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# THEORY OF LOW-SCALE MHD WAVES IN THE NEAR EQUATORIAL REGION OF THE EARTH PLASMASPHERE

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Study of low-scale MHD waves in the inner Earth magnetosphere was performed using the standard set of ballooning mode equations obtained naturally for finite pressure plasmas in general magnetic field geometries. Flute- and ballooning-type low-scale disturbances in the near equatorial plasmasphere where they may have maximum growth rates were studied. The ballooning mode stability condition was obtained, the wave excitation spectrum was investigated. Corresponding dispersion equations for low-frequency wave modes were formulated and analysed. Dependencies of eigenfrequencies and growth rates on plasma pressure and McIlwain parameter were discussed.

## INTRODUCTION

Our approach to the low-scale MHD disturbances is based on the dipole model of the geomagnetic field sketched on Figure 1. This field with the components

$$B_\theta = \frac{M\cos\theta}{r^3}, \qquad B_r = -\frac{2M\sin\theta}{r^3}$$

and with module

$$B = \frac{M\sqrt{a(\theta)}}{r^3},$$

where $a(\theta) = 1 + 3\sin^2\theta$, $M$ — the Earth's magnetic dipole moment.

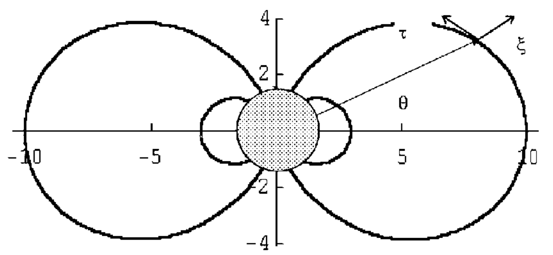



Fig. 1. Sketch of the dipole geomagnetic field geometry

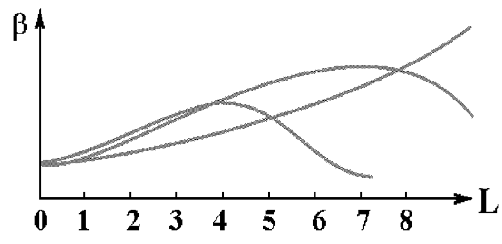



Fig. 2. Different possible radial dependencies of plasma $\beta$

Figure 2 represents the characteristic radial dependencies of $\beta$ which is the ratio of the gas-kinetic plasma pressure to the magnetic one. It results from the most general reasons, the $\beta$ is higher the plasma energy power is higher and its ability to deform the magnetic field equilibrium configuration containing the plasma is higher. Maxima of the presented curves are of the order of unit, so $\beta$ is not in the least a small value. At the same time it must be noted that the generally accepted results were derived predominatly for the case of plasma with $\beta \ll 1$. So the aim of this work is to extend the derived results on the finite $\beta$ domain.

## THE BALLOONING MODES EQUATIONS

To obtain the required ballooning equations which we are interested in one can start from ballooning-type equations in the magnetic field of arbitrary configuration, as it was provided in [1] and [4]. Also one can start from the linearized MHD equations for the plasma volume element displacement, as it was done in [2] and [5]. The principal fact is that either approach leads to the same system of equations. After the transformation to the dimensionless form which was used in [6] this system has the form:

$$\Omega^2\hat{\xi} + \frac{4}{a(\theta)\cos^4\theta}\left(\frac{\alpha\beta}{\gamma}\hat{\xi}+F\right) + \frac{a(\theta)}{\cos^{13}\theta}\frac{d}{d\theta}\left[\frac{1}{a(\theta)\cos\theta}\frac{d\hat{\xi}}{d\theta}\right] = 0, \tag{1}$$

$$\Omega^2\hat{\tau} + \frac{1}{\cos^7\theta}\frac{dF}{d\theta} = 0, \tag{2}$$

where

$$F = \frac{\beta\cos^2\theta}{a(\theta) + \beta\cos^{12}\theta}\left\{\cos^3\theta\frac{d\hat{\tau}}{d\theta} - \frac{2\sin\theta\cos^2\theta}{a(\theta)}\,[4+5a(\theta)]\,\hat{\tau} - \frac{4\hat{\xi}}{a(\theta)}\right\}, \tag{3}$$

$\alpha = -(L/p)dp/dL$ is the dimensionless pressure gradient. System of Equations (1)—(2) complemented by corresponding uniform boundary conditions at the upper ionosphere border $r(\theta) = 1$ (we consider ionosphere to be an ideal conductor) is the boundary eigenvalue problem. If we solve this problem we define it's eigenfrequencies or growth rates of aperiodic solutions and corresponding eigenfunctions. The equations forming this system are the linear differential equations with the variable coefficients, so there is no hope to solve this eigenvalue problem analytically. This problem may be solved only by applying some numerical method. Such approach enables us to derive any desired eigenvalue dependencies (i. e. squared eigenfrequencies or squared growth rates) upon the parameters but it has not desirable predicting features.

