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UNSTABLE AXIALLY SYMMETRIC MHD FLOW BETWEEN ROTATING BOUNDARIES

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For cylindrical and spherical liquid layers confined between two rotating axially symmetric shells the conditions for azimuth flow instability creating meridional component rise were obtained. Methods for solution of nonlinear equation describing steady-state flow for large Reynolds numbers were proposed.

This report is related with hydromagnetic dynamo problem in the geophysics. It is supposed that magnetic field of the Earth appears due to differential rotation in electroconductive liquid core caused by difference between angular velocities of inner core and mantle [1]. In order to clarify general mechanisms and specific features of such processes it is purposeful to investigate some model examples in simplified formulation. First of all we should explain how flat differential rotation can excite poloidal component of the flow, for example, between coaxial surfaces.

In this section the flow of incompressible viscous liquid is examined that in the fixed frame of reference is described by Navier-Stokes equation

$$\frac{\partial v}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho_m}\mathrm{grad}(p + U) + \frac{\tilde{\eta}}{\rho_m}\Delta\mathbf{v}, \qquad \mathrm{div}\mathbf{v} = 0, \tag{1}$$

where $\mathbf{v}$, p, U, ρ_m, $\tilde{\eta}$ are respectively velocity, pressure, gravitation potential, mass density and viscosity coefficient. This equation in the frame of reference rotating together with the Earth at angular velocity Ω takes for total velocity $\mathbf{V}_\Sigma$ the following form:

$$\frac{\partial \mathbf{V}_\Sigma}{\partial t} - [\mathbf{V}_\Sigma\times\mathrm{rot}\mathbf{V}_\Sigma] + 2\,[\Omega\times\mathbf{V}_\Sigma] = -\mathrm{grad}\left\{\frac{p+U}{\rho_m} + \frac{\mathbf{V}_\Sigma^2}{2} - \frac{1}{2}[\Omega\times\mathbf{r}]^2\right\} + \frac{\tilde{\eta}}{\rho_m}\Delta\mathbf{V}_\Sigma \tag{2}$$

where $\mathbf{r}$ is radius vector of observation point. Application of rotor operator to (2) results in Helmholtz equation

$$\frac{\partial \Gamma_\Sigma}{\partial t} - \mathrm{rot}\,[\mathbf{V}_\Sigma\times\Gamma_\Sigma] + 2\mathrm{rot}[\Omega\times\mathbf{V}_\Sigma] = \frac{\tilde{\eta}}{\rho_m}\Delta\Gamma_\Sigma,$$

where $\Gamma_\Sigma = \mathrm{rot}\mathbf{V}_\Sigma$. If z — axis is directed along vector Ω and accepted system of units is such that $\Omega = 1$ and characteristic length $r_0 = 1$ this equation takes a compact dimensionless form

$$\frac{\partial \Gamma_\Sigma}{\partial t} = \mathrm{rot}\,[\mathbf{V}_\Sigma\times\Gamma_\Sigma] + 2\frac{\partial \mathbf{V}_\Sigma}{\partial z} + \eta\Delta\Gamma_\Sigma. \tag{3}$$

Differential rotation in the liquid is characterized by dependence of circular velocity $\mathbf{V}_\varphi(\rho)$ on radius ρ. Density of the pulse moment in flow is connected with rho by relation

$$\mu(\rho) = \rho V_\varphi(\rho).$$

Application of the energetic principle makes possible to express increment γ of local instability (for unstable profiles) with $\mu(\rho)$ by simple formula

$$\gamma^2 = \frac{4}{\rho^3}\frac{d}{d\rho}\mu^2(\rho). \tag{4}$$

For our purposes small increments are of specific interest. If increment is rather small and constant in definite interval of rho then we have situation which is called below overcritical regime. The neutral stability takes place if $V_\varphi(\rho)$ in rotating frame of reference has profile

$$V_0(\rho) = \frac{1}{\rho} - \rho. \tag{5}$$

The overcritical regime can be expressed by additional term

$$V_\alpha(\rho) = \frac{1-\rho^4}{2\rho} \tag{6}$$

and subsequently total velocity for this regime can be represented as

$$\mathbf{V}_\Sigma = (V_0 + a^2 V_\alpha)\,\mathbf{e}_\varphi + \mathbf{V}, \tag{7}$$

where $\alpha^2 \ll 1$ — overcritical parameter,

$$\mathbf{V} = V_\rho \mathbf{e}_\rho + V_\varphi \mathbf{e}_\varphi + V_z \mathbf{e}_z$$

is small perturbation of the flow velocity to be found, $\mathbf{e}_\rho$, $\mathbf{e}_\varphi$, $\mathbf{e}_z$, — vector basis of cylindrical coordinate system.

