

PITCH-ANGULAR DIFFUSION OF HIGH-ENERGY PARTICLES IN PLASMA OF THE MAGNETOSPHERE

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In this work some problems of dynamics of magnetospheric charged particles of high energies (1—1000 MeV) are considered. The coefficients of a pitch-angular and radial diffusion of protons and electrons in a dipole magnetic field are defined. The calculations were grounded on the basic kinetic equation with use of a method by G. M. Zaslavsky [1]. Our calculations were based on the results of the modern theory of nonlinear oscillations in dynamic systems [1, 2]. The aim of the work was to define a role of the mechanism of the breaking down of the first adiabatic invariant in shaping pitch-angular distribution of particles in the magnetosphere. It is shown, that the considered mechanism of scattering reduces in a strong pitch-angular diffusion of protons and heavier ions, while the beams of polar electrons are very stable, can be long-lived in time and oscillate between points of reflection, which are in the polar zones. On the basis of the obtained results it is possible to explain and to interpret origin and stability of auroras, and also radio-frequency radiation from magnetospheres of planets as secondary effect of a stability of polar electron beams.

1. INTRODUCTION

In work [3] on the basis of exact equations of motion two sharply distinguished one from other regimes of motion of particles, entrapped by magnetic field of dipole were explored. The first, basic regime is a condition of gyrorotation and corresponds to the well known Alfvén approximation, based on the drift theory of guiding centre motion of a particle. The second regime of motion arises in an equatorial zone for particles with small pitch-angles in a magnetic field of a dipole, and closely corresponds to the central Stormer's trajectory. The trapped particles have only a gyrorotational trajectory, or have elements as Stormer's central trajectory in an equatorial zone, and elements of a trajectory with fast gyrorotation in polar zones. In work [3] the conditions of transition between these modes of motion are defined. Such transitions are characteristic for motion of particles in nonuniform magnetic fields with distinct from zero curvature of force lines. For plane-parallel magnetic fields this appearance is missed.

The transition from a mode of gyrorotation to a mode of a central trajectory actually corresponds to break down of the first adiabatic invariant, and subsequent inverse transition, i.e., its restitution. Such phase conversions on a trajectory of a particle correspond to saddle points on phase portraits of a nonlinear oscillator [2]. The saddle point arises from a point of a type centre at a change of motion parameter, which is in our case a pitch-angle of a particle.

In work [3] it was marked, that the transitions between regimes of motion occur for values of gyrophase equal $\pi/2$ and $3\pi/2$. This deduction is well correlated with outcomes of work [4], where it was shown, that the bifurcations in Hamiltonian systems take place in saddle points at the mentioned above values of phases.

The existence of regions of parameters and phase space with the destroyed integrals of motion, in particular, of the first adiabatic invariant, is a typical physical situation, which is accompanied by occurrence of random dynamics. For one bounces-period a particle with high-latitude points of reflection and the high energy four times transits near saddle bifurcation points on the trajectory. The trajectories of such particles in an equatorial zone between these points go close to a separatrix, where the parameters of trajectories also gain stochastic properties [1,2].

In this work we do not consider in detail modification of a trajectory of a particle at transitions through bifurcation points. We shall be restricted, basing on the strongest assumption about a possibility

of averaging casual phases for an evaluation of diffusion coefficients as in the case of good phase intermixing. The intermixing on phases occurs much faster than slow evolution on a variable action [1]. Basing on this, we suppose, that the operation of averaging phases can be carried out irrespective of evolution of action not only for a regime of fast gyrorotation, but also for a unstable regime of a slow gyrorotation at transiting bifurcation points and stochastic layers in an equatorial zone by a particle.

2. BASIC KINETIC EQUATION

Motion of a charged particle, entrapped by magnetic field of a magnetosphere is in many respects similar to the behaviour of a nonlinear oscillator. The basic kinetic equation for a distribution function $f(I, t)$ of nonlinear oscillator is as [1]

$$\frac{\partial f(I, t)}{\partial t} = \frac{1}{2} \frac{\partial}{\partial I} D_{II} \frac{\partial f(I, t)}{\partial I}, \quad (1)$$

where D_{II} is coefficient of a diffusion on a variable action I , $D_{II} = \langle (\Delta I)^2 \rangle / T$, where ΔI is change of action on interval of time T , the angular brackets mean averaging for this interval, which by virtue of our supposition about ergodicity of process is equivalent to average on phases. According to this the change of the action ΔI should be determined for the same interval of time. We find the magnitude dI/dt from the Hamilton equation

$$\frac{dI}{dt} = - \frac{\partial H_1(I, \varphi)}{\partial \varphi}, \quad H_1(I, \varphi) = mv^2 \sqrt{1 - \mu^2} u_d (\cos \varphi + \sin \varphi)$$

