Космічна наука і технологія. ДОДАТОК. 2001. Т 7. № 2. С. 84—88.

UDC 530.145

POSSIBLE PECULIARITIES OF SYNCHROTRON RADIATION IN STRONG MAGNETIC FIELDS

B. I. Lev[1], A. A. Semenov[1], C. V. Usenko[2]

[1]Institute of Physics, NAS of Ukraine, 46, Nauky pr., Kiev 03022, Ukraine
[2]Taras Shevchenko Kiev University, Physical Department, 6, Academician Glushkov pr., Kiev 03127, Ukraine

Relativistic quantum effects on physical observables of scalar charged particles are studied. Possible peculiarities of their behavior that can be verified in an experiment can confirm several fundamental conceptions of quantum mechanics. For observables independent of charge variable, we propose the relativistic Wigner function formalism that contains explicitly the measurement device frame. This approach can provide the description of charged particles gas (plasma). It differs from the traditional one but is consistent with the Copenhagen interpretation of quantum mechanics. The effects that are connected with this approach can be observed in astrophysical objects, i.e., neutron stars.

INTRODUCTION

The wave function nature has been considered as philosophical rather than physical question for a long time. However, it is very actual now because of the recent theoretical and experimental progress in quantum information [12].

In 1980, the specific behavior of the quantum systems that are described by the Einstein — Podolsky — Rosen (EPR) paradox was confirmed in an experiment [1]. It is very important that EPR correlation «distributes» in a space instantly. Nevertheless, if one adheres to the Copenhagen interpretation, there is no a casualty principle breaking.

In contrast, if one can try to hold an objective and deterministic description of quantum mechanics, then classical understanding of the casualty principle should be broken, because of the conflict between quantum mechanics and special relativity [2], [6]. In [4] the relativistic classical and quantum mechanics that generalizes the casualty principle was constructed. Such an approach follows from the Eberhard and Bell idea that correct description of quantum mechanics should contain a preferred frame.

This theory has a well definite position operator. In the preferred frame it coincides with the Newton — Wigner coordinate [7]. It means that measurement of the coordinate does not create a particle-antiparticle couple because there is no an odd part.

Hence, it is very important to find situations when the odd part of the position operator could manifest itself. However, unfortunately, such experiments are very difficult on the Earth because of necessity of a very strong field. There are such fields near the astrophysical objects, i.e., neutron stars. Therefore, it is interesting to find how the odd part of the position operator can influence the observable variables in a multy-body system (in a gas of charged particles or plasma).

The Wigner function formalism [14] is a convenient method to describe such systems. However, there are several problems of generalization for the relativistic case. The first problem is that the time is not a dynamic variable in the Weyl rule. In [5] this problem was solved by the generalization of the spatial integration over the whole space-time domain. The Wigner function formulated in a framework of the stochastic interpretation of quantum mechanics is also the Lorentz invariant [10].

Formalism of the matrix-valued Wigner function for spin S particles was developed in [8] by usual Weyl rule. Certainly, such equations are not the Lorentz invariant.

Next problem is the absence of well-defined position operator. In [13] the Wigner function formalism was developed by using the Newton — Wigner coordinate. The results of this approach differ from the standard one. However, one can connect them with [4] where the correct definition of the position operator is possible.

The aim of this work is a formulation of the Wigner formalism for scalar charged particles in the approach [8]. In addition we try to find several peculiarities of the behavior of relativistic quantum system including the cases with the complicated structure of the position operator.

WIGNER FUNCTION FOR A FREE PARTICLE

To develop the Wigner function formalism one needs to formulate the Weyl rule. Following [8] one should take into account that classical variables are matrixes. However, we restrict ourselves with those proportional to the identity matrix. For a convenience we shall use the Feshbach — Villars representation [7]. The Weyl rule is defined by usual way:

$$A_\alpha^\beta = \int A(p, q)\widehat{W}_\alpha^\beta(p, q)dpdq,$$

where α, $\beta = \pm 1$, $A(p, q)$ is the classical variable, $\widehat{A}_\alpha^\beta$ is the corresponding classical variable, $\widehat{W}_\alpha^\beta$ is the density operator that can be presented by the displacement operator:

$$\widehat{W}_\alpha^\beta(p, q) = \frac{1}{(2\pi h)^{2d}} \int \widehat{D}_\alpha^\beta(P, Q)e^{\frac{i}{h}(Qp - Pq)}dQdP. \tag{1}$$