Substitution of (7) in (3) brings equation for $\mathbf{V}$

$$\frac{\partial \Gamma}{\partial t} = \mathrm{rot}[\mathbf{V}\times\Gamma] - \frac{2}{\rho^2}\Gamma_\rho \mathbf{e}_\varphi - -\alpha^2\left[2\rho^2\frac{\partial\,\mathbf{V}}{\partial\,z} - 4\rho V_\rho \mathbf{e}_z + \left(\rho^2 + \frac{1}{\rho^2}\right)\Gamma_\rho \mathbf{e}_\varphi\right] \tag{8}$$

with $\Gamma = \mathrm{rot}\mathbf{V}$ by $\mathrm{div}\mathbf{V} = 0$.

Linearization of (8) allows to obtain sufficiently simple system for V_ρ, V_φ, V_z depending on time t according to $e^{\gamma t}$:

$$\begin{gathered}
\gamma\frac{\partial V_\varphi}{\partial z} = 2\alpha^2\rho^2\frac{\partial V_\rho}{\partial z},\\
\gamma\left(\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial \rho}\right) = \frac{2}{\rho^2}\frac{\partial V_\varphi}{\partial z},\\
\gamma\left(\frac{\partial V_\varphi}{\partial \rho} + \frac{1}{\rho}V_\varphi\right) = 2\alpha^2\rho^2\left(\frac{\partial V_\rho}{\partial \rho} + \frac{3}{\rho}V_\rho\right),\\
\frac{\partial V_\rho}{\partial \rho} + \frac{1}{\rho}V_\rho + \frac{\partial V_z}{\partial z} = 0.
\end{gathered} \tag{9}$$

Elimination $\partial V_\varphi/\partial z$ from the first and second equations yields

$$\frac{\partial V_z}{\partial \rho} = \left[1 - \left(\frac{2\alpha}{\gamma}\right)^2\right]\frac{\partial V_\rho}{\partial z}. \tag{10}$$

Taking this into account we obtain wave equation for V_ρ:

$$\frac{\partial^2 V_\rho}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial V_\rho}{\partial \rho} - \frac{V_\rho}{\rho^2} + \left[1 - \left(\frac{2\alpha}{\gamma}\right)^2\right]\frac{\partial^2 V_\rho}{\partial z^2} = 0,$$

which allows separation of variables by substitution

$$V_\rho = X(\rho)\,Y(z).$$

Finally an explicit expressions for increment and modal column are obtained:

$$\gamma_{\mathrm{nk}}^2 = \alpha^2 \frac{k^2\pi^2}{b^2} \frac{1}{\lambda_{\mathrm{n}}^2 + \frac{k^2\pi^2}{b^2}}. \tag{11}$$

$$\begin{aligned} V_{\mathrm{nk}}^{(\rho)} &= \mathrm{e}^{\gamma_{\mathrm{nk}}\mathrm{t}} Z_1(\lambda_{\mathrm{n}}\rho)\cos\left(\frac{k\pi z}{b}\right), \\ V_{\mathrm{nk}}^{(\varphi)} &= \mathrm{e}^{\gamma_{\mathrm{nk}}\mathrm{t}} \frac{2\alpha}{\gamma_{\mathrm{nk}}/\alpha} \rho^2 Z_1(\lambda_{\mathrm{n}}\rho)\cos\left(\frac{k\pi z}{b}\right), \\ V_{\mathrm{nk}}^{(z)} &= -\mathrm{e}^{\gamma_{\mathrm{nk}}\mathrm{t}} \lambda_{\mathrm{n}} \frac{b}{k\pi} Z_0(\lambda_{\mathrm{n}} r)\sin\left(\frac{k\pi z}{b}\right). \end{aligned} \tag{12}$$

Here λ_{n}^2 ($n = 1, 2, 3, \ldots$) are eigenvalues of characteristic equation system $Z_1[(1 - a)\lambda_{\mathrm{n}}] = 0$, $Z_1[(1 + a)\lambda_{\mathrm{n}}] = 0$ for tube domain ($1 - a \le \rho \le 1 + a$, $0 \le z \le b$).