Therefore, it seems absolutely necessary to enunciate a simpler problem, in a certain sense similar to the original one, which will permit the analytical solution.

## NEAR-EQUATORIAL REGION AND CONSTANT COEFFICIENT PROBLEM

The most natural simplification for the original system (1)—(2) is the conversion from the variable coefficients to the constant ones. On the strength of the reasons of symmetry let us take their values at $\theta = 0$, i. e. in the geomagnetic equatorial plane. After the substitution of $F$ into the Equations (1) and (2), calculating of all the necessary derivatives and taking coefficient values at zero we will obtain the system of equations with the constant coefficients

$$\beta\frac{d^2\hat{\tau}}{d\theta^2} + [\Omega^2(1+\beta) - 18\beta]\,\hat{\tau} - 4\beta\frac{d\hat{\xi}}{d\theta} = 0, \tag{4}$$

$$\frac{d^2\hat{\xi}}{d\theta^2} + \left[\Omega^2 + 4\frac{\alpha\beta}{\gamma} - 16\frac{\beta}{1+\beta}\right]\hat{\xi} + 4\frac{\beta}{1+\beta}\frac{d\hat{\tau}}{d\theta} = 0. \tag{5}$$

This system of equations is good enough approximation of the original system (1)—(2) at close to zero values $\theta$. Is it possible to adduce the meaningful physical interpretation for such a task? It is possible if we find the near-equatorial region with the physically essentially different properties. The radiation belt of the Earth may be considered as such region. Indeed, in the radiation belt the $\beta$ values ($\beta$ is the ratio of gaskinetic plasma pressure to the pressure of the magnetic field) multifoldly preponderate over $\beta$ outside the belt. Moreover, the advanced $\beta$ leads to the advanced gaskinetic pressure gradient $\alpha$, which, as will be shown below, affects decisive the stability of the disturbances under consideration. During the

Solar activity elevation periods high-energy particle concentration increases exactly within the radiation belts. As for our problem such growth of concentration means the highly essential increase of $\beta$ and $\alpha$, which gives rise swing and transformation into unstable modes of low-scale waves.

With the boundary conditions on the radiation belt border we may expect that such a problem may be applicable to description of the low-scale disturbances. Furthermore, the properties of constant coefficient problem (4)—(5) solutions may be the desirable starting point for the interpretation of the original problem (1)—(2) solutions.

One may make sure that the system (4)-(5) has the nontrivial solutions

$$\hat{\tau}(\theta) = \begin{cases} \tau_0 \sin\mu_k\theta, & \text{even } k, \\ \tau_0 \cos\mu_k\theta, & \text{odd } k, \end{cases} \qquad \hat{\xi}(\theta) = \begin{cases} \xi_0 \cos\mu_k\theta, & \text{even } k, \\ \xi_0 \sin\mu_k\theta, & \text{odd } k, \end{cases} \tag{6}$$

with the boundary conditions

$$\frac{d\hat{\tau}}{d\theta}(\pm\theta_0) = 0, \; \hat{\xi}(\pm\theta_0) = 0, \tag{7}$$

where $2\theta_0$ is the angular size of the advanced $\beta$ domain, and also

$$\hat{\tau}(\theta) = \begin{cases} \tau_0 \cos\mu_k\theta, & \text{even } k, \\ \tau_0 \sin\mu_k\theta, & \text{odd } k, \end{cases} \qquad \hat{\xi}(\theta) = \begin{cases} \xi_0 \sin\mu_k\theta, & \text{even } k, \\ \xi_0 \cos\mu_k\theta, & \text{odd } k, \end{cases} \tag{8}$$

with the boundary conditions

$$\hat{\tau}(\pm\theta_0) = 0, \qquad \frac{d\hat{\xi}}{d\theta}(\pm\theta_0) = 0, \tag{9}$$

where $\mu_k = \dfrac{\pi}{2\theta_0}$ , $k = 0, \pm 1, \pm 2, \ldots$

(It is necessary to note that the system (4)—(5) with the simplest Dirichlet boundary conditions

$$\hat{\tau}(\pm\theta_0) = 0, \qquad \hat{\xi}(\pm\theta_0) = 0 \tag{10}$$

has not nontrivial solutions at all - their existence is precluded by the $d\hat{\xi}/d\theta$-consistent term in the $\hat{\tau}(\theta)$-equation and $d\hat{\tau}/d\theta$-consistent term in the $\hat{\xi}(\theta)$-equation).

