

## MODEL OF THE INTERNAL GRAVITY WAVES EXCITED BY LITHOSPHERIC GREENHOUSE EFFECT GASES

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The satellite and near-ground study demonstrated IR temperature anomalies associated with fault system of the crust in the seismic region of the Earth. The amplitude of anomalies is about  $\Delta T = 3$  K. Such change of temperature needs about 100 % increase of CO<sub>2</sub> concentration. Internal gravity waves (IGW) excited by greenhouse effect gases could cause some ionospheric disturbances including density irregularities. The accurate numerical model of 2D «lithospheric greenhouse effect gas antenna» with heat, mass and concentration sources of IGW in the atmosphere is built in the present work. The system of hydrodynamics equations for IGW excitation in the atmosphere by the source in the form of near-ground layer of lithospheric greenhouse effect gases is reduced to the system of two equations for pressure and vertical velocity component. Corresponding effective boundary conditions are obtained by means of limit pass to the case of very thin layer and absolutely rigid lithosphere. Periodical boundary conditions in the horizontal directions are used to avoid computations with continuous spectrum and numerical convergence of the model is checked carefully. It is shown that at the altitude  $Z = 200$  km the value of vertical velocity of IGW with period 1 hour could reach the value of order 4 m/s what is enough for plasma bubbles formation in accordance with previously published data. In distinction to the qualitative model of Gohberg et al., reactive modes in the IGW spectrum are taken into account. It is shown that, although these modes are non-propagating, they influence significantly the IGW excitation and, as a result, the characteristics of propagating modes. In particular, non-taking into account reactive modes could cause under definite conditions overestimating of energy flow by at least two orders of value. This result is analogous to the well known in microwave technique effect of the influence of reactive modes on scattering of electromagnetic waves on inhomogeneities in microwave waveguides.

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### INTRODUCTION

Study of satellite thermal images [1, 2] demonstrated IR anomalies associated with largest linear structures and fault system of the crust in the seismic region of the Earth. Typical dimensions of these anomalies are 700 km length and 50 km width. They appear 6-24 days before shock and are sensitive to crust earthquakes with magnitude more than 4.7. The amplitude of observed temperature anomalies is about  $\Delta T = 3$  K. In accordance with [3], such change of temperature needs about 100 % increase of CO<sub>2</sub> concentration. It is possible to expect that IGW excited by greenhouse effect gases could cause both periodical changes of the intensity of optical glows of the ionosphere and a shift of the average level of the glows intensity. IGW could also cause different ionosphere disturbances and density irregularities. The spectrum of variations before earthquake of intensity of red and green lines of oxygen optical emissions (with wavelength 630 nm and typical altitude 270 km and wavelength 557.7 nm and typical altitude 97 km, respectively) includes periods from some minutes to two hour [4, 5]. Such periods are also typical for IGW and acoustic-gravity waves (from the side of shorter waves) [6]. It is supposed in [7] that IGW could be responsible also for the observed change of electromagnetic wave characteristics in the waveguide «Earth-Ionosphere» before earthquake.

The model of 2D «lithospheric greenhouse effect gas antenna» with heat, mass and concentration sources of IGW in the atmosphere is built in the present work. The system of hydrodynamics equations for the IGW excitation in the atmosphere by the gas source in the form of thin near-ground layer of lithospheric greenhouse effect gases is reduced to the system of two equations for pressure and vertical velocity component. Corresponding effective boundary conditions are obtained by means of limit pass to the case of very thin layer and absolutely rigid lithosphere. Periodical boundary conditions in the

horizontal directions are used to avoid problems with continuous spectrum, and numerical convergence of the model is checked carefully.

We suppose in accordance with [3] that the release of lithospheric gases is modulated in time due to two factors. The first one is the modulation of the number of elementary processes of lithospheric gas release by the deformation processes in the earth crust during the earthquake preparation. The second one is the modulation of the lithospheric gas release by the seismogravity oscillation in the system «lithosphere-atmosphere». This modulation develops during last days and hours before earthquake [3], [8]. The typical period of seismogravity oscillations is 1 hour [8]. As it was mentioned above, the periods of the same order have been observed in the spectrums of modulations of the airglow intensity before earthquakes [4]. At the same time, these periods are typical for the IGW in the atmosphere [6].