It can be seen that $V^{(\rho)}$ and $V^{(z)}$ are larger in order than $V^{(\varphi)}$. This exhibits resonance nature of poloidal component generation by azimuth differential rotation. Another important subsequence of this analysis consists in specific dependence of increment on axial k and radial n wave numbers. The largest increment is observed in short wave part of axial spectrum but in long wave of radial one. This general features take place for spherical layers also as we show further.

Starting analysis of the sphere let us pay attention to one obstacle which prevents application of $V_\alpha(\rho)$ expressed by (6). The sense of this caution consists in circumstance that neighborhood of axis does not belong to the tube domain in cylindrical case but is present in polar zones in spherical layer. So one should change the form of overcritical additional term by $V_\alpha(\rho) = -\rho$.

For azimuth component F of vector potential we obtain the following equation:

$$\Delta_1 F + \frac{2\alpha^2}{\gamma^2\rho^2}\Gamma_\alpha(\rho)\frac{\partial^2 F}{\partial z^2} = 0, \tag{13}$$

where

$$\Delta_1 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + \frac{\partial^2}{\partial z^2} - \frac{1}{\rho^2},$$

$$\Gamma_\alpha(\rho) = \frac{1}{\rho}\frac{\partial}{\partial\rho}[\rho V_\alpha(\rho)] = -2.$$

Correspondingly for potential

$$f(r, \theta) = F(\rho, z)\Big|_{\rho\ =\ \mathrm{r}\sin\theta,\ \mathrm{z}\ =\ \mathrm{r}\cos\theta}.$$

Expressed in spherical coordinates r, θ, φ equation (13) takes the form

$$\Delta_1 f - \frac{4\alpha^2}{\gamma^2 r^2\sin^2\theta}\hat{L}f = 0, \tag{14}$$

$$\Delta_1 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) - \frac{f}{r^2\sin^2\theta}, \tag{15}$$

$$\hat{L}f = \frac{\partial^2 f}{\partial r^2}\cos^2\theta - \frac{2}{r}\frac{\partial^2 f}{\partial r\partial\theta}\sin\theta\cos\theta + \frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}\sin^2\theta + \frac{1}{r}\frac{\partial f}{\partial r}\sin^2\theta +$$

$$+ \frac{2}{r^2}\frac{\partial f}{\partial\theta}\sin\theta\cos\theta. \tag{16}$$

which is inhibitory to separation of variables. Thereby eigenfunction $f_\sigma = f_\sigma(r, \theta)$ and corresponding eigenvalues $\gamma = \gamma_\sigma$ are represented as linear combination

$$f_\sigma = \sum_{\mathrm{nl}} C^\sigma_{\mathrm{nl}} u_{\mathrm{nl}}(r, \theta) \tag{17}$$

of normalized in metric $\langle f, g\rangle = \int_a^1 \int_0^\pi f(r, \theta)g(r, \theta)r^2 \sin\theta dr d\theta$ spherical harmonics

$$u_{\mathrm{nl}} = \frac{R_{\mathrm{n}}(k_{\mathrm{nl}}r)}{||R_{\mathrm{nl}}||}\frac{P_{\mathrm{n1}}(\cos\theta)}{||P_{\mathrm{n1}}||}\ (a \le r \le 1, 0 < \theta < \pi),$$

where

$$R_{\mathrm{n}}(k_{\mathrm{nl}}r) = R^{(+)}_{\mathrm{n}}(k_{\mathrm{nl}})R^{(-)}_{\mathrm{n}}(k_{\mathrm{nl}}r) - R^{(-)}_{\mathrm{n}}(k_{\mathrm{nl}})R^{(+)}_{\mathrm{n}}(k_{\mathrm{nl}}r), \tag{18}$$

$$||R_{\mathrm{nl}}|| = \left(\int_a^1 R^2_{\mathrm{n}}(k_{\mathrm{nl}}r)r^2 dr\right)^{1/2},$$

$$||P_{\mathrm{n1}}|| = \sqrt{\frac{2n(n+1)}{2n+1}},$$

$$R^{(+)}_{\mathrm{n}}(x) = \sqrt{\frac{p}{2x}}\, J_{\frac{2n+1}{2}}(x),$$

$$R^{(-)}_{\mathrm{n}}(x) = (-1)^{\mathrm{n}}\sqrt{\frac{p}{2x}}\, J_{-\frac{2n+1}{2}}(x).$$