### DISPERSION EQUATION AND ITS SOLUTIONS

The condition of the existence of nontrivial solutions (its physical sense — the dispersion relation) has almost the same form for both variants of the boundary conditions:

$$[\Omega^2(1+\beta) - 18\beta - \beta\mu_k^2]\left[\Omega^2 - \mu_k^2 + 4\frac{\alpha\beta}{\gamma} - 16\frac{\beta}{1+\beta}\right] - 16\frac{\beta^2\mu_k^2}{1+\beta} = 0. \tag{11}$$

Let us explain the sense of this assertion. Equation (11) with respect to $\Omega^2$ always has two solutions. Meantime the $k = 0$ case (accordingly to $\mu_k^2 = 0$) needs the special consideration. In fact, for the boundary conditions (7) at $k = 0$ we obtain the solutions

$$\hat{\tau}(\theta) \equiv \tau_0 \neq 0, \qquad \hat{\xi}(\theta) \equiv 0. \tag{12}$$

After their substitution into the constant coefficients system (4)—(5) we obtain the condition

$$\Omega^2(1+\beta) - 18\beta = 0 \tag{13}$$

from the $\hat{\tau}$-equation, while the $\hat{\xi}$-equation is satisfied identically. In the same way for the boundary conditions (9) and solutions

$$\hat{\tau}(\theta) \equiv 0, \qquad \hat{\xi}(\theta) \equiv \xi_0 \neq 0 \tag{14}$$

one can obtain the condition

$$\Omega^2 + 4\frac{\alpha\beta}{\gamma} - 16\frac{\beta}{1+\beta} = 0. \tag{15}$$

The solutions of the eigenproblem (4)—(5) with boundary conditions (7) and (9) at $k = 0$ have a common property: all the solutions are the identical constants and are absolutely independent of the variable $\theta$, i. e. they are the purely flute solutions. At $k \neq 0$ the dependencies on $\theta$ are essential, so the corresponding solutions are the ballooning ones.

Considering the latter remark about dispersion relations which result from the constant coefficient system (4)—(5) solvability, one may say:

— for the ballooning eigenfunctions satisfying both boundary conditions (7) and (9) the dispersion relation (11)

$$[\Omega^2(1+\beta) - 18\beta - \beta\mu_k^2]\left[\Omega^2 - \mu_k^2 + 4\frac{\alpha\beta}{\gamma} - 16\frac{\beta}{1+\beta}\right] - 16\frac{\beta^2\mu_k^2}{1+\beta} = 0.$$

is valid at the nonzero $k$ and $\Omega^2$ has two roots;

— for the flute eigenfunctions the dispersion equations are different for different boundary conditions, namely:
for the boundary conditions (7)

$$\Omega^2 = 18\frac{\beta}{1+\beta}, \tag{16}$$

is valid; for the boundary conditions (9)

$$\Omega^2 = 16\frac{\beta}{1+\beta} - 4\frac{\alpha\beta}{\gamma}, \tag{17}$$

is fulfilled and $\Omega^2$ has only one root.

Thereby, the set of dispersion equation solutions in a $(\beta, \Omega^2)$ plane consists of the only flute curve (16) or (17) (depending on boundary conditions) and of the infinite set of the ballooning curves pairs — for every nonzero $k$. These curves behaviour is demonstrated by Figure 3. The bold-typed curves are the ballooning solutions pair corresponding to $k = 3$, the upper thin curve is the flute solution for the boundary conditions (7), the lower one — for the conditions (9). It was noted that the ballooning dispersion equation solutions are completely identical for the both kinds of boundary conditions.

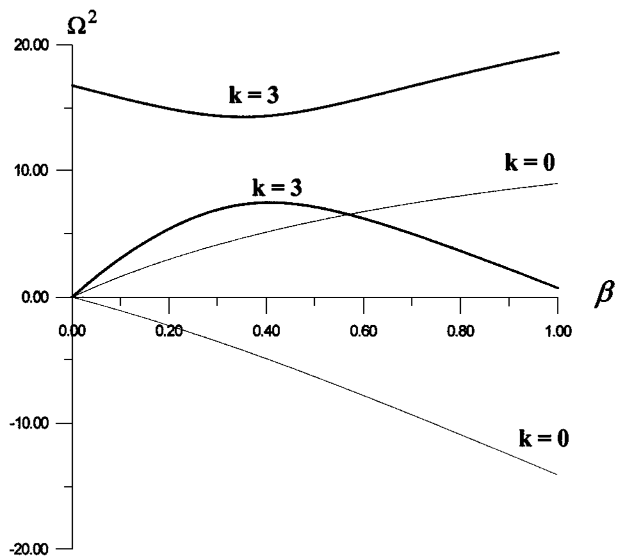



Fig. 3. $\beta$-dependent squared dimensionless eigenfrequencies $\Omega^2$ (constant coefficients problem)

