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THE OVERFLOW OF DENSITY SINGULARITY BY SHOCK GENERATED BY STRONG EXPLOSION

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Cosmological nature of the GAMMA-Ray Bursts means that energy discharged from the GRB is greater than energy emitted from supernova explosion, and is enough to make shock reach neighbouring stars or gas clouds and remain strong. It implies the possibility of using the Kompaneets strong explosion approximation for analysis of «hypernovae» remnants shapes, which may correspond to the Gamma-Ray Bursts. Forms of the shock generated by a strong explosion in a medium with quadratic law of density decrease and coming to constant on big distances are analysed. The overflow of density singularity is observed. Obtained results are compared with observational data about hypernova explosions.

The revelation of GAMMA-Ray Bursts cosmological nature in the recent couple of years means that in a nature the explosions of stars are much more powerful than the explosions of supernovae (with the energy approximately 10^{53} erg) [7]. In its turn, it means that like supernovae remnants existing during tens of thousands years, the remnants of these more powerful explosions — hypernovae — are also to exist too. At present the objects NGC5471B, MF83 are considered as pretenders [1, 9].

The energy emitted by hypernova explosion in principle is sufficient to make the blast wave reach neighbouring stars and remain strong enough. In doing so the rounding of obstacle [6] (star, cloud) has to arise, reminding characteristic detail in the image of presupposed remnant of hypernova [1].

The case of enlarging of the shock wave in a radially stratified medium with quadratic density decrease (similar to peripheral regions of sun corona) and coming to the constant corresponding to average density of an interstellar medium is the most interesting. Direct solution of the problem for this case is difficult, but it is possible to obtain it using transformation suggested by V. Kontorovich and S. Pimenov [4, 5], for shock front generated by a strong blast in the flat-stratified medium with positive exponential index and coming to the constant in negative z region:

$$\varphi(z) = \beta\, e^{\alpha z} + c. \tag{1}$$

So, using transformation [4, 5]

$$\ln\frac{R}{a} = \frac{z}{z_0},\ z_0\chi = r,\ \Psi(R) = \varphi\left(z_0 \ln\frac{R}{a}\right)\frac{z_0^2}{R^2}, \tag{2}$$

where $\Psi(R)$ is the dependence of density on distance to density singularity, a is the distance from singularity to the explosion in radially stratified medium, $\varphi(z)$ is the density function in flat-stratified medium, z_0 is the scale of the medium nonhomogeneity, general form of the density function from distance to singularity is obtained:

$$\varphi(z) = \beta\, e^{\alpha z} + c \Leftrightarrow \Psi(R) = \frac{\beta\, z_0^2}{a^{\alpha z_0}} R^{\alpha z_0 - 2} + c\,\frac{z_0^2}{R^2}. \tag{3}$$

One can see from (3), $\alpha z_0 = 2$ corresponds to coming to the constant in (1). In this way obtain the law of the dependence of density on distance, required for solution of the problem ($z_0 = 1$, $\alpha = 2$):

$$\Psi(R) = \frac{\beta}{a^2} + \frac{c}{R^2}. \tag{4}$$

Solutions for the blast wave in the medium (1) for equation proposed by Kompaneets,

$$\left(\frac{\partial r}{\partial y}\right)^2 - \frac{1}{\varphi(z)}\left[\left(\frac{\partial r}{\partial z}\right)^2 + 1\right] = 0, \tag{5}$$

where $r = r(z, y)$ is the function describing the form of the shock wave front, and generalized «time» y of the shock wave distribution is introduced accordingly to

$$dy = \sqrt{\frac{\lambda(\gamma^2 - 1)E}{2\rho_0}}\,\frac{dt}{\sqrt{V(t)}}, \tag{6}$$

where t is the time, $V(t)$ is the volume bounded by the shock front in time moment t, γ is the adiabat index, λ is the constant, taking into account the deviation of pressure directly beyond the front from average pressure by volume, built analogously [3—5, 8] by division of variables method:

$$\frac{\partial r}{\partial y} = \xi, \qquad \frac{\partial r}{\partial z} = \pm\sqrt{\xi^2(\beta e^{\alpha z} + c) - 1}, \tag{7}$$

$$r = \int_0^z \sqrt{\xi^2(\beta e^{\alpha z} + c) - 1}\,dz + \xi y + b(\xi),$$

$$\frac{\partial r}{\partial \xi} = 0 = y + \int_0^z \frac{\xi(\beta e^{\alpha z} + c)}{\sqrt{\xi^2(\beta e^{\alpha z} + c) - 1}}\,dz + \frac{db}{d\xi}. \tag{8}$$

Integrating by using condition of sphericity of the shock wave in small y and z ($b(\xi) = 0$) analogously to Silich and Fomin [8], and applying transformation (2) to obtained solutions, we obtain complete parametrical form of solution for the required density dependence:

$$\chi(R, \xi) = \frac{1}{\sqrt{1 - c\xi^2}}\left[\arccos\sqrt{\frac{1 - c\xi^2}{\xi^2\beta}} - \arccos\left(\frac{a}{R}\sqrt{\frac{1 - c\xi^2}{\xi^2\beta}}\right)\right],$$

$$y(R, \xi) = \frac{1}{\xi}\left[\sqrt{\xi^2(\beta + c) - 1} - \sqrt{\left(\xi\frac{R}{a}\right)^2\beta + c\xi^2 - 1}\right] + \chi c\xi, \tag{9}$$

in region

$$R \le R_-, \qquad \frac{1}{\beta(R/a)^2 + c} \le \xi^2 < \infty; \tag{10}$$

$$\chi(R, \xi) = \frac{1}{\sqrt{1 - c\xi^2}}\left[\arccos\left(\frac{a}{R}\sqrt{\frac{1 - c\xi^2}{\xi^2\beta}}\right) - \arccos\sqrt{\frac{1 - c\xi^2}{\xi^2\beta}}\right],$$

$$y(R, \xi) = \frac{1}{\xi}\left[\sqrt{\xi^2(\beta + c) - 1} - \sqrt{\left(\xi\frac{R}{a}\right)^2\beta + c\xi^2 - 1}\right] + \chi c\xi, \tag{11}$$

in region

$$R \ge R_+, \qquad \frac{1}{\beta + c} \le \xi^2 < \infty. \tag{12}$$

In region

$$R_- \ge R \ge R_+,$$

$$\eta(z)\frac{1}{\beta + c} + \eta(z)\frac{1}{\beta(R/a)^2 + c} \le \xi^2 < \infty, \tag{13}$$

the front of the shock is described by solution (with $b(\xi) \neq 0$, where $b(\xi)$ is chosen from the continuity condition of χ and y):

$$\chi(R,\xi) = \frac{1}{\sqrt{1-c\xi^2}}\left[\arccos\left(\frac{a}{R}\sqrt{\frac{1-c\xi^2}{\xi^2\beta}}\right) + \arccos\sqrt{\frac{1-c\xi^2}{\xi^2\beta}}\right],$$
$$y(R,\xi) = \frac{1}{\xi}\left[\sqrt{\xi^2(\beta+c)-1} + \sqrt{\left(\xi\frac{R}{a}\right)^2\beta + c\xi^2 - 1}\right] + \chi c\xi. \tag{14}$$

The bounds of the regions R_+ and R_- follow from real-validity and continuity of χ and y [8], and are defined by equations

$$y = \sqrt{\beta}\left[\sqrt{1-\left(\frac{R_-}{a}\right)^2} + \frac{c}{\beta}\frac{a}{R_-}\arccos\frac{R_-}{a}\right], \tag{15}$$

$$y = \sqrt{\beta}\left[\sqrt{\left(\frac{R_-}{a}\right)^2 - 1} + \frac{c}{\beta}\arccos\frac{a}{R_+}\right], \tag{16}$$

which follow from (9) and (11) correspondingly.