Since the Bessel function of half-integer index can be expressed via trigonometric function it is possible to fulfil calculations by means of recurrent procedure as follows

$$R^{(+)}_{\mathrm{n}}(x) = A_{\mathrm{n}}(x)\cos x + B_{\mathrm{n}}(x)\sin x,$$

$$R^{(-)}_{\mathrm{n}}(x) = -A_{\mathrm{n}}(x)\sin x + B_{\mathrm{n}}(x)\cos x,$$

where

$$A_{\mathrm{n}+1}(x) = \frac{n}{x}A_{\mathrm{n}}(x) - A'_{\mathrm{n}}(x) - B_{\mathrm{n}}(x),$$

$$B_{\mathrm{n}+1}(x) = \frac{n}{x}B_{\mathrm{n}}(x) - B'_{\mathrm{n}}(x) + A_{\mathrm{n}}(x)$$

with initial data $A_1(x) = -1/x$, $B_1(x) = 1/x^2$.

The construction (18) satisfies boundary condition $u_{\mathrm{nl}}(1, \theta) \equiv 0$ which expresses the fact of impenetrability of the liquid through external sphere. The same demand at inner sphere of radius $r_{\mathrm{i}} = a$ leads to characteristic equation

$$\mathrm{tg}[k(1-a)] = \frac{B_{\mathrm{n}}(k)A_{\mathrm{n}}(ka) - A_{\mathrm{n}}(k)B_{\mathrm{n}}(ka)}{A_{\mathrm{n}}(k)A_{\mathrm{n}}(ka) + B_{\mathrm{n}}(k)B_{\mathrm{n}}(ka)}.$$

For every index $n = 1, 2, 3$ ... the infinite sequence of wave-numbers k_{nl} corresponds. Finally we obtain $u_{\mathrm{nl}} = u_{\mathrm{nl}}(r, \theta)$ and k_{nl} satisfying standard demands:

$$\langle u_{\mathrm{n'l'}}, \Delta_1 u_{\mathrm{nl}}\rangle = -k_{\mathrm{n'l'}}k_{\mathrm{nl}}\delta_{\mathrm{n'l',\,nl}}, \qquad \langle u_{\mathrm{n'l'}}, u_{\mathrm{nl}}\rangle = d_{\mathrm{n'l',\,nl}}, \qquad u_{\mathrm{nl}}\big|_{(S_{\mathrm{i}})} = 0, \qquad u_{\mathrm{nl}}\big|_{(S_{\mathrm{e}})} = 0.$$

The next step consists in resolution of infinite system

$$\sum_{\nu}(m_{\nu'\nu}-\lambda_\sigma^2\delta_{\nu'\nu})f_\nu^\sigma=0\ (\nu,\nu'=1,2,...),$$

where

$$\nu=\{n,l\},\qquad \nu'=\{n',l'\},\qquad m_{\nu'\nu}=\frac{M_{\nu'\nu}}{k_{\nu'}k_\nu},\qquad f_\nu^\sigma=k_\nu C_\nu^\sigma,$$

$$M_{\mathrm{n'l',\ nl}}=\langle u_{\mathrm{n'l'}},\frac{-4}{r^2\sin^2\theta}\hat{L}u_{\mathrm{nl}}\rangle\ .$$

The method applied consisted in cutting off $(N\times N)$ matrix from the infinite one and subsequent step by step raise of number N. Ordering in sequence of $\nu=\{n,l\}$ by raise of k_ν according to table 1 we obtain as a result of calculations table 2 representing dependence of $\lambda_{\max}$ on N.