In this representation the displacement operator can be expanded by eigenvectors of the momentum operators:

$$\widehat{D}_\alpha^\beta(P, Q) = \int |p + P/2\rangle R_\alpha^\beta\left(p + \frac{P}{2}, p - \frac{P}{2}\right) e^{-\frac{i}{h}Qp}dp\,\langle p - P/2|. \tag{2}$$

In contrast to [13] and for non-relativistic case there is the matrix-valued variable in (2):

$$R_\alpha^\beta(p_1, p_2) = \varepsilon(p_1, p_2)\delta_\alpha^\beta + \chi(p_1, p_2)\tau_{1_\alpha}^\beta.$$

It contains even and odd parts and is expressed by the relativistic energy of a free particle $E(p)$:

$$\varepsilon(p_1, p_2) = \frac{E(p_1) + E(p_2)}{2\sqrt{E(p_1)E(p_2)}}, \qquad \chi(p_1, p_2) = \frac{E(p_1) - E(p_2)}{2\sqrt{E(p_1)E(p_2)}}. \tag{3}$$

Combining (1) and (2) we obtain now the formula for the density operator expansion:

$$\widehat{W}_\alpha^\beta(p, q) = \frac{1}{(2\pi h)^d} \int |p + P/2\rangle R_\alpha^\beta\left(p + \frac{P}{2}, p - \frac{P}{2}\right) e^{-\frac{i}{h}Pq}dP\langle p - P/2|. \tag{4}$$

The Wigner function is the average of this operator for an arbitrary state:

$$W(p, q) = \sum_{\alpha,\ \beta} \langle\psi_\beta|\widehat{W}_\alpha^\beta(p, q)|\psi^\alpha\rangle.$$

This expression contains four terms. Two of them are the average of the even part of the density operator and two others are the average of the odd part. Now one can introduce the symbols:

$$W_\alpha^\beta(p, q) = \langle\psi_\beta|\widehat{W}_\alpha^\beta(p, q)|\psi^\alpha\rangle. \tag{5}$$

It should be noted that $W_\alpha^\beta(p, q)$ is not the matrix-valued Wigner function in the sense of [8].

Using the expressions (4) and (5) the components of the Wigner function are obtained in the form:

$$W_\alpha^\alpha(p, q) = \frac{1}{(2\pi h)^d} \int \varepsilon\left(p + \frac{P}{2}, p - \frac{P}{2}\right)\psi_\alpha^*\left(p + \frac{P}{2}\right)\psi^\alpha\left(p - \frac{P}{2}\right) e^{-\frac{i}{h}Pq}dP,$$

$$W_{\alpha}^{-\alpha}(p, q) = \frac{1}{(2\pi h)^{d}} \int \chi\left(p + \frac{P}{2}, p - \frac{P}{2}\right)\psi_{\alpha}^{*}\left(p + \frac{P}{2}\right)\psi^{-\alpha}\left(p - \frac{P}{2}\right)e^{-\frac{i}{h}Pq}dP.$$

One can obtain the quantum Liouville equation by standard way [14]:

$$\begin{aligned} \frac{\partial W_{\alpha}^{\alpha}(p, q, t)}{\partial t} &= \alpha \frac{2}{h} E(p) \sin\left\{-\frac{h}{2} \overleftarrow{\partial_{p}}\overrightarrow{\partial_{q}}\right\} W_{\alpha}^{\alpha}(p, q, t), \\ \frac{\partial W_{\alpha}^{-\alpha}(p, q, t)}{\partial t} &= i\alpha \frac{2}{h} E(p) \cos\left\{-\frac{h}{2} \overleftarrow{\partial_{p}}\overrightarrow{\partial_{q}}\right\} W_{\alpha}^{-\alpha}(p, q, t). \end{aligned} \tag{6}$$

The equation for the even part of the Wigner function coincides with the similar expression in the Newton — Wigner coordinate approach [13]. Hence dynamics of the distribution function in both cases is identical. The difference is in the constraints for the initial conditions. The physical variables that contain higher moments of the coordinate (for example, dispersion) differ from those in [13]. For one particle problem these peculiarities were developed in [3].