The real gas source of IGW has rather wide frequency spectrum. It is also possible to expect that the lithospheric gas source includes several non-correlated in space (and, possibly, in time) elementary gas sources of IGW. To search the effect of separate spectrum components of lithospheric gas source, we consider in the present article the problem in the simplest possible formulation. Namely, we consider the «model problem» of IGW excitation in the atmosphere by lithospheric gas source correlated in space and harmonic in time and search this process under different source frequencies.

#### MODEL DESCRIPTION

The model of 2D «lithospheric greenhouse effect gas antenna» of IGW in the atmosphere is built in the present work. Heat, mass and concentration sources of IGW are taken into account. The system of hydrodynamics equations for acoustic-gravity waves [6] excitation and propagation has the following form:

$$\begin{cases} \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \rho \mathbf{g} + \rho \mathbf{F}_z, \\ \frac{d\rho}{dt} + \rho \operatorname{div} \mathbf{v} = q, \\ \frac{dp}{dt} + \mathbf{v} \nabla p = c^2 \left( \frac{\partial \rho}{\partial t} + \mathbf{v} \nabla \rho \right) + A\rho \end{cases} \quad (1)$$

where  $\rho$ ,  $\mathbf{v}$ ,  $p$ ,  $\mathbf{g}$  are density, velocity, pressure and gravity acceleration, respectively,  $d/dt \equiv \partial/\partial t + \mathbf{v} \nabla$ ,  $\gamma$  is adiabatic constant,  $\mathbf{F}_z = F_z \mathbf{e}_z$  is the force function,  $q(\mathbf{r}, t)$  and  $A(\mathbf{r}, t)$  are densities of the mass and heat sources which are determined by gases released from the lithosphere.

After linearization of system of equation (1), we could obtain (from corresponding homogeneous system of equation) well-known dispersion equation for acoustic-gravity waves [6] in the form:

$$\omega^2 = \frac{c^2}{2} \left( k_x^2 + k_z'^2 + \frac{1}{4H^2} \right) \pm \sqrt{\frac{c^4}{4} \left( k_x^2 + k_z'^2 + \frac{1}{4H^2} \right) - (\gamma - 1)g^2 k_x^2}, \quad (2)$$

where  $\omega_b = g\sqrt{\gamma - 1}/c$  is the Brunt—Vaisala frequency,  $H$  is the atmospheric scale height,  $k_z' = -i/(2H) + k_z$ ,  $k_x$  and  $k_z$  are horizontal wave number and the real part of vertical wave number, respectively,  $\omega$  is the frequency of the lithospheric gas source, and 2D problem is considered,  $c$  is acoustic speed,  $\partial/\partial y = 0$ . This corresponds physically to the case of the lithospheric gas source associated with long (linear) crust fault. In the case  $\omega \ll \omega_b$  the dispersion equation of acoustic-gravity waves (2) reduced to the dispersion equation for gravity waves (or internal gravity waves, IGW) [6]:

$$k_z = -\frac{i}{2H} \pm k_z' = -\frac{i}{2H} \pm \sqrt{-\frac{1}{4H^2} + \left( \frac{\omega_b^2}{\omega^2} - 1 \right) k_x^2}, \quad (3)$$

All numerical calculations below in the present paper are presented for the case of IGW, when

dispersion equation is in the form (3).

For  $\frac{1}{4H^2} > \left(\frac{\omega_b^2}{\omega^2} - 1\right)k_x^2$ , the wave number  $k_z$  is purely imagine and corresponding modes are «reactive». These modes could not propagate in the atmosphere. At the same time, it is well know in electrodynamics of inhomogeneous transmission lines (including antennas systems) that reactivities could modify remarkably the coefficients of reflection and transmission as well as the effectiveness of excitation of electromagnetic waves. It is shown in the present paper that analogous situation takes place for IGW excitation. It is necessary to take into account reactive modes otherwise the mistake of determination in the energy flow at the altitude 200 km could exceed two order of value. In the qualitative model [3] the influence of the reactive modes on the excitation of gravity wave in the atmosphere has not been taken into account.

To avoid the problems with continuous spectrum the discrete Fourier transform is used in horizontal directions  $x$ ,  $y$  and calculations are done in horizontal region with big, but finite dimensions  $L_x \gg l_x$ ,  $L_y \ll l_y$  and with periodical boundary conditions. Here  $l_{x,y}$  are horizontal dimensions of the IGW lithospheric source. The values  $L_x$ ,  $L_y$  are chosen in such a way, that their increase does not cause any remarkable change of the IGW field in the region of interest. In our case the region of interest (where the characteristics of IGW are calculated) has the dimension  $L_x = 10^4$  km (the approximation of plane Earth and atmosphere is used) and the number of Fourier modes is  $N_x = 500...600$ . The horizontal wave numbers corresponding to these Fourier modes are  $k_x(l_x) = 2\pi l_x / (N_x L_x)$ , where  $l_x = 0, N_x - 1$ .

