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RADIATION SPECTRUM OF A RELATIVISTIC ELECTRON MOVING IN CURVED LINES OF MAGNETIC FIELD

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Radiation of ultra relativistic charged particles moving with small pitch angles along the spiral trajectory winded around curved magnetic force line is considered. The general radiation formulae, which can reduce to the formulae for either synchrotron or curvature radiation in the limiting cases, have been obtained. When the velocity of cyclotron rotation and the drift velocity are similar it is necessary to take into account the curvature of magnetic force line.

1. INTRODUCTION

Synchrotron radiation has broad application in investigation of laboratory plasma, plasma of solid bodies, in designing new apparatuses [7, 8]. Synchrotron radiation is among the most important in radio astronomy and astrophysics. Power law spectra of extragalactic radio sources corresponds to synchrotron radiation of ultrarelativistic electrons in cosmic magnetic fields [2]. The synchrotron radiation formulae are derived assuming a uniform and straight magnetic field. In the magnetosphere of a pulsar (Jupiter) charged particles are moving along curved magnetic field lines. Their trajectories are likely to be spiral curves along dipolar field lines. However, on calculating radio emission, many authors used the synchrotron radiation formulae for charged particles motion in uniform and straight magnetic field lines which may be correct or incorrect.

New radiation formulae for an ultrarelativistic charged particle motion along spiral trajectory with a small pitch angle to curved magnetic force line have been obtained in [1, 5, 6]. The case for which in the reference frame moving with the particle along magnetic force line the particle's energy is ultrarelativistic has been considered in [1, 4, 6]. This radiation was called as synchrotron curvature in [1]. The case when transverse energy is nonrelativistic in the particle system, and radiation is gathering from many particle cycles around force line was considered in [5] and called as the undulator curvature. The word «curvature» was added to distinguish between synchrotron and undulator radiation from charged particles in straight field lines and that from charged particles in curved magnetic field lines. Synchrotron radiation of relativistic electrons moving along spiral trajectory in tokamak was considered in [4]. When the drift velocity and cyclotron velocity of relativistic electrons are comparable it is necessary to take into account the curvature of magnetic field lines [6].

The radiated spectrum, polarization characteristics of the radiation emitted by the ultrarelativistic charged particle moving along spiral trajectory with small pitch angles in the curved magnetic field lines are obtained in the paper.

In the region in which radiation goes from magnetic field the lines are approximated by circular force lines with curvature radius R. The particle velocity along force line is equal to the light velocity, $v_{\parallel} \to c$. Emitted energy losses are not taken into account.

2. SYNCHROTRON CURVATURE RADIATION

Spectral angular distribution of energy radiated by the charged particle in the wave zone (energy radiated in the range of solid angles between o and $o + do$, and in the unit frequency interval of the frequency

ω) can be written as [3]

$$dEdo\,d\omega = \frac{cR_0^2}{4\pi^2}|E(\omega)|^2, \tag{1}$$

where $E(\omega)$ is the Fourier component of an electric field in the wave zone

$$E(\omega) = \frac{-i\omega e}{cR_0}\exp\frac{i\omega R_0}{c}\int_{-\infty}^{+\infty}[n[n,\beta]]\exp i\omega(t - nr/c)dt. \tag{2}$$

Here R_0 is a distance to the observer, n is the unit vector pointing the observer, $\beta = v/c$, v is the particle velocity, r is the particle position vector, for which (in the approximation $r_B/R \ll 1$, where r_B is a Larmor radius) we have expression [6]

$$r = [-2\delta r_B\sin\omega_B t\cos\Omega t + (R + r_B\cos\omega_B t)\sin\Omega t]\mathbf{i} + [2\delta r_B\sin\omega_B t\sin\Omega t + (R + r_B\cos\omega_B t)\cos\Omega t]\mathbf{j} + \\ + [v_D t - r_B\sin\omega_B t]\mathbf{k}, \tag{3}$$

where $\omega_B = eB/mc\gamma$, $\gamma \gg 1$ is a Lorentz factor, $\delta = \Omega/\omega_B \ll 1$, $\Omega \equiv v_{\|}/R$ is an angular velocity of the motion along circular force line, $v_D = -\Omega^2R/\omega_B$ is the drift velocity, **i**, **j**, **k** are the unit vectors of a coordinate system (magnetic force lines are localized in (x, y)-plane, and the magnetic surface axis is directed along z-axis), the magnitude (B) of the magnetic field is constant.