PARTICLES IN A HOMOGENEOUS MAGNETIC FIELD

The particles in external electromagnetic fields are more sensitive to the odd part of the coordinate. For example, in a uniform electric field the odd part of the position in the Hamiltonian of interaction results in the effects of particles creation from the vacuum [9]. The origin of this peculiarity is that even and odd parts of the Wigner function are entangled in equations like (6).

Here we shall study the behavior of particles in a time-independent and homogeneous magnetic field that is more typical for astrophysical objects. Following [11] we shall use the energy representation and so we have to consider quasi-particles rather than particles. Both the position and momentum operators have odd parts in this approach.

Further we do not take into account the particle motion along the magnetic field and consider only relativistic rotator.

Following the previous paragraph one can write the displacement operator in the energy representation:

$$D_{n,m;\alpha}^{\beta}(P, Q) = (\varepsilon_{n,m}\delta_{\alpha}^{\beta} + \chi_{n,m}\tau_{1_{\alpha}}^{\beta})D_{n,m}(P, Q),$$

where $D_{n,m}(P, Q)$ are the matrix elements of the usual displacement operator on the eigenfunctions of the harmonic oscillator [15]. $\varepsilon_{n,m}$, $\chi_{n,m}$ are defined like (3), but with the spectrum of the relativistic rotator in place of the energy of a free particle. Then, the density operator and the Wigner function are defined in the way presented in the previous paragraph. The final expressions for the even and odd components of the Wigner function are

$$W_{\alpha}^{\alpha} = \sum_{m,n} \varepsilon_{m,n} C_{n;\alpha}^{\mathrm{sup}} * C_{m;}^{\alpha} T_{m,n}(p, q),$$

$$W_{\alpha}^{-\alpha} = \sum_{m,n} \varepsilon_{m,n} C_{n;\alpha}^{\mathrm{sup}} * C_{m;}^{-\alpha} T_{m,n}(p, q),$$

Here $C_{n;\alpha}$ is the wave function in the energy representation, $T_{m,n}(p, q)$ is the matrix elements of the usual displacements operator

$$\hat{T}(p, q) = \frac{1}{(2\pi h)^{2d}} \int \hat{D}(P, Q) e^{-\frac{i}{h}(Pq - Qp)} dPdQ.$$

The equations for the Wigner function (12) can be obtained in the standard way too. Here the different components are not entangled. Hence there are no effects connected with vacuum instability [9]:

$$\frac{\partial W_{\alpha}^{\alpha}(p,q,t)}{\partial t} = \alpha \frac{2}{h} E(p,q) \sin\left\{\frac{h}{2}(\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_q)\right\} W_{\alpha}^{\alpha}(p,q,t),$$
$$\frac{\partial W_{\alpha}^{-\alpha}(p,q,t)}{\partial t} = i\alpha \frac{2}{h} E(p,q) \cos\left\{\frac{h}{2}(\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_q)\right\} W_{\alpha}^{-\alpha}(p,q,t). \tag{7}$$

In this expression we introduce the Weyl symbol for the Hamiltonian of the relativistic rotator in the energy representation (it should be redefined):

$$E(p,q) = mc^2\sqrt{1 + \frac{2}{mc^2}\left(\frac{p^2}{2m} + \frac{\omega_c^2 m}{2} q^2\right)},$$

where $\omega_c = eB/m$ is the cyclotron frequency.

The odd part in the Wigner function definition describes interference effects between particles and antiparticles. Furthermore, the value ε_{nm} defines the specification of the initial conditions.

In [11] we studied dynamics of the one particle problem. It was shown that in fields less than critical ($h\omega_c < mc^2$) the mean radius of the trajectory oscillates with the frequency

$$\Omega = \frac{h\omega_c^2}{mc^2}.$$

This is essencially quantum relativistic effect. It can be observed as a low frequency modulation of synchrotron radiation. However, unfortunately such dynamical process is identical to both approaches and does not reveal the complicated structure of the position operator.

