

GENERATION OF THE KINETIC ALFVEN WAVE AND LOWER HYBRID WAVE IN SPACE PLASMA

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Satellite observations show close relationship between the whistlers and lower-hybrid waves in space plasmas. Intense whistler waves generated by lightning discharges [Kelley, 1990] or by quasimonochromatic VLF (very low frequency) transmitters can be unstable and cause the three-wave parametric interaction. Work [Bell, 1994] demonstrates experimental results of excited lower hybrid waves by VLF whistler mode waves in the topside ionosphere and near magnetosphere. The Kinetic Alfvén waves are often observed by satellites [Louran], [Volokitin, 1989]. In this paper we analytically consider a possible mechanism of this Relationship, i.e., parametric interaction of whistler pump waves with lower-hybrid and the Alfvén waves in magnetized plasma with small plasma parameter. In the dynamics of the Alfvén waves the kinetic effects (finite ion Larmor radius and electron inertia) are taken into account. A nonlinear dispersion equation describing three-wave interaction is obtained in the framework of two-fluid magnetohydrodynamics. The instability growth rates and the time of instability development are found. Our theoretical investigation shows, that the whistler mode will be an effective source of the lower hybrid and the Alfvén waves in the magnetospheric plasma. This nonlinear process can take place in the Earth magnetosphere and in the Sun atmosphere. The products of the decay, i.e., the lower-hybrid and the kinetic Alfvén waves, can effectively interact with magnetospheric and sun plasmas.

INTRODUCTION

The interest to the three-wave resonant (TWR) parametric decay closely is related to the study of the origin of the waves observed in the laboratory and space plasmas. Also, different wave modes damp with different rates, and a mutual transformation of the wave modes caused by TWR can significantly change the rate of plasma heating, often providing an explanation for enhanced (or reduced) plasma heating. Therefore, the parametric interactions is of great interest ([Berger, 1976], [Shukla, 1978], [Murtaza, 1984], [Stenflo, 1990], [Guha, 1991], [Leyser, 1991], [Yukhimuk, 1992, 1998]). In introduction we shall be interested in processes, which can be generalized by the same physical phenomenon, i.e., the parametric interaction of whistler mode waves with ionospheric and magnetospheric plasmas. [Sharma, 1992] analyzed the effect of parametric excitation of the electrostatic whistler waves by electron plasma waves. The paper [Chian, 1994] presented a new excitation mechanism of the auroral Langmuir and the Alfvén waves in the planetary magnetosphere. It was shown that a large-amplitude electromagnetic whistler wave propagating along the magnetic field lines can nonlinearly generate the Langmuir and the Alfvén waves by three wave parametric instability. In the paper [Tara, 1987] about generation of the oblique Alfvén waves by the parametric instability of the whistlers in the near Earth plasma decay process with participation of two whistler waves spreading under angles to an external magnetic field and the oblique Alfvén wave is considered. It is shown that parametric instability whistlers should reduce to the rapidly growing oblique Alfvén waves with perpendicular wave length. The work of [Grach, 1975] is also very interesting. In his paper a parametric instability of quasi-monochromatic VLF waves in the upper ionosphere is considered. It is shown that the initial signal scatters into a low-frequency plasma wave and decays into low frequency and ion-cyclotron waves most effectively. A parametric interaction between

two whistlers and ion-cyclotron wave is considered. The above process also may be responsible for excitation of the low-hybrid resonance in the ionosphere and magnetosphere of the Earth. The processes of whistlers scattering at big angles to an external magnetic field were analyzed in the paper [Tara, 1989]. In the paper [Shukla, 1978] the decay of UHW (upper hybrid waves) into the whistler and KAW was studied. In the paper [Murtaza, 1984] the decay of LHW into the UHW and electromagnetic wave was studied. Later [Leyser, 1991] used theoretical results of Murtaza and Shukla for interpretation of the nature of downshifted maximum, and Stenflo for interpretation of stimulated electromagnetic emission in the ionosphere. In the paper [Yukhimuk, 1998], the nonlinear interaction of kinetic Alfvén wave with upper hybrid wave has been investigated, and the results have been used for the interpretation of long-period geomagnetic pulsations.