THE STABILITY OF FLUTE AND BALLOONING DISTURBANCES

Study of stability of disturbances under consideration is of a special interest. Indeed, $\Omega^2$ in Equations (1)—(2) arises from time dependency of the form $\exp(-i\Omega t)$ when the Fourier method is used. It is obvious that the obtained solutions will not increase with time only under condition $\Omega^2 \geq 0$, otherwise the factor $\exp(\Omega_i t)$ where $\Omega_i = \mathrm{Im}(\Omega)$ will approve itself.

As to flute disturbances the stability question may be solved without any difficulties. For the boundary conditions (7) we have

$$\Omega^2 = 18\frac{\beta}{1+\beta} \geq 0, \tag{18}$$

i. e., the flute solutions are absolutely stable. For the boundary conditions (9) the stability criterion will have the form

$$\alpha \leq \frac{4\gamma}{1+\beta}, \tag{19}$$

and in the ultimate case of very small $\beta$ it descends into classical flute disturbances stability criterion (note, that isentropic exponent value is $\gamma = 5/3$).

$$\alpha \leq 20/3.$$

To obtain the ballooning disturbances stability criterion it is necessary to write down the conditions of dispersion equation (11) solutions reality which sooner or later can be expressed in the form

$$\alpha \leq \frac{\mu_k^2}{\beta}\frac{\gamma}{4} + \frac{4\gamma}{1+\beta}\frac{1}{1+\mu_k^2/18}. \tag{20}$$

It must be noted that in the ultimate case $\mu_k^2 = 0$ expression (20) coincides with the condition of flute disturbances stability for the boundary conditions (9). It is also evident that at small enough $\beta$ the ballooning disturbances are stable because with decrease of $\beta$ the restrictions for $\alpha$ become more and more weak. It results from the monotonous increase $\mu_k^2$ with increasing $k$ that the $k$ is higher, the mode is more stable. The $\beta$-dependent flute and ballooning modes growth rates behaviour is shown on Figure 4, where the thin curve corresponds to a flute disturbance (boundary conditions (9)) and the bold-typed curve corresponds to a ballooning one (incidentally $\alpha > 20/3$.

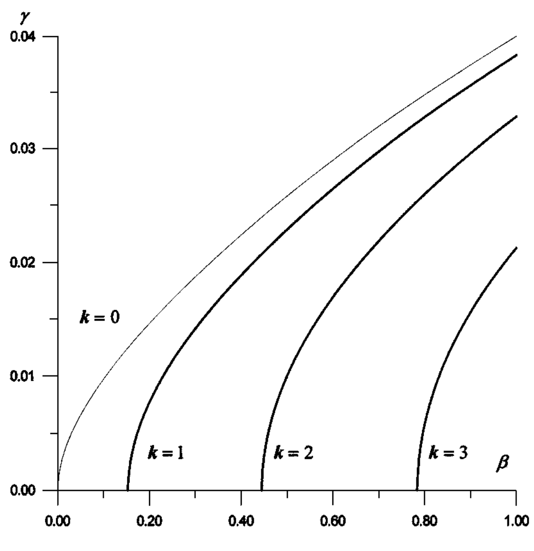



Fig. 4. $\beta$-dependent flute mode growth rate ($k$ = 0) and ballooning mode growth rates ($k$ = 1, 2, 3) (constant coefficients problem)

## THE ANALYSIS OF NEAR-EQUATORIAL DISTURBANCES SPECTRUM

Let us study the features of dispersion equation (11) solutions at $k \neq 0$. If we eliminate the term $16\beta^2\mu_k^2/(1+\beta)$ from the dispersion equation then the residuary equation is factorised and has the solutions

$$\Omega^2 = (\mu_k^2 + 18)\frac{\beta}{1+\beta} \tag{21}$$

and

$$\Omega^2 = \mu_k^2 + 16\frac{\beta}{1+\beta} - 4\frac{\alpha\beta}{\gamma}, \tag{22}$$

which are shown on Figure 5 by the dotted lines: the branch (21) corresponding to slow magnetic sound $\Omega^2_{MS}(\beta)$ monotonously increases and the branch (22) corresponding to poloidal shear Alfven mode