In order to calculate the spectral angular distribution of radiated energy given by equations (1), (2), we shall do as in the paper [6]. It is known that only a small part of the trajectory is effective in producing the narrow cone (with an apex angle $\propto 1/\gamma$) of radiation observed in the direction of the relativistic particle velocity. So at a given point the trajectory can be replaced by a circle, and the radiation formulae of the circular motion can be used.

The unit vectors in the plane perpendicular to the vector **n** pointing to the observer are defined as

$$\mathbf{e}_\sigma = [\mathbf{b}, \mathbf{n}]/|[\mathbf{b}, \mathbf{n}]|, \mathbf{e}_\pi = [\mathbf{n}[\mathbf{b}, \mathbf{n}]]/|[\mathbf{b}, \mathbf{n}]|, \tag{4}$$

where **b** is the unit vector of the binormal to the trajectory (3). The unit vector $\mathbf{e}_\sigma$ is perpendicular to the (**v**, **n**)-plane, and the vector $\mathbf{e}_\pi$ is placed in this plane.

To calculate the power radiated by a relativistic electron in the frequency range between the frequencies ω and $\omega + d\omega$, it is necessary to integrate the expression (1) over the solid angle and average it in time. The radiation is concentrated in the angular width $\delta\chi \propto 1/\gamma$ with respect to the surface being formed by the velocity unit vector $\tau = \mathbf{v}/v$, when the phase is changed in the interval $0 \le \omega_B t \le 2\pi$. The angle χ is counted in the direction perpendicular to this surface, the second angle μ is counted along the curve drawn by the end of the vector τ in the $(\mathrm{v}_y, \mathrm{v}_z)$-plane. The solid angle is $do = d\chi d\mu$, where $d\mu = = (1/v)(dv_y^2 + dv_z^2)^{1/2} = |v_D/v|\ (1 + q^2 + 2q\cos\omega_B t)^{1/2}d\omega_B t$, $q \equiv \omega_B^2 r_B/\Omega^2 R$ is the ratio of cyclotron velocity to the magnitude of the drift velocity. The cyclotron period $2\pi/\omega_B$ was taken as a time interval.

After that, the spectral power radiated by the charged particle in π- and σ-polarization can be written as

$$\begin{aligned} \frac{dP_\pi}{d\omega} &= \frac{3\sqrt{3}}{8\pi}\frac{1}{\gamma^3}\int_0^{2\pi}\frac{d\omega_B t}{2\pi}\frac{W}{kc}y\left[\int_y^\infty dxK_{5/3}(x) - K_{2/3}(y)\right], \\ \frac{dP_\sigma}{d\omega} &= \frac{3\sqrt{3}}{8\pi}\frac{1}{\gamma^3}\int_0^{2\pi}\frac{d\omega_B t}{2\pi}\frac{W}{kc}y\left[\int_y^\infty dxK_{5/3}(x) + K_{2/3}(y)\right], \end{aligned} \tag{5}$$

where $y = \omega/\omega_S$, $\omega_S = (3/2)\gamma^3 kc$, $W = (2/3)(e^2/c)k^2c^2\gamma^4$ is a total power radiated by the electron, which carries out circular motion with the curvature radius $1/k$, $kc = \Omega(1 + q^2 + 2q\cos\omega_B t)^{1/2}$.