Whistlers is one of the most widespread type of waves in a magnetized plasma. The generation mechanisms are related to flashes, particle beams, anisotropic velocity distributions of particles, and nonlinear processes in space plasmas. In the Earth magnetosphere, whistlers can propagate from one hemisphere to another, and under favorable conditions can be detected by the ground instruments and satellites in «magnetoconjugated» points. The low-hybrid waves have been often observed simultaneously with whistlers ([Barrington, 1963], [Brice, 1964], [Gurnet, 1966], [Scarf, 1972], [Bell, 1991, 1994]). Satellite observations have shown a close relationship between whistlers and LHWs ([Titova, 1984], [Bell, 1988]), and it was supposed that the whistlers are the source of the lower hybrid wave.

However, despite of the mentioned above observational data, an exact mechanism of such a close relation between lower hybrid emissions and whistlers is still uncertain. In the present paper the nonlinear interaction of the whistlers and the lower hybrid waves has been investigated. The lower hybrid wave is excited as a result of parametric decay of whistlers into KAWs and LHW. A high efficiency of the interaction among KAWs and whistlers is caused by the presence of the longitudinal component of KAW electric field.

BASIC EQUATIONS

Parametric instability, where the whistlers wave decays into the LHW and the kinetic Alfvén wave

$$W = LHW + KAW$$

is considered.

The whistler pump wave

$$\mathbf{E}_0 = (E_{0x}\mathbf{e}_x + E_{0y}\mathbf{e}_y)\exp i\psi_0 + c.c. \quad (1)$$

where $\psi_0 = -\omega_0 t + k_{0x}x + k_{0z}z$, $\omega_0 = (k_0^2 c^2 / \omega_{pe}^2) |\cos\theta \omega_{Be}|$ propagates in the homogeneous magnetized plasma ($\mathbf{B}_0 = B_0 \mathbf{e}_z$). It is assumed that the wave synchronism condition is satisfied

$$\omega_0 = \omega + \omega_1, \mathbf{k}_0 = \mathbf{k} + \mathbf{k}_1, \quad (2)$$

where ω_0, \mathbf{k}_0 are frequency and wave vector of the whistler pump wave, ω, \mathbf{k} are frequency and wave vector of the kinetic Alfvén wave, ω_1, \mathbf{k}_1 are frequency and wave vector of the lower hybrid wave. Let us consider that all wave vectors are in Oxz plane (Fig. 1).

To study three-waves parametric interaction we use two-fluid magnetohydrodynamics (MHD):

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} = \frac{1}{m_\alpha} (e_\alpha \mathbf{E} + \mathbf{F}_\alpha) + (\mathbf{v}_\alpha \times \boldsymbol{\omega}_{B\alpha}) - \frac{T_\alpha}{m_\alpha n_\alpha} \nabla n_\alpha, \quad (3)$$

$$\frac{\partial n_\alpha}{\partial t} = -\nabla \cdot (n_\alpha \mathbf{v}_\alpha), \quad (4)$$

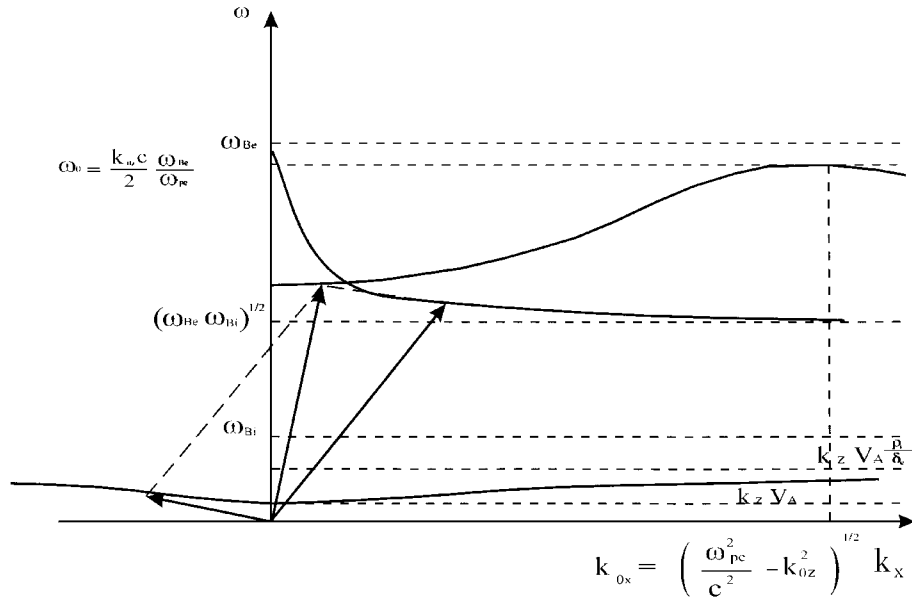


Fig. 1.

