипативній іоносфері. Установлено, що вони генеруються неоднорідностями (широтними варіаціями) геомагнітного поля в іоносфері та обертанням Землі. Хвилі поширюються вздовж паралелей в обох напрямках. У Е-області швидкі хвилі мають фазові швидкості 2—20 км/с і частоти 0.1—100 мГц; повільні хвилі поширюються зі швидкостями локальних вітрів і мають частоти 1—100 мкГц. У F-області швидкі хвилі мають фазові швидкості від десятків до кількох сотень кілометрів за секунду і частоти 10—0.001 Гц. Повільна мода утворюється динамо електричного поля; вона є узагальненням типових хвиль Россбі в іоносфері, яка обертається, і зумовлена ефектом Холла в Е-шарі. Швидкі збурення — це нові моди, які асоціюються з осциляціями іоносферних електронів, вмерзлих у геомагнітне поле, і пов'язані з виникненням великомасштабного внутрішнього вихрового електричного поля в іоносфері. Великомасштабні хвилі затухають слабо. Теоретичні характеристики узгоджуються із спостережуваними характеристиками великомасштабних УНЧ-осциляцій і магнітоіоносферних хвильових збурень. Установлено, що завдяки силі Коріоліса й електромагнітній силі генерація повільних планетарних електромагнітних хвиль на певній широті в іоносфері може викликати зміну напрямків локальних вітрів і зміну напряму загальної іоносферної циркуляції. Розглянуто ще один тип хвиль, названих повільними магнітогідродинамічними хвилями, на які не впливає неоднорідність сил Коріоліса й Ампера. Ці хвилі виникають як суміш повільних альвенівських пертурбацій і пертурбацій типу вістлера і породжують геомагнітне поле порядку 10—100 нТл і більше. Установлено, що УНЧ-хвилі під час взаємодії з локальними зональними вітрами можуть локалізуватися у вигляді нелінійних одиночних вихорів, що переміщуються вздовж кіл широти як у західному, так і у східному напрямі зі швидкістю, відмінною від фазової швидкості відповідних лінійних хвиль. Вихори затухають повільно й мають довгий термін існування. Вони зумовлюють геомагнітні пульсації, на порядок більші, ніж лінійні хвилі. Вихрові структури переносять захоплені частки навколишнього середовища, а також енергію й теплоту, і тому можуть бути елементами сильної макротурбуленції іоносфери.

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БАЛЛОННЫЕ МОДЫ ВО ВНУТРЕННЕЙ МАГНИТОСФЕРЕ ЗЕМЛИ С УЧЕТОМ КОНЕЧНОЙ ПРОВОДИМОСТИ ИОНОСФЕРЫ

Досліджується проблема генерації власних МГД-збурень балонного типу у внутрішній магнітосфері Землі у дипольній геометрії геомагнітного поля з урахуванням граничних умов на іоносфері. Остання розглядається як тонкий шар зі скінченною провідністю. Основна увага у роботі приділена вивченню впливу провідності іоносфери на стійкість вказаних збурень. Показано, що у наближенні ізолюючої іоносфери у магнітосферній плазмі збуджуються жолобкові збурення, для яких отриманий аналітичний критерій стійкості. У випадку ідеальної провідності іоносфери основним джерелом нестійких МГД-збурень є балонні моди, умова збудження яких є жорсткішою, ніж умова збудження жолобкових мод. Показано, що стійкі тороїдальні альвенівські хвилі слабо затухають за рахунок скінченної провідності іоносфери.

ИСХОДНЫЕ УРАВНЕНИЯ

Ранее в работах [1, 3] было показано, что баллонные возмущения являются естественным видом МГД-возмущений внутренней магнитосферной плазмы и описываются следующими уравнениями малых колебаний:

$$\Omega^2\xi + \frac{a(\theta)}{\cos^{13}\theta}\frac{\partial}{\partial\theta}\left[\frac{1}{a(\theta)\cos\theta}\frac{\partial\xi}{\partial\theta}\right] +$$