The total radiated power, calculated from equations (5) [6], is

$$\frac{dP}{d\omega} = \frac{3\sqrt{3}}{4\pi}\frac{1}{\gamma^3}\int_0^{\pi}\frac{d\omega_B t}{\pi}\frac{W}{kc}\, y\int_y^{\infty} dxK_{5/3}(x). \quad (6)$$

It should be noted that the formula (6) can be derived from the expression (15) of the paper [4], in which the method based on the consideration of the rate at which the electron accomplishes work in the electromagnetic field was used. Thus, the formula (6) has been obtained by using two methods: on calculating the flux of the radiated power in the wave zone and by considering the work of the radiation field.

On performing the integration over the frequency in equations (5), we obtain $P_\pi = (1/8)P$ and $P_\sigma = (7/8)P$, where the total radiated power is given by

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^4\Omega^2(1 + q^2). \quad (7)$$

It is shown from equation (7) that the total power radiated by an electron in the curved magnetic field consists of both the curvature radiation losses (the first term in (7)) and the synchrotron radiation losses (the second term in (7)).

The formulae (5), (6) can be simplified by changing the order of integration, and they are reduced to

$$\frac{dP_i}{d\omega} = \frac{P}{\omega_1} f_i(y_1, q),\ i=\pi, \sigma, \text{tot}, \quad (8)$$

where $f_i(y_1, q)$ is given by

$$f_i(y_1, q) = \frac{(1+q)^2}{1+q^2}\frac{9\sqrt{3}}{8\pi}y_1\left\{\int_{y_1\frac{1+q}{|1-q|}}^{\infty} dxF_i(x) + \int_{y_1}^{y_1\frac{1+q}{|1-q|}} dxF_i(x)\frac{2}{\pi}\arcsin\frac{1+q}{2\sqrt{q}}\sqrt{1-\frac{y_1^2}{x^2}}\right\}. \quad (9)$$

In which $y_1 = \omega/\omega_1$, $\omega_1 = (3/2)\gamma^3\Omega(1 + q)$, $F_{\pi,\sigma}(x) = (1/2)(K_{5/3}(x) \pm dK_{2/3}(x)/dx)$ (plus is related to π-polarization), $F_{tot} = F_\pi + F_\sigma = K_{5/3}$. The function (9) has property $f_i(y_1, q) = f_i(y_1, 1/q)$.

Therefore, the universal function of synchrotron radiation in a uniform magnetic field [3, 7]

$$f(y) = \frac{9\sqrt{3}}{8\pi}y\int_y^{\infty} dxK_{5/3}(x) \quad (10)$$

is replaced by expression (9).

The figure gives a graphical representation of the function (9) for different values of the parameter q.

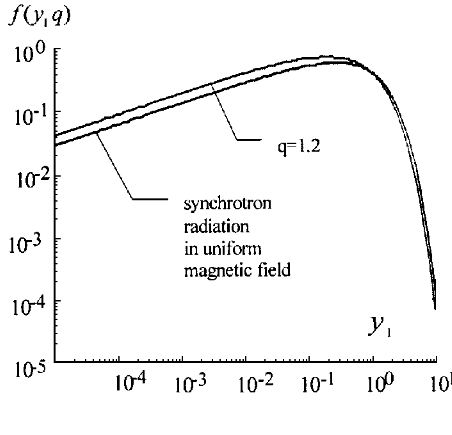

Figure. The function of synchrotron radiation in curved magnetic field

3. DIFFERENT CASES OF SYNCHROTRON RADIATION

Let's classify different cases of the radiation in curved magnetic field with respect to the parameter q and the ratio of the Lorentz factors γ and $\gamma_{\parallel}$, where

$$\gamma = (1/\gamma_{\parallel}^2 - \beta_{\perp}^2 - \beta_{\mathrm{D}}^2)^{-1/2}, \qquad \gamma_{\parallel} = (1 - v_{\parallel}^2/c^2)^{-1/2},$$

$$\beta_{\perp} = |\omega_{\mathrm{B}}| r_{\mathrm{B}}/c, \qquad \beta_{\mathrm{D}} = |v_{\mathrm{D}}|/c.$$

The charged particle is moving at the constant angle to the guiding center trajectory. The angle between this trajectory and the magnetic force line is $\sim \beta_{\mathrm{D}} = \Omega/\omega_{\mathrm{B}}$.