Let us consider now the dispersion of the orbit radius for the nonlinear coherent state [11]:

$$\overline{\Delta R^2} = \frac{h}{m\omega_c}\left[1 - |C_0|^2 \sum_n \frac{R^{2n}}{2^n n! [\varepsilon_n]!}\chi_n^2\right],$$

where C_0 is the normalization factor, and Q, P are the mean value of the position and momentum of the wave packet. It contains both the standard and additional terms. They result in the appearance of the states with formally broken uncertainty relation. One can expect that such effects take place for multi-body systems too.

CONCLUSION

The odd part of the position operator results in the non-standard behavior of the physical observables. The whole system has also peculiarities. However, it is observed not for all physical variables. For example, behavior of energy does not contain such peculiarities. Hence, one can expect these effects for the quadratic and higher moments of the coordinate and momentum.

Especially one should notice the effects connected with interference between particles and antiparticles. They result from the odd part of the Wigner function and can be observed in systems of particles with opposite charge signs.

Finally we briefly note that relativistic quantum mechanics in a Wigner formulation contains the measurement device frame. Actually, one can write the equation (6) and (7) using only four dimensional Lorentz invariant symbols. To make it possible one should incorporate into equations a certain time-like vector. It can be interpretated as a four-velocity of the frame where a wave packet reduction happens relative to the second (immobile) observer. It is very important that quantum mechanics equations contain explicitly the observer characteristics. This fact can serve as an additional argument in favor of the Copenhagen interpretations of quantum mechanics.

REFERENCES

1. Aspect A., Roger G., et. al. Time Correlations between Two Sidebands of the Resonance Fluorescence Triplet // Phys. Rev. Lett.—1980.—**45**.—P. 617;
Aspect A., Grangier P., Roger G. Experimental Test of Realistic Local Theories via Bell's Theorem // Phys. Rev. Lett.—1981.—**47**.—P. 460.
2. Bell J. S. Speakable and Unspeakable in Quantum Mechanics. — Cambridge: Cambridge Univ. Press, 1987.
3. Blokhintzev D. I. On localization relativistic micro particles in space and time // JINR, R-2631, Dubna, 1966 (in Russian).
4. Caban P., Rembielinski J. Lorentz-covariant quantum mechanics and preferred frame // Phys. Rev. A.—1999.—**59**.—P. 4187.
5. De Groot S. R., van Leeuwen W. A., Weert Ch. G. Relativistic kinetic theory. Principles and Applications. — Amsterdam: North-Holland, 1980.
6. Eberhard P.H. Bell's Theorem and the Different Concepts of Locality. // Nuov. Cim.—1978.—**46B**.—P. 392.
7. Feshbach H., Villars F. Elementary Relativistic Wave Mechanics of Spin 0 and Spin S Particles // Rev. Mod. Phys.—1958.—**30**.—P. 24.
8. Gerard P., Markovich P. A., Mauser N. J., et al. Homogenization Limits and Wigner Transforms // Comm. Pure Appl. Math.—1997.—**50**.—P. 0323.
9. Grib A. A., Mamaev S. G., Mostapenko V. M. Vacuum quantum effects in strong fields. — Moscow: Energoatomizdat, 1988 (in Russian).
10. Holland P. R., Kyprianidis A., Maric Z., et al. Relativistic generalization of the Wigner function and its interpretation in the casual stochastic formulation of quantum mechanics // Phys. Rev. A.—1986.—**33**.—P. 4380.
11. Lev B. I., Semenov A. A., Usenko C. V. Behaviour of $\pi\pm$ mesons and synchrotron radiation in a strong magnetic field // Phys. Lett. A.—1997.—**230**.—P. 261.
12. Menskii M. B. Quantum mechanics: new experiments, new applications, new formulations // Uspekhi fizicheskikh nauk.—2000.—**170**, N 6.—P. 631.
13. Mourad J. The Wigner-Weyl formalism and the relativistic semi-classical approximation. — E-print of LANL: /hep-th/9307135.
14. Moyal J. E. Quantum Mechanics as a Statistical Theory // Proc. Cambr. Phil. Soc.—1949.—**45**.—P. 99.
15. Perelomov A. M. Generalized coherent states and their applications. — Berlin: Springer, 1996.