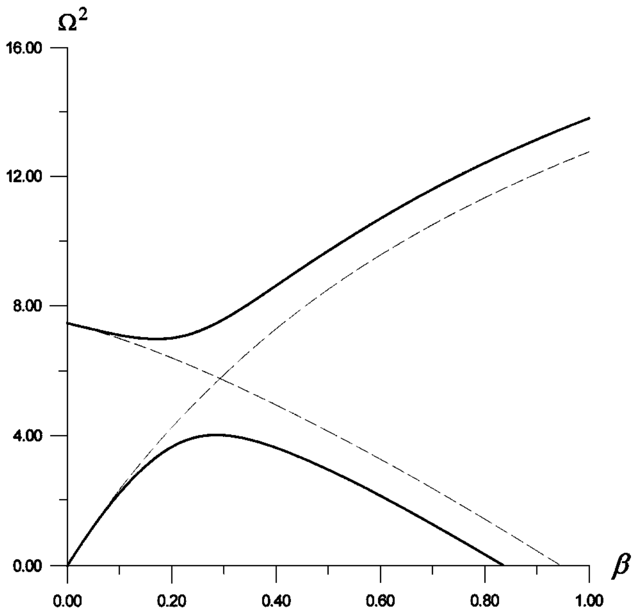



Fig. 5. $\beta$-dependent squared dimensionless eigenfrequencies $\Omega^2$ for the complete dispersion equation (bold-faced curves) and for the «truncated» dispersion equations (dotted curves) (constant coefficients problem)

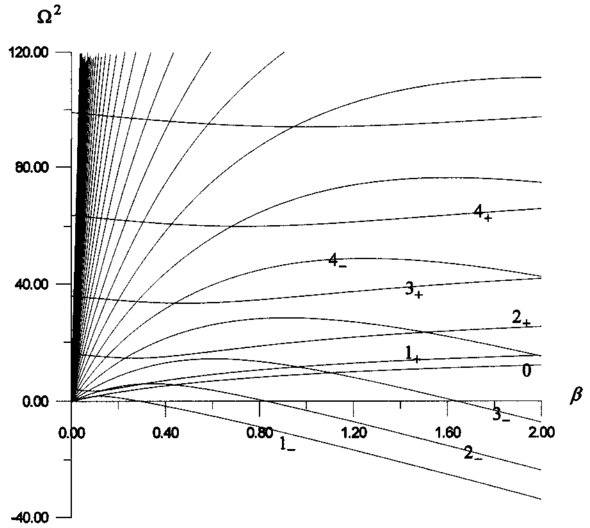
$\Omega^2_A(\beta)$ monotonously decreases. Both these branches intersect at $\beta_0$ which equals to

$$\beta_0 = \frac{1}{12\alpha}\sqrt{(6\alpha+5)^2 + 60\alpha\mu_k^2} - (6\alpha+5), \tag{23}$$

and

$$\Omega^2(\beta_0) = \Omega^2_A(\beta_0) = \Omega^2_{MS}(\beta_0) = (\mu_k^2 + 18)\frac{\beta_0}{1+\beta_0}. \tag{24}$$

The dispersion equation (11) derived above describes the effect of coupling slow nagnetosonic and poloidal shear Alfven waves. It is well-known [3] that the small disturbances of the dispersion equations eliminate the degeneration first of all exactly near the intercection of the branches of oscillations. As such disturbance the term $-16\beta^2\mu_k^2/(1+\beta)$ may be considered which leads to the gap between the upper and lower branches in the $k$-th mode oscillation spectrum (Figure 5, bold-typed curves). The width of this gap at $\beta = \beta_0$ equals to



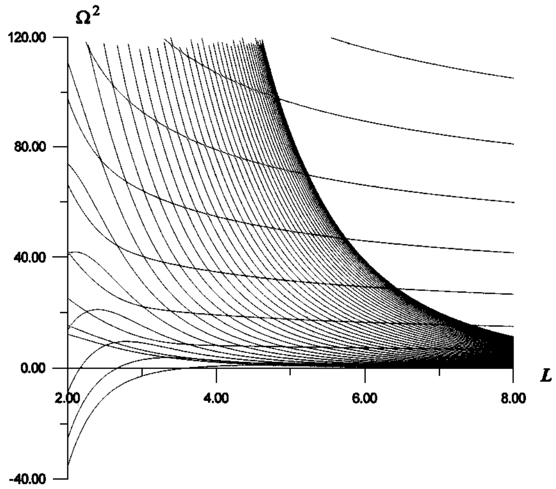




Fig. 6. $\beta$-dependent squared dimensionless eigenfrequencies $\Omega^2$ (constant coefficients problem)