Ultra relativistic shock wave is formed in the initial stage of expansion in the case of analysis of the shock waves, generated by energy emission of hypernova order. It's velocity of spreading in any point of the front equals light speed.

Blandford — Mac-Kee formula, which describes the behaviour of the ultra relativistic shock wave front in the homogeneous medium, is used for analysis of initial stage of shock wave expansion:

$$\Gamma^2 = \frac{E}{\rho\, c^2 V}, \tag{17}$$

where $V = \frac{4\pi}{3}(ct)^3$ is the volume bounded by shock front. So, the time expended on the ultra relativistic stage of expansion (at $\Gamma^{-2} \geq 1$) is expressed in energy of blast and density of medium:

$$t_{\mathrm{ur}} = \sqrt[3]{\frac{3E}{4\pi c^5 \rho}}, \tag{18}$$

and full time of the shock front expansion presented in form of a sum of times expended to ultra relativistic and non-relativistic stages of expansion accordingly to (6) and (18):

$$t_{\mathrm{f}} = t_{\mathrm{ur}} + \sqrt{\frac{2\rho_0}{\lambda(y^2-1)E}}\int_{y_{\mathrm{ur}}}^{y}\sqrt{V(y)}\,dy, \tag{19}$$

where y_{ur} — such «time» moment y, that $V(y) = \frac{4\pi}{3}(ct_{\mathrm{ur}})^3$.

In Fig. 2 one can see the remnant of hypernova blast and calculated form of shock front in the «time» instant $y = 8.5$ corresponding to the real time $t \sim 30000$ years, passed since the moment of the blast with energy $E \sim 10^{48}$ J, which took place on a distance 500 light years from the density singularity.

In Fig. 2 it is noticeable the correspondence of the forms of the shock waves overflowing density decrease, which in this case can correspond to the dense cloud of the interstellar gas with size $R \sim 0.5$ light years and density $\rho \sim 10^{-21}$ kg/m^3 [2]. Also, there are many formations similar to one described above here in the left-hand side of

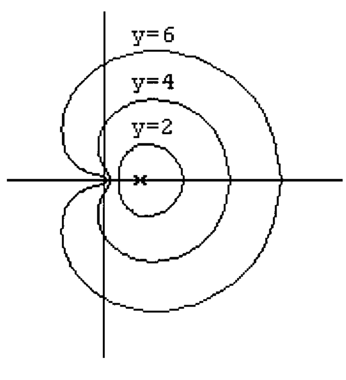

Figure 1. Cross section of a shock wave front in thee successive «time» moments y

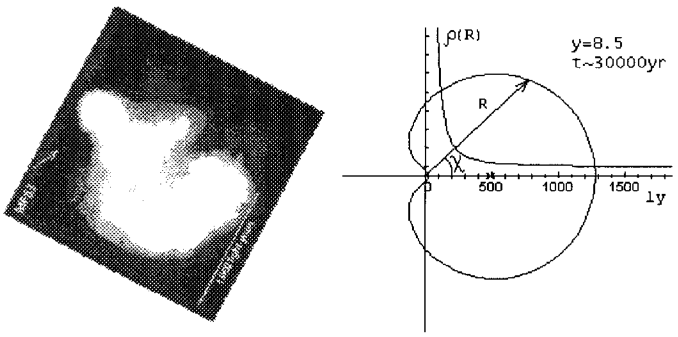

Figure 2. Image of the intended hypernova remnant MF83 (on the left side) and calculated form of a shock wave front (in the right side). Object, which can correspond to density singularity is marked by arrow (in the right figure density singularity is situated in the coordinate center), and overflow of a shock wave front is observed around it, shown in the right-hand side of the figure. Red color on the left figure corresponds to ionized sulfur, green — H_α-emission. Explosion point R = 500 light years is marked up by cross.

Fig. 2. These formations may be shown as a result of the overflow of the shock wave front on stars and interstellar gas clouds situated nearby the hypernova explosion.

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