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (5)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (6)$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad (7)$$

where

$$\mathbf{j} = e(n_i \mathbf{v}_i - n_e \mathbf{v}_e),$$

$$\rho = e(n_i - n_e),$$

$$\mathbf{F}_\alpha = \frac{e_\alpha}{c} (\mathbf{v}_\alpha \times \mathbf{B}) - m_\alpha (\mathbf{v}_\alpha \nabla) \mathbf{v}_\alpha.$$

Index $\alpha = i, e$ which correspond to the ion and electron components of plasma respectively. Electron density and velocity, electric and magnetic field are presented in the form:

$$\begin{aligned} n_e &= n_0 + \tilde{n}_A + \tilde{n}_1, & \mathbf{v}_e &= \mathbf{v}_0 + \mathbf{v}_A + \mathbf{v}_1, \\ \mathbf{E} &= \mathbf{E}_0 + \mathbf{E}_A + \mathbf{E}_1, & \mathbf{B} &= B_0 \mathbf{e}_z + \mathbf{B}_A, \end{aligned} \quad (8)$$

where n_0 is the mean value of plasma density, index 0 in the expressions for \mathbf{v}_e and \mathbf{E} denotes the variable related to the pump wave and indexes A and 1 denote variables related to KAW and UHW respectively.

DISPERSION EQUATION OF THE ALFVEN WAVES

To obtain dispersion equation we use plasma approximation

$$\tilde{n}_e = \tilde{n}_1, \quad (9)$$

since the Alfvén waves are slow. Here \tilde{n}_e and \tilde{n}_i are the perturbations of electron and ion density respectively. From the motion equation (3) and continuity equation (4) we find the expression for \tilde{n}_e , \tilde{n}_i :

$$\frac{\tilde{n}_e}{n_0} = \left(1 - \frac{v_{\text{ph}}^2}{v_{\text{Te}}^2}\right)^{-1} eT_e \left[\varphi - A + \frac{k_x}{k_z^2} \frac{\omega}{\omega_{\text{Be}}} \frac{1}{e} \left(i \frac{\omega}{\omega_{\text{Be}}} F_x + F_y \right) + \frac{F_z}{iek_z} \right], \quad (10)$$

$$\frac{\tilde{n}_i}{n_0} = -\frac{e}{T_e} \frac{\mu_i}{1 + \mu_i} \left(\varphi + \frac{\omega_{\text{Bi}}^2 k_z^2}{\omega^2 k_x^2} A \right)$$

where $v_{\text{ph}} = \omega/k_z$, $A = \frac{\omega}{k_z c} A_z$, $\mu = k_y^2 \rho^2$, $\rho_i = v_{\text{Ti}}/\omega_{\text{Bi}}$ is the ion Larmor radius, φ and A_z are scalar and vector potentials of KAW electric field respectively. The first relation between φ and A follows from (10) = (11):

$$A = \left[1 + \frac{T_e}{T_i} \frac{\mu_i}{1 + \mu_i} \left(1 - \frac{V_{\text{ph}}^2}{V_{\text{Te}}^2} \right) \right] \varphi - \frac{k_x^2}{k_z^2} \frac{\omega^2}{\omega_{\text{Be}}^2} \frac{1}{iek_x} \bar{F} + \frac{1}{iek_z} F_z - \frac{T_e}{e} \frac{V_{\text{ph}}^2}{V_{\text{Te}}^2} \frac{\mathbf{k}}{\omega} \left(\frac{n}{n_0} \mathbf{V} \right)_e \quad (12)$$

where

$$\bar{F} = F_x - i \frac{\omega_{\text{Be}}}{\omega} F_y.$$

We can find the second relation between φ and A from the perpendicular projection of Ampere law:

$$-k^2 k_z A_z = \frac{4\pi}{c} k_x j_x. \quad (13)$$

The linear part of the transverse current j_x is determined by the plasma ion component, but the nonlinear perpendicular current is generated by the beating of LHW and whistler in electron motions:

$$j_x = en_0 V_{\text{ix}}^{\text{L}} + j_{\text{ex}}^{\text{NL}}. \quad (14)$$

Inserting the expression for ion velocity and nonlinear electron current we get the second relation between KAW potentials:

$$A = \frac{V_{\text{ph}}^2}{V_{\text{A}}^2} \frac{1}{1 + \mu_i} \varphi + (k_z^2 \delta_i^2)^{-1} \frac{m_i}{e} \frac{\omega}{k_x} \frac{n_e^{\text{L}}}{n_0} V_{\text{ex}}^{\text{L}} + (k_z^2 \delta_i^2)^{-1} \frac{m_i}{e} \frac{\omega^2}{\omega_{\text{Be}}^2} \times$$

$$\times \left[\left(\frac{V_{\text{ph}}^2}{V_{\text{Te}}^2} - 1 \right)^{-1} \frac{1}{im_e k_z} F_z + \left(\frac{V_{\text{ph}}^2}{V_{\text{Te}}^2} - 1 \right)^{-1} \frac{\omega}{k_z^2} \mathbf{k} \left(\frac{n}{n_0} \mathbf{V} \right) + \frac{1}{im_e k_x} \bar{F} \right]. \quad (15)$$

where $\delta_i^2 = c^2/\omega_{\text{pi}}^2$. Equating (12) = (15) we obtain the nonlinear dispersion equation for KAW:

$$\left[\frac{V_{\text{ph}}^2}{V_{\text{A}}^2} (1 + \chi_e) - (1 + \bar{\mu}_i) \right] \frac{1}{1 + \mu_i} \varphi = \frac{1}{iek_z} F_{ez} - \frac{m_e}{m_i} \frac{V_{\text{ph}}^2}{V_{\text{A}}^2} (1 + \chi_e) \frac{1}{iek_x} \left(F_{ex} - i \frac{\omega_{\text{Be}}}{\omega} F_{ey} \right) -$$

$$- \frac{m_i}{e} (k_z^2 \delta_i^2)^{-1} \frac{\omega}{k_x} (1 + \chi_e) \frac{n_e^{\text{L}}}{n_0} V_{\text{ex}}^{\text{L}}, \quad (16)$$

where $\chi_e = k^2 \delta_e^2$, $\bar{\mu}_i = (1 + T_e/T_i) \mu_i$. From the motion equation we find electron velocity components in the field of whistler wave:

$$v_{0x} = -i \frac{eE_{\text{ox}}}{m_e(\omega_0 - \omega_{\text{Be}})}, \quad v_{0y} = \frac{eE_{\text{ox}}}{m_e(\omega_0 - \omega_{\text{Be}})}, \quad (17)$$

and components of the pump wave magnetic field from (6):

$$\begin{aligned}
 b_{0x} &= -i \frac{ck_{0z}}{\omega_0} E_{0x}, \\
 b_{0y} &= \frac{ck_{0z}}{\omega_0} E_{0x}, \\
 b_{0z} &= i \frac{ck_{0x}}{\omega_0} E_{0x}.
 \end{aligned} \tag{18}$$

Velocity components in the LHW field can be obtained from electron motion equation:

$$\begin{aligned}
 v_{1x} &= -\frac{ek_{1x}\omega_1}{m_e(\omega_1^2 - \omega_{Be}^2)} \varphi_1, \\
 v_{1y} &= -i \frac{ek_{1x}\omega_{Be}}{m_e(\omega_1^2 - \omega_{Be}^2)} \varphi_1, \\
 v_{1z} &= -\frac{ek_{1z}}{m_e\omega_1} \varphi_1.
 \end{aligned} \tag{19}$$