Fig. 7. McIlwaine parameter $L$ dependent squared dimensionless eigenfrequencies $\Omega^2$ (constant coefficients problem)

$$\Delta\,\Omega^2(\beta_0) = 8\,\mu_k^2\frac{\beta_0}{1+\beta_0}.$$

If we write explicitly the dispersion equation (11) solutions $\Omega_+^2(\beta)$ and $\Omega_-^2(\beta)$ — the upper and lower bold-typed curves on Figure 5 accordingly, one may see for oneself that the solutions behave at $\beta = 0$ as follows:

$$\Omega_+^2(0) = \mu_k^2,\ \frac{d\Omega_+^2}{d\beta}(0) = 4(4 - \alpha/\gamma) \tag{25}$$

and

$$\Omega_-^2(0) = 0,\ \frac{d\Omega_-^2}{d\beta}(0) = \mu_k^2 + 18, \tag{26}$$

and at $\beta \to \infty$ as

$$\Omega_+^2(\beta) \to \mu_k^2 + 18 \tag{27}$$

and

$$\Omega_-^2(\beta) \to \mu_k^2 + 16 - 4\frac{\alpha}{\gamma}\beta. \tag{28}$$

Let us now look at the fragment of the spectrum of problem (4), (5), (7). It is shown on Figure 6. The numerical solution of the generalised eigenvalue problem which is the finite-difference approximation of the initial problem is presented here. The curves corresponding to the $k$-th mode are marked by $k_\pm$ so the sign «plus» means the upper branches $\Omega^2{}_+(\beta)$ and the sign «minus» means the lower ones $\Omega_-^2(\beta)$. The obtained spectrum may be easily interpreted as a set of multitude pairs (in general, infinite) of curves, one of them is shown on Figure 5.

If we are interested in eigenfrequency spatial dependency then using the fact that $\beta$ depends on McIlwain parameter $L$ employed in our model we can obtain the $L$-dependent squared dimensionless frequencies which are shown on Figure 7 and the same dependence of dimensional frequencies $f(L)$ which are shown on Figure 8.

We can see that on all three latter Figures 6, 7, and 8, where the dependencies $\Omega^2(\beta)$, $\Omega^2(L)$ and

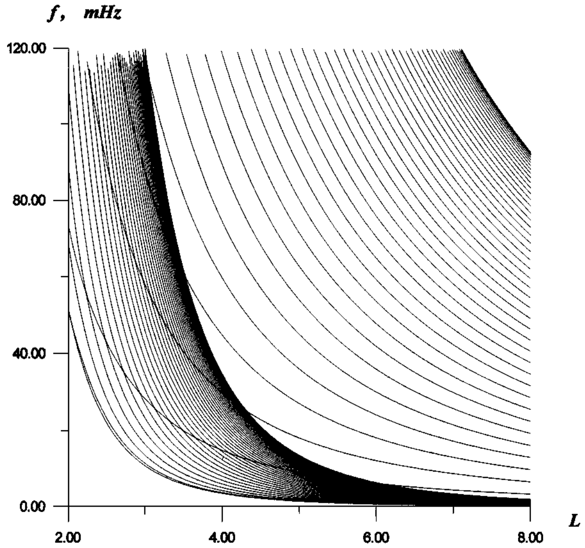
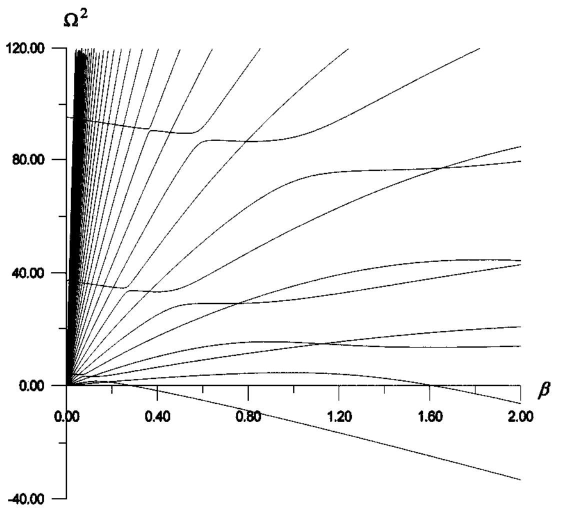



Fig. 8. McIlwaine parameter $L$ dependent dimensioned eigenfrequencies $f$ (constant coefficients problem)

$f(L)$ are shown, the spectrum consists of two families of branches. Let us consider the $\Omega^2(\beta)$ dependence. The ballooning component of spectrum consists of the family $\Omega^2_+(\beta)$, which has not any common point and of families $\Omega^2_-(\beta)$, which have the only one common point — limiting point — the origin of coordinates. At the same time curves pertaining to the different families have the manifold crossing-points (the same is true for the spectra on Figures 7 and 8).