For calculations we use expression for a precise Hamiltonian $H(I, \varphi)$, obtained in work [5]. Let's obtain expression for derivative dI/dt within the first order of accuracy

$$dI/dt = E \sqrt{2(1 - \mu^2)} u_d (\sin \varphi - \cos \varphi), \quad (2)$$

Where $E = \text{const}$ is the complete kinetic energy of a particle, L is a parameter of the Mc-Ilvain drift, μ is cosine of an pitch-angle of a particle, u_d is normalized relative to a velocity v magnitude of a cross velocity of a drift of a particle [3, 6]

$$u_d = \frac{q}{|q|} \frac{vL^2}{a_0 \Omega_0} \frac{1 + \langle \mu^2 E \rangle}{2} \frac{(1 + \cos^2 \theta) \sin^5 \theta}{(1 + 3 \cos^2 \theta)^2}, \quad (3)$$

q is charge of a particle, L is the parameter of the Mc-Ilvain drift envelope, θ is polar angle in a spherical frame, bound with a magnetic dipole, a_0 is the radius of a planet, Ω_0 is gyrofrequency on equator of a planet.

Using (2), (3), obtain expression for D_{II} as

$$D_{II} = \left\langle \left(\frac{dI}{dt} \right)^2 \right\rangle T = 2TE^2 (1 - \langle \mu^2 \rangle - u_d^2) u_d^2. \quad (4)$$

In work [3] we have shown, that for a polar group of particles with small pitch-angles in an equatorial region, i.e. for a quasiconstant phase mode the relation $1 - \langle \mu^2 \rangle = 2u_d^2$ is valid. Due to a strong dependence of D_{II} on a polar angle θ an essential value of a diffusion coefficient on a variable action is maintained only near an equatorial plane. As an interval for average T for the Stormer mode with a slow phase we select bouns-period $T_b = 2\pi/\omega_b$ for particles with zero equatorial pitch-angle. Here ω_b is bouns-frequency, for which we take the expression obtained in [6] for these particles: $\omega_b = v \cos^4 \lambda_* / (r_e |\sin \lambda_*|)$, where $\lambda_* \approx 45^\circ$ is the effective geomagnetic latitude of reflection for such particles, r_e is distance from center of the dipole to a cross point of an equatorial plane along a force line, on which the particle is located.

3. PITCH-ANGULAR DIFFUSION OF PARTICLES IN A MAGNETOSPHERE

The diffusion reduces on a variable action to pitch-angle and radial diffusions of particles. However, as tentative estimation of coefficients of a radial diffusion shows, the path length of an entrapped particle on a drift envelope and before complete scattering is about 10^{18} cm, and reference time for scattering is more than one year. So, the radial diffusion at the expense of violation of the first adiabatic invariant is incidental. It is known [7], that for process of a radial diffusion the most essential mechanism is the betatron, which effectively works at sudden impulses of a field during geomagnetic disturbances.

Thus, further we consider only process of a pitch-angular diffusion occurring at the expense of violations of the first adiabatic invariant. Passing from a variable of action to a variable $\xi = \mu^2$, i.e., quadrate of a cosine pitch-angle of a particle, we shall obtain the following equation instead of (1)

$$\frac{\partial f(\xi, t)}{\partial t} = \frac{\partial}{\partial \xi} v_{\xi\xi} \frac{\partial f(\xi, t)}{\partial \xi}, \quad (5)$$

where $v_{\xi\xi}$ is the effective collision frequency. For a polar group of particles with small equatorial pitch-angles (mode of a slow phase) we have obtained expression

$$v_{\xi\xi\xi} = 4\pi\sqrt{2}v^3L^3/(a_0^3\Omega_0^2). \quad (6)$$

4. CONCLUSION

The estimate of an effective collision frequency $v_{\xi\xi}$ and reference time of scattering τ^S for a polar group of particles having small equatorial pitch-angles and a velocity $v \sim 10^{10}$ cm s⁻¹ on an envelope $L = 5$ gives: for electrons $v_{\xi\xi}^e \approx 2 \cdot 10^{-7}$ s⁻¹, $\tau^e \approx 5 \cdot 10^6$ s, and for protons $v_{\xi\xi}^p \approx 1$ s⁻¹, $\tau^p \approx 1$ s.

On the basis of these estimates it is possible to make the conclusion that beams of polar protons are scattering fast, because of an operation of the mechanism of equatorial break down of the first adiabatic invariant. At the same time electron beams of polar particles are very stable relative to the mechanism of scattering. The considerable selectivity of an operation of the surveyed mechanism of scattering relative to polar electrons and protons reduces in the important physical consequences for dynamics of trapped radiation in a magnetosphere. In particular, it can explain why the emptying of electrons is the basic reason of auroras.

The dynamic stability of electron beams in magnetospheric plasma can play the important role in origin of collective plasma effects such, as a beam instability and swing of plasma oscillations with all following from this physical consequences well known in the theory of plasma [8].

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