#### BOUNDARY CONDITIONS

The wave equation for the elastic waves propagating in the earth crust (which is considered as isotropic elastic media) has the form [9]:

$$\rho \ddot{\mathbf{U}} = \mu \Delta \mathbf{U} + (\lambda + \mu) \text{grad div} \mathbf{U}, \quad (4)$$

where  $\rho$  is the density of media,  $\mathbf{U}$  is volume element displacement,  $\mathbf{v} = \partial \mathbf{U} / \partial t = -i\omega \mathbf{U}$ ,  $\lambda$  and  $\mu$  are elastic constants [9].

The general solution of the equation (4) has the form:

$$\mathbf{U} = \begin{pmatrix} U_x \\ U_y \end{pmatrix} = A_1 \begin{pmatrix} -\frac{k_{z1}}{k_x} \\ 1 \end{pmatrix} + A_2 \begin{pmatrix} \frac{k_x}{k_{z2}} \\ 1 \end{pmatrix},$$

where  $k_x$  and  $k_{z1,2}$  are  $x$ - and  $z$ -components of wave numbers of elastic modes. Indexes 1 and 2 correspond to the transverse and longitudinal elastic modes with dispersion equations having the forms, respectively [9]:

$$\omega^2 = \left(\frac{\mu}{\rho}\right)(k_x^2 + k_{z1}^2), \quad \omega^2 = \frac{\lambda + 2\mu}{8}(k_x^2 + k_{z2}^2).$$

We suppose for simplicity that lithospheric gases occupy the near-Earth region of the atmosphere  $z_a \leq z \leq z_b$ , which is called below the «lithospheric gases layer» (Figure 1). Here  $z_a = 0$ ,  $z_b = \Delta z_0$ ,  $\Delta z_0$  is the thickness of the lithospheric gas layer.

As one can see from Figure 1

$$\begin{cases} \Delta V_{z,s} = V_z^a - V_z^b, \\ \Delta P_s = P^a - P^b \end{cases} \quad (6)$$

where  $\Delta V_{z,s}$  and  $\Delta P_s$  are amplitudes of velocity and pressure generated by IGW source, respectively,  $P^{a,b}$ ,  $V^{a,b}$  are pressure and wave velocity at the points  $z = z^b = \Delta z_0$  and  $z = z^a = 0$ , respectively.

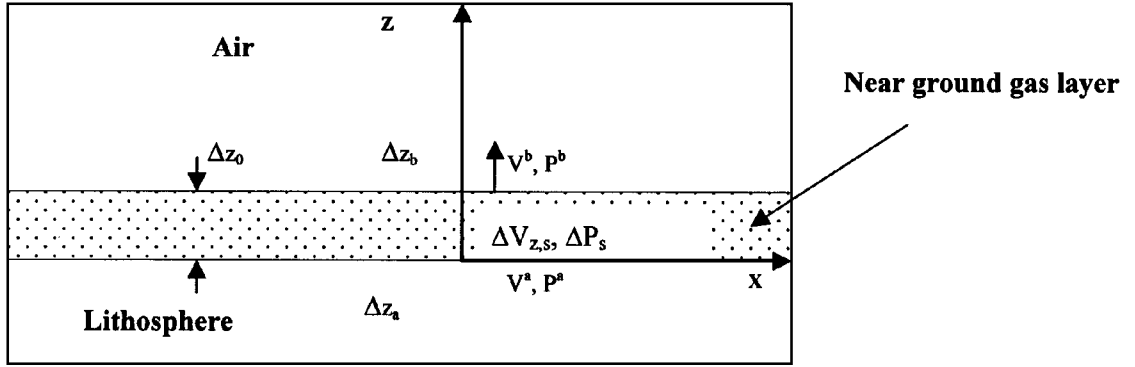


Fig. 1. Gas source of AGW generation in near-ground layer of the atmosphere

The boundary conditions at the both surfaces  $z = z_a$  and  $z = z_b$  of the lithospheric gas layer have the form:

$$\begin{cases} \sigma_{ij} n_j |_{z=z_{a,b}-0} = \sigma_{ij} n_j |_{z=z_{a,b}+0}, \\ V_z |_{z=z_a-0} = V_z |_{z=z_a+0}, \\ P |_{z=z_a-0} = P |_{z=z_a+0}, \end{cases} \quad (6)$$

where  $n_j$  is  $j$ -th component of normal to the corresponding surface. The stress tensor  $\sigma_{ij}$  inside the lithosphere ( $z < z_a$ ) has the form

$$\begin{aligned} \sigma_{ij} &= 2\mu U_{ij} + \lambda \text{div} \mathbf{U} \delta_{ij}, \\ U_{ij} &= \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right). \end{aligned}$$

In the gas media both inside lithosphere gas layer,  $z_a < z < z_b$ , and in the atmosphere above the lithospheric gas layer,  $z > z_b$ , the stress tensor is

$$\sigma_{ij} = -p \delta_{ij},$$

where  $p$  is the pressure in the corresponding gas media.

Using equations, which we have obtained after Fourier transform of the system (1) and boundary conditions (6), we get the following expression for  $\Delta v_{z,s}$ ,  $\Delta P_s$ :

$$\Delta v_{z,s} = \int_0^{\Delta z_0} \left( \frac{A}{gH} + \gamma \frac{q}{\rho_0} \right) dz, \quad (7)$$

$$\Delta P_s = \frac{1}{gH} \int_0^{\Delta z_0} \left( F_z + D_t^{-1} \frac{A}{\gamma H} \right) dz, \quad (8)$$

$$R_p(k_x) = \frac{P}{v_z} = - \frac{D_t - gD_t^{-1} \left( D_z - \frac{1}{H} \right)}{gH \left( D_z - \frac{1}{H} \right) + gD_t^{-2} D_x^2},$$

where  $D_t = i\omega$ ,  $D_z = -ik_z$ ,  $D_x = -ik_x$ ,  $R = \tilde{\rho}/\rho_0$ ,  $P = \tilde{p}/p_0$ ,  $P$  and  $V_z$  are the pressure and vertical component of the particles velocity in the atmosphere above the lithospheric gas layer ( $z > z_b$ ),  $\tilde{\rho}$ ,  $\tilde{p}$ ,  $\rho_0$ ,  $p_0$  are variable and stationary parts of the atmospheric density and pressure, respectively.

It is possible to obtain Fourier-transform of  $v_z$  and  $P$  from equations (2)—(5):

$$v_z^b = \frac{p_0 \Delta P_{\text{жк}} + \frac{S}{\omega(\beta_1 + a)} \Delta v_{z,s}}{p_0 R_p + \frac{S}{\omega(\beta_1 + a)}}, \quad (9)$$

$$P^b = R_p v_z^b,$$

where

$$\beta_1 = -\frac{k_x}{k_{z1}}, \quad \alpha_1 = \frac{k_x}{k_{z2}}, \quad a = A_2'/A_1 = -\frac{k_{z1} + k_x \beta_1}{k_{z2} \alpha_1 + k_x},$$

$$S = \lambda(k_x + k_{z1} \beta_1) + 2\mu k_{z1} \beta_1 + a[\lambda(k_x \alpha_1 + k_{z2}) + 2\mu k_{z2}].$$

In the limit of absolutely rigid lithosphere the equation (9) reduces to the form:

$$v_z^a = 0, \quad v_z^b = \Delta v_{z,s}. \quad (10)$$

Later we shall use this approximation.

The analysis of equation (7), (8) shows that the IGW are excited at most by the heat source with temperature amplitude  $\Delta T$  (numerical estimations show that the mass and force sources presented in eqs. (1), (7), (8) give negligibly small contribution to IGW excitation in comparison with the heat source). In accordance with [3], IGW source amplitude  $\Delta v_{z,s} = \frac{A}{\gamma H} \Delta z_0$  is related to the amplitude  $\Delta T$  of the observed heat anomaly before earthquake as follows:

$$\Delta v_{z,s} = \Omega \Delta z_0 \frac{\Delta T}{2\gamma T} \exp(z/2H),$$

where  $\Omega$  is frequency of temperature oscillation (determined by above mentioned processes of lithospheric gas release modulation before earthquake),  $\gamma$  is adiabatic constant,  $T$  is an air temperature (we use the approximation of isothermic atmosphere here),  $z$  is vertical coordinate.