## ORIGINAL PROBLEM SPECTRUM

Meantime, it would be useful to search the dispersion equation (or the system of differential equations (4)—(5)) disturbance which is able to remove the degeneration near the points of the branch crossings. We have a chance to jump out the non-common state case by perturbing the system (4)—(5) coefficients. Actually, the non-simplified system spectrum with $\theta$-dependent coefficients (1)—(2) has the form shown on Figures 9 and 10.



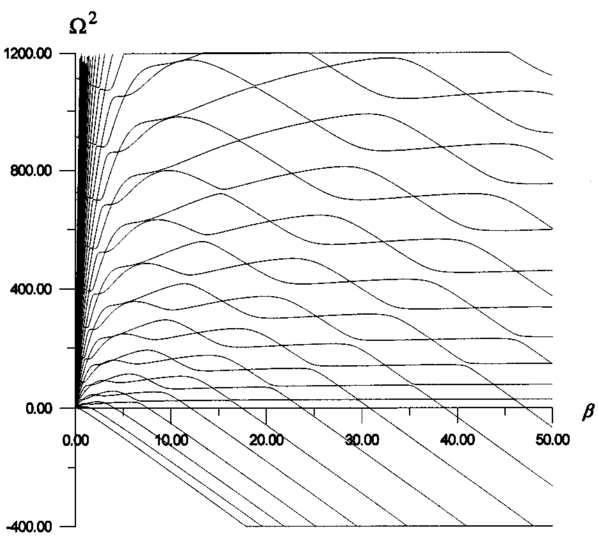




Fig. 9. $\beta$-dependent squared dimensionless eigenfrequencies $\Omega^2$ (variable coefficients problem)

Fig. 10. $\beta$-dependent squared dimensionless eigenfrequencies $\Omega^2$ (variable coefficients problem)

One can see that the part of branch crossings is splitted and another part is not splitted. The ballooning modes are subdivided into the branch pairs which interact between themselves but do not interact with the components of another pairs. One can see very well the deflected gaps — the ways from infinite $\beta$ to the origin of coordinates without crossings with any branch of the spectrum.

We will try to explain such a behaviour of the spectrum. Let us simplify original problem again but by the less thoroughgoing way then before. Let us present the system of equations (1)—(2) in the form:

$$\begin{aligned}\Omega^2 F(\theta)\,\hat{\tau} &= A(\theta)\,\hat{\tau}'' + B(\theta)\,\hat{\tau}' + C(\theta)\,\hat{\tau} + D(\theta)\,\hat{\xi}' + E(\theta)\,\hat{\xi},\\ \Omega^2 f(\theta)\,\hat{\xi} &= a(\theta)\,\hat{\xi}'' + b(\theta)\,\hat{\xi}' + c(\theta)\,\hat{\xi} + d(\theta)\,\hat{\tau}' + e(\theta)\,\hat{\xi}.\end{aligned} \tag{29}$$

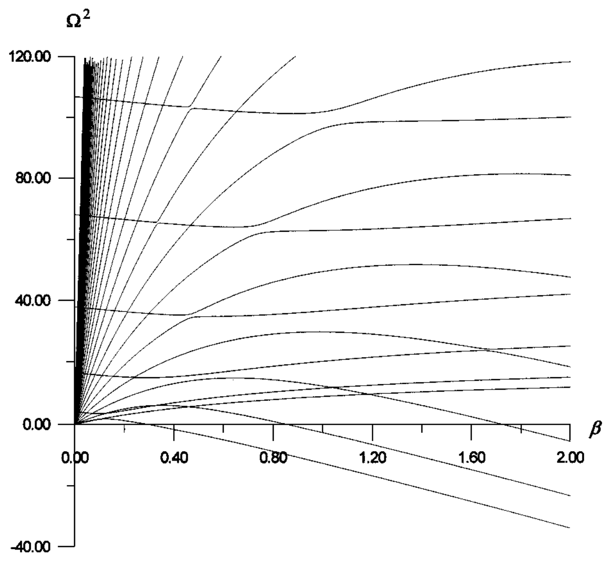
Let us replace all its coefficients noted as $K(\theta)$ by $K(\theta;\sigma)$ where

$$K(\theta;\sigma) = (1-\sigma)\,K(0) + \sigma\,K(\theta). \tag{30}$$

It is clear, that at $\sigma = 0$ we obtain the constant coefficient system (4)—(5) and at $\sigma = 1$ we obtain the original one (1)—(2). We have complete information concerning the system (4)—(5) solutions because they can be obtained analytically, the original system (1)—(2) solutions can be obtained only numerically.