We have the following limiting cases:

1. When $q = |v_{\perp}/v_{\mathrm{D}}| > 1$ and $\gamma \gg \gamma_{\parallel} \gg 1$, the radiation is concentrated between two surfaces which have the apex angles $\propto 1/\gamma_{\parallel}$. The angular width between these surfaces is $1/\gamma$. The picture is analogous to the straight magnetic field. If $\gamma \sim \gamma_{\parallel}$ and $\beta_{\perp} < 1/\gamma_{\parallel}$, we have the case of undulator curvature radiation [5].

2. When $q \sim 1$ and $\gamma \gg \gamma_{\parallel} \gg 1$, the apex angle of the radiation cones is $\propto 1/\sqrt{2}\gamma_{\parallel}$ with respect to the direction of the guiding center trajectory. The angular width between cones is $\propto 1/\gamma$. The curvature of magnetic field lines is essential, the radiated spectral power is described by equations (8), (9). For $\gamma \sim \gamma_{\parallel}$ one needs to consider an undulator radiation case.

3. When $q < 1$, the apex angle of the radiation cone is less than the angle between the force line and the guiding center trajectory. If $\gamma \gg \gamma_{\parallel} \gg 1$, $\beta_{\mathrm{D}} \sim 1/\gamma_{\parallel}$, the apex angle of the radiation cone is $1/\gamma_1 = (1/\gamma_{\parallel}^2 - \beta_{\mathrm{D}}^2)^{1/2} \ll 1/\gamma_{\parallel}$. If also $\beta_{\perp} \sim 1/\gamma_1$, then $\gamma \gg \gamma_1$, and we have the usual picture of synchrotron radiation. The case $\beta_{\perp} < 1/\gamma_1$ needs additional consideration. If $\gamma \approx \gamma_{\parallel}$ it is the case of curvature radiation.

Therefore, there exist the correct formulae of synchrotron radiation of a relativistic charged particle moving at small pitch angles along curved magnetic field lines. The universal function (10) of synchrotron radiation in a uniform magnetic field is replaced by formula (9), which can be identical to either synchrotron or curvature radiation in certain parametric regions. The curvature of magnetic force line is essential when the parameter $q = |v_{\perp}/v_{\mathrm{D}}| = \omega_{\mathrm{B}}^2 r_{\mathrm{B}}/\Omega^2 R \sim 1$.

REFERENCES

1. Cheng K. S., Zhang J. L. General radiation formulae for a relativistic charged particle moving in curved magnetic field lines: the synchrocurvature radiation mechanism // Astrophys. J.—1996.—**463**, N 1.—P. 271—283.
2. Ginzburg V. L. Theory physics and astrophysics. — M.: Nauka, 1987.—488 p.
3. Landau L. D., Lifshits E. M. The field theory. — M.: Nauka, 1973.—504 p.
4. Pankratov I. M. Towards analyses of runaway electrons synchrotron radiation spectra. // Fizika Plazmy.—1999.—**25**, N 2.—P. 165—168.
5. Sobolev Ya. M. Towards radiation theory of a relativistic charged particle in curved magnetic field // Radio Physics and Radio Astronomy.—2000.—**5**, N2.—P. 137—147.
6. Sobolev Ya. M. Drift trajectory and synchrotron radiation of an ultrarelativistic electron moving in magnetic field with curved force lines // Voprosi Atomnoj Nauki i Tekhniki. Ser.«Plasma electronics and new acceleration methods»—2000.—N 2.—P. 27—30.
7. Sokolov A. A., Ternov I. M. Relativistic electron. — M: Nauka, 1983.—304 p.
8. Ternov I. M., Mikhailin V. V. Synchrotron radiation. Theory and experiment. — M.: Energoatomizdat, 1986.—96 p.