#### MODEL SOURCE AND THE FIELD OF GRAVITY WAVES IN THE ATMOSPHERE

Consider one-dimensional thermal source with the space distribution in the form  $\Delta T = \Delta T_0 \cdot \text{ch}^{-2}(x_1/l)$ , where  $\Delta T_0$  is the amplitude of observed temperature anomaly (it is equal to 3 K in the present calculations),  $l$  is the lithospheric source width. Such source corresponds to the longitudinal lithospheric fault which has the length much bigger than its width [2]. Corresponding vertical velocity at the earth

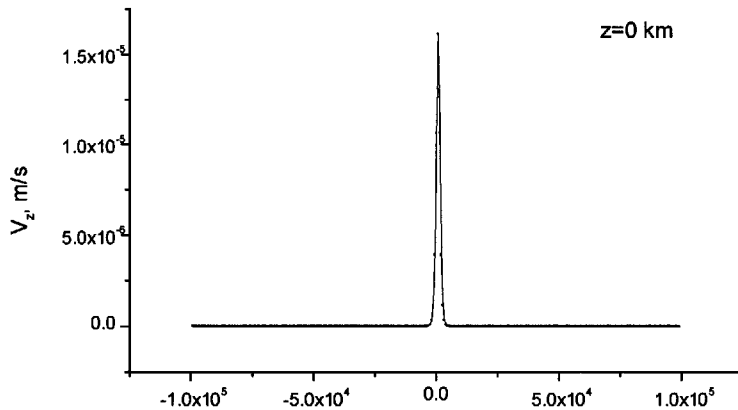


Fig. 2. Vertical velocity profile  $V_z$  (m/s) at the level of the Earth crust ( $z = 0$ ), period is equal to 15 min, number of modes is 500,  $\Delta T = 3$  K, source size is 100 km

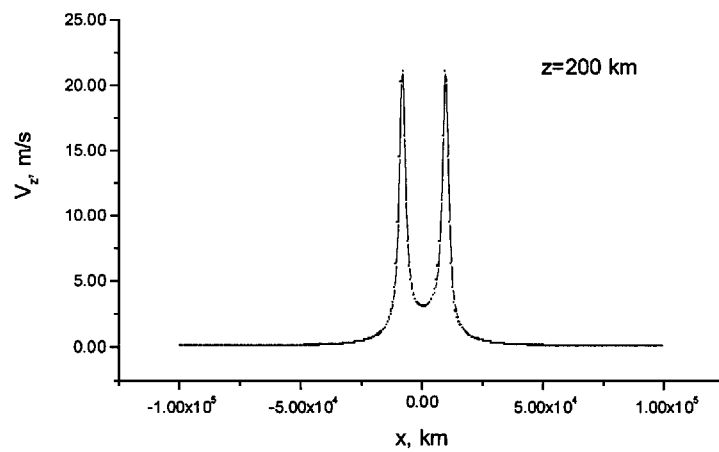


Fig. 3. Vertical velocity profile at the 200 km without reactive modes, period is equal to 15 min, number of modes is 600,  $\Delta T = 3$  K, source size is 100 km

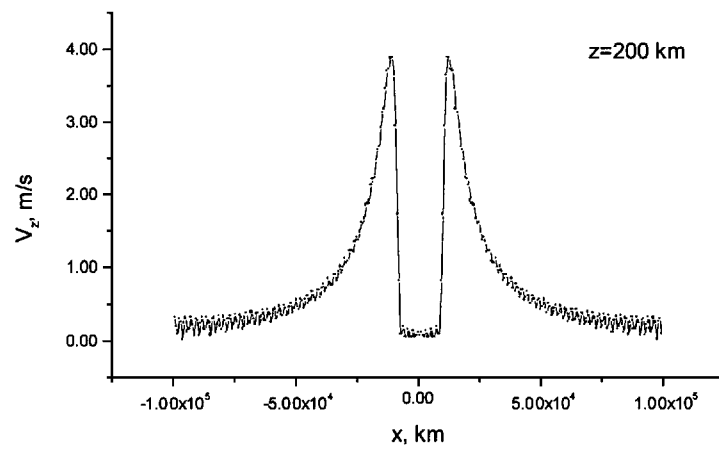


Fig. 4. Vertical velocity profile at the 200 km with reactive modes, period is equal to 15 min number of modes is 500,  $\Delta T = 3$  K, source size is 100 km