Let us attempt to solve the boundary eigenvalue problem with very small but positive $\sigma$. On the one hand, the presence of the nonzero $\sigma$ which perturbs the constant coefficients is sufficient for the elimination of degeneration. On the other hand, such solutions and consequently the spectrum will slightly differ from the solutions and spectrum of problem with constant coefficients.

The spectrum of the problem with perturbed (lightly variable) coefficients ($\sigma = 0.01$) is shown on Figure 11. Let us enumerate the clearly visible branch splittings: $5_+$ and $9_-$ , $5_+$ and $7_-$ , $4_+$ and $8_-$ , $4_+$ and $6_-$ , $3_+$ and $5_-$ , $2_+$ and $4_-$ (twice). The assumption that splitting occurs at branch $m_+$ with branch $(m + 2l)_-$ , $l = 1, 2, \ldots$ crossing points seems to be verisimilar hypothesis. But why do the modes engage one another with step equal to two and why don't the adjoining modes interact? Let us decompose all the coefficients of system (29) into the Fourier series. If we go through this we will reveal that all the even coefficients are decomposable by cosines either by even powers or by odd powers and the odd



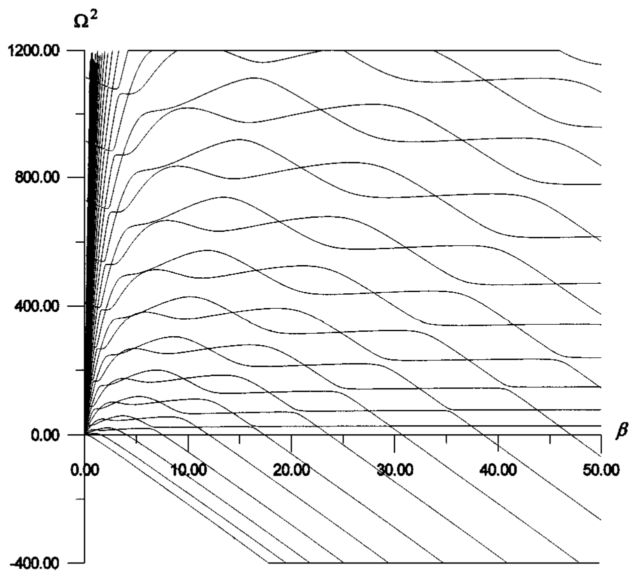




Fig. 11. $\beta$-dependent squared dimensionless eigenfrequencies $\Omega^2$ (lightly variable coefficients problem, $\sigma = 0.01$)

Fig. 12. $\beta$-dependent squared dimensionless eigenfrequencies $\Omega^2$ (variable coefficients Dirichlet problem (10))

coefficients are decomposable by sines of the same powers. This fact turns out to be forcible argument for the efforts to confirm the rule for the splitting and to estimate $\sigma$-dependent splitting parameter. Since the centres of global gaps pass through the crossing points of the splitted branches one can attempt to obtain them, at the least numerically. The successful realization of this scenario offers a valid explanation to the reason of (see Figure 10) why the initial problem spectrum has resolved into the set of disjoint «queues».

It might help in the interpretation of the original problem spectrum with Dirichlet-type boundary conditions (10)

$$\hat{\tau}(\pm\theta_0) = 0,$$
$$\hat{\xi}(\pm\theta_0) = 0,$$

which seem to be very natural but do not permit to simplify the original system. Figure 12 shows that the Dirichlet boundary conditions problem spectrum is similar enough to the boundary conditions (7) problem spectrum.

## CONCLUSIONS

The system of equations which describes the MHD waves development in the dipole magnetic field was obtained there.

The system obtained was used for the description of disturbances in the near-equatorial region of plasmasphere.

It was noted that such disturbances first of all must occur at the radiation belts.

It was shown that advanced plasma pressure leads to the essential modification of spectrum:

— maghetosonic oscillation branch appears;

— unstable flute and ballooning modes appear.

It was shown that flute instability growth rate preponderate over the ballooning instability ones.

The flute mode and ballooning modes stability criterion was derived for the case of pressure which is not small.

There was obtained the $\beta$-dependent and $L$-dependent spectra of MHD modes suitable for the whole magnetic force line not only in the near-equatorial region.

It was shown that oscillation spectrum contains the gaps between magnetosonic modes and poloidal shear Alfven ones.

*A* lot of local gaps resulting from elimination of degeneration at non-constant coefficients of equations and the global gaps in the $\beta$-space were described.

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