Using expressions (17)–(19) and from (16), we find dispersion equation for the Alfvén wave:

$$\varepsilon_A \varphi = \mu_A (E_{0x} \varphi_1^*), \tag{20}$$

where μ_A is the coupling coefficient, that defined by the expression:

$$\begin{aligned}
 \mu_A &= i \frac{e}{m_e} \frac{k_{1x}}{\omega_{Be}} \frac{k_{0z}}{k_z} \frac{k_z^2 V_A^2}{\omega_0} (1 + \mu_i) \left[1 + \frac{m_e}{m_i} \frac{k_z}{k_x} \frac{\omega_{Be}^2}{\omega_1 \omega} \frac{k_{1z}}{k_{1x}} (1 + \bar{\mu}_i) \right], \\
 \varepsilon_A &= \omega^2 - k_z^2 V_A^2 (1 + \bar{\mu}_i).
 \end{aligned}$$

In the calculation of the coupling coefficient μ_A , we have taken account of the ponderomotive force created by scattering whistlers pump wave on LHW. Since we consider magnetosphere, where the plasma parameter $m_e/m_i \ll \beta \ll 1$, in the dispersion equation for KAW we keep terms with finite-Larmor-radius effects.

DISPERSION EQUATION OF LHW

Dispersion equation of LHW can be found from Poisson equation:

$$\Delta \varphi_1 = -4\pi e (\tilde{n}_i - \tilde{n}_e). \tag{21}$$

Expression for \tilde{n}_i , \tilde{n}_e we find from the motion equation (3) and continuity equation (4):

$$\tilde{n}_i = \frac{en_0}{m_i} \left(\frac{k_{1x}^2}{\omega_1^2 - \omega_{Bi}^2} + \frac{k_{1z}^2}{\omega_1^2} \right) \varphi_1, \tag{22}$$

$$\begin{aligned}
 \tilde{n}_e &= -\frac{en_0}{m_e} \left(\frac{k_{1x}^2}{\omega_1^2 - \omega_{Be}^2} + \frac{k_{1z}^2}{\omega_1^2} \right) \varphi_1 - \frac{n_0}{m_e} \frac{k_{1x}\omega_{Be}}{\omega_1(\omega_1^2 - \omega_{Be}^2)} \times \\
 &\times \left[i \frac{\omega_1}{\omega_{Be}} F_{1x} + F_{1y} + i \frac{k_{1z}}{k_{1x}} \frac{(\omega_1^2 - \omega_{Be}^2)}{\omega_1 \omega_{Be}} F_{1z} \right] - \frac{\nabla \langle \tilde{n}_A \mathbf{V}_0 \rangle}{-i\omega},
 \end{aligned} \tag{23}$$

where components of ponderomotive force are defined by interaction of pump wave and Alfvén wave. From (21)–(23) we find the dispersion equation for LHW:

$$\varepsilon_1 \varphi_1 = \mu_1 (E_{0x} \varphi^*), \quad (24)$$

where coupling coefficient μ_1 is defined by the expression:

$$\mu_1 = i \frac{e}{m_e} \frac{\mu_s}{1 + \mu_1} \frac{\omega}{k_z} \frac{1}{V_{Te}^2} \frac{k_{0z}}{\omega_0} \frac{\omega_1 \omega_{Be}}{k_{1x}} \left[1 + \frac{k_{1z}}{k_{1x}} \frac{k_z}{k_{0z}} \frac{k_x}{\omega_1} \frac{\omega_0}{\omega^2} \frac{\omega_{Bi}^2}{V_s^2} (1 + \tilde{\mu}_i) + \frac{\omega_0}{\omega} \frac{k_z}{k_{0z}} \right],$$

$$\varepsilon_1 = \omega_1^2 - \frac{\omega_{pi}^2}{1 + \frac{\omega_{pe}^2}{\omega_{Be}^2}} \left(1 + \frac{m_i}{m_e} \frac{k_{1z}^2}{k_1^2} \right).$$