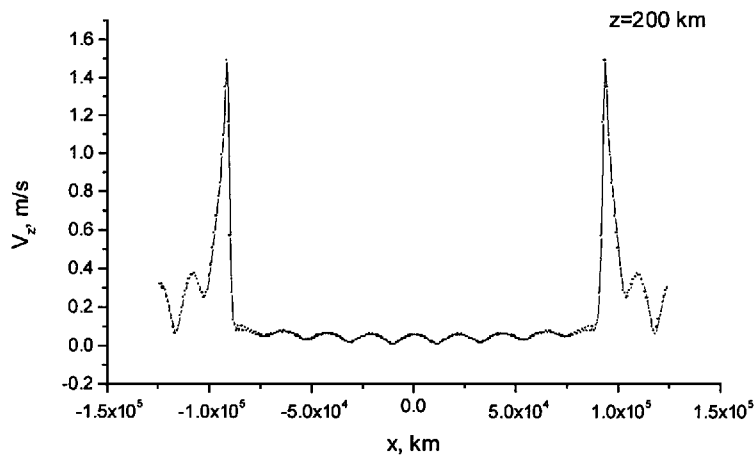


Fig. 5. Vertical velocity profile at the 200 km with reactive modes, period is equal to 60 min, number of modes is 600,  $\Delta T = 3$  K, source size is 100 km

surface is  $V_{z,s}^{\text{mod}}(x_1) = \Delta v_{z,s} \text{ch}^{-2}(x_1/l)$  (see Figure 2).

As it follows from the solution of the system of equations (1), the velocity and pressure of IGW excited in the atmosphere by the lithospheric gas source have the following form:

$$v_z(x_1, z) = e^{z/(2H)} \sum_{m=1-N_x/2}^{N_x/2} v_z(k_m) \cdot e^{-ik_m x_1 - ik'_z z},$$

$$v_x(k_m) = -\frac{gH}{\omega} k_m R_p v_z(k_m),$$

$$P(x_1, z) = e^{z/(2H)} \sum_{m=1-N_x/2}^{N_x/2} R_p v_z(k_m) \cdot e^{-ik_m x_1 - ik'_z z},$$

where  $k_m = 2\pi m/L$ ,  $m = \overline{1 - N_x/2, N_x/2}$ ,  $x_1 = \frac{l}{N_x} L$ ,  $l = \overline{1 - N_x/2, N_x/2}$ ,  $N_x$  is the number of modes taken into account in discrete Fourier transform,  $k'_z$  is real part of  $k_z$ ,  $v_z(k_m)$  is Fourier amplitude of the  $m$ -th harmonic of IGW vertical velocity.

The distribution of the vertical velocity  $V_z$  taking into account reactive modes are shown on the Figures 4, 5 for the altitude 200 km and the values of the periods  $T = 15$  min and  $T = 60$  min respectively. Distribution of the  $V_z$  with the spectrum where reactive modes are omitted is shown in the Figure 3.

## RESULTS AND CONCLUSION

1. It is shown (see Figures 3—5) that it is necessary in general case to take into account the reactive modes in the spectrum, in distinction to the model [3]. For example, for the period  $T = 15$  min difference of the  $V_z$  amplitudes computed by means of the present model and of the model [3] reaches more than 10 times and IGW peak energy density reaches more than  $10^2$  times at the altitude 200 km (see Figures 3, 4). In distinction to the model [3], the present model demonstrates much wider distribution with the side lobes in horizontal direction of the IGW vertical velocity. On the other hand, for higher frequencies the difference between the energy flow density computed with and without taking into account the presence of reactive modes in the IGW spectrum is rather small.

2. The gases released from the lithosphere before earthquake are rather effective source of IGW. Namely, for  $T = 60$  min the value of vertical component of IGW velocity  $V_z$  could reach 4 m/s at the altitude  $Z = 200$  km (Figure 5). The IGW with such velocity could excite ionospheric plasma bubbles [10] and influence the intensity of the ionospheric glows before an earthquake [11]. As a result microinstabilities in the ionosphere could be excited on the gradient of electron density [12]. These processes are nonlinear and therefore very sensitive for the amplitude and space distribution of the initial disturbances in the ionosphere. The present accurate model of IGW excitation by the lithospheric gas source is important for calculation of such disturbances excited in ionosphere due to lithospheric processes before an earthquake.

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