NONLINEAR DISPERSION EQUATION FOR THREE-WAVE INTERACTION

From equations (20) and (24) we find the nonlinear dispersion equation describing three-wave interaction (decay of whistler pump wave into the KAW and LHW):

$$\varepsilon_A \varepsilon_1^* = \mu_A \mu_1^* |\varphi_0|^2.$$

Assuming in (25)

$$\omega = \omega_r + i\gamma_1, \quad \omega_1 = \omega_{1r} + i\gamma_1,$$

(where $|\gamma_1| \ll \omega_r, \omega_{1r}$) and decomposing ε_A and ε_1 to the Taylor series with respect to the small parameter $i\gamma_1$ we obtain the expression for the instability growth rate

$$\gamma_1 = \frac{\mu_A \mu_1^* |E_{0x}|^2}{\frac{\partial \varepsilon_A}{\partial \omega}} \frac{\partial \varepsilon_1}{\partial \omega_1} \Big|_{\omega = \omega_r, \omega_1 = \omega_{1r}}, \quad (26)$$

where ω_r and ω_{1r} could be found from the equations

$$\varepsilon_A(\omega_r, \mathbf{k}) = 0, \quad \varepsilon_1(\omega_1, \mathbf{k}_1) = 0.$$

Substituting values of derivatives

$$\frac{\partial \varepsilon_A}{\partial \omega} = 2\omega, \quad \frac{\partial \varepsilon_1}{\partial \omega_1} = 2\omega_1,$$

and coefficients μ_A and μ_1 into (26) we obtain

$$\gamma^2 = \frac{W}{4} \frac{\omega_{pe}^2}{\omega_0^2} \mu_s V_A^2 k_{0z}^2 \left[1 + \frac{m_e}{m_i} \frac{k_z}{k_x} \frac{\omega_{Be}^2}{\omega_1 \omega} \frac{k_{1z}}{k_{1x}} (1 + \tilde{\mu}_i) \right] \left[1 + \frac{k_{1z}}{k_{1x}} \frac{k_z}{k_{0z}} \frac{k_x}{\omega_1} \frac{\omega_0}{\omega^2} \frac{\omega_{Bi}^2}{V_s^2} (1 + \tilde{\mu}_i) + \frac{\omega_0}{\omega} \frac{k_z}{k_{0z}} \right],$$

where $W = \frac{|E_{0x}|^2}{4\pi n_0 T_e}$.

DISCUSSION AND APPLICATION

In this paper we analytically consider nonlinear parametric interaction of whistlers with LHW and the Alfvénic wave in a low- β plasma. The kinetic effects (finite ion Larmor radius and electron inertia) are taken into account for dispersion law, when we investigate the dynamics of the Alfvén wave. Such type of the Alfvén wave is usually called the kinetic Alfvén waves. They have some properties different from the MHD Alfvénic waves. They include the presence of nonzero parallel electric field E_z and can propagate in xz -plane. Due to this properties KAW interacts effectively with the whistler and amplifies the lower hybrid waves. The considered nonlinear three-wave parametric interaction can be observed in

laboratory and space plasmas with low- β plasmas. In space plasmas such processes can take place in the ionosphere and magnetosphere of the Earth and in the some regions of Sun atmosphere where the plasma parameter is small.

In our paper we consider magnetospheric plasma in the capacity of supplement (or application). Typical parameters for magnetospheric plasma at $(3...4)R_E$ are: $V_{Te} \approx 10^7 \text{ s}^{-1}$, $\omega_{pe} \approx 10^5 \text{ s}^{-1}$, $\omega_{Be} \approx 10^5 \text{ s}^{-1}$, $\omega_{Bi} \approx 10^2 \text{ s}^{-1}$. We substitute plasma parameters and whistlers intensity $W \approx 10^{-5}$ into (27). The time of instability development is $\tau \approx \gamma_{\max}^{-1} = 0.01 \text{ s}$.

Our theoretical investigation shows, that whistler wave while propagating in the magnetosphere will be a source of the lower hybrid and the Alfvén waves. Theoretical results can explain satellite observations of the coexistence of the whistler, lower hybrid and Alfvén waves.

Nonlinear parametric processes considered in this paper can take place in the Solar corona as well, where whistlers are generated by energetic electrons in magnetic traps on the Sun. Whistlers will excite the lower hybrid and Alfvén waves while propagating through the Solar atmosphere. The lower hybrid wave can effectively accelerate electrons to high energy. The kinetic Alfvén waves interact effectively with plasma particles and participate in the heating and acceleration of space plasma particles owing to presence of the longitudinal component of electric field. Thus, i. e., the products of the decay — the lower hybrid and kinetic Alfvén waves, can affect the magnetospheric and Sun plasmas stronger than the initially excited whistler waves.

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