Finally we are going to obtain governing equation for stationary axial symmetric MHD flow in conductive liquid. Initial equations are taken in form

$$\begin{gathered}[\mathbf{V}\times\Gamma]-[\mathbf{H}\times\mathbf{J}]+\varepsilon\eta\Delta\mathbf{V}=\mathrm{grad}W,\\ \mathrm{rot}[\mathbf{V}\times\mathbf{H}]+\varepsilon\eta_{\mathrm{m}}\Delta\mathbf{H}=0\qquad(0\le\varepsilon\ll1),\\ [\mathbf{V}\times\mathbf{H}]-\varepsilon\eta_{\mathrm{m}}\mathbf{J}=\mathrm{grad}\psi,\\ \mathrm{rot}[\mathbf{V}\times\Gamma]-\mathrm{rot}[\mathbf{H}\times\mathbf{J}]+\varepsilon\eta\Delta\Gamma=0.\end{gathered}\tag{19}$$

where grad$W$ — generalized potential force, Γ = rot$\mathbf{V}$, $\mathbf{J}$ = rot$\mathbf{H}$, div$\mathbf{V}$ = 0, div$\mathbf{H}$ = 0, $\Delta\mathbf{H}=-$rot$\mathbf{J}$, ψ — scalar potential of electric field. In cylindrical system all components of $\mathbf{V}$, $\mathbf{H}$, Γ, $\mathbf{J}$ can be expressed via four scalar functions A, V, B, H according to

$$V_\rho=-\frac{1}{\rho}\frac{\partial A}{\partial z},\qquad V_\varphi=\frac{1}{\rho}V,\qquad V_z=\frac{1}{\rho}\frac{\partial A}{\partial\rho},$$

$$H_\rho=-\frac{1}{\rho}\frac{\partial B}{\partial z},\qquad H_\varphi=\frac{1}{\rho}H,\qquad H_z=\frac{1}{\rho}\frac{\partial B}{\partial\rho},$$

$$\Gamma_\rho=-\frac{1}{\rho}\frac{\partial V}{\partial z},\qquad \Gamma_\varphi=-\frac{1}{\rho}\hat{\Lambda}A,\qquad \Gamma_z=\frac{1}{\rho}\frac{\partial V}{\partial\rho},$$

$$J_\rho=-\frac{1}{\rho}\frac{\partial H}{\partial z},\qquad J_\varphi=-\frac{1}{\rho}\hat{\Lambda}B,\qquad J_z=\frac{1}{\rho}\frac{\partial H}{\partial\rho},$$

where $\hat{\Lambda}=\rho\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}$ in cylindrical system and $\hat{\Lambda}=\frac{\partial^2}{\partial r^2}+\frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$ in spherical one. Taking into consideration that grad$_\varphi$ = 0 in axial symmetric flow one can represent (19) as follows:

Table 1. The wave numbers k_{nl} of elementary harmonics

l	n							
	1	2	3	4	5	6	7	8
1	5.37	6.16	7.16	8.25	9.38	10.52	11.62	12.60
2	10.08	10.60	11.32	12.22	13.24	14.34	15.45	16.71
3	14.91	15.27	15.80	16.48	17.30	18.24	19.22	20.35

Table 2. Dependence of $\lambda_{\max}$ on number N of involved elementary harmonics

Evenness of number n	N					
	1	2	3	4	5	6
even	2.06	3.92	4.66	4.67	5.25	5.56
odd	3.29	4.34	4.92	4.97	6.12	6.39

$$
\begin{aligned}
&[\mathbf{V}\times\mathbf{H}]_\varphi - \varepsilon\eta_{\rm m} J_\varphi = 0,\\
&[\mathbf{V}\times\Gamma]_\varphi - [\mathbf{H}\times\mathbf{J}]_\varphi + \varepsilon\eta\Delta_1 V_\varphi = 0,\\
&\mathrm{rot}_\varphi[\mathbf{V}\times\mathbf{H}] + \varepsilon\eta_{\rm m}\Delta_1 H_\varphi = 0,\\
&\mathrm{rot}_\varphi[\mathbf{V}\times\Gamma] - \mathrm{rot}_\varphi[\mathbf{H}\times\mathbf{J}] + \varepsilon\eta\Delta_1\Gamma_\varphi = 0.
\end{aligned} \tag{20}
$$

By means of bilinear skew symmetric differential form

$$
\{f, g\} = \frac{\partial f}{\partial z}\frac{\partial g}{\partial \rho} - \frac{\partial f}{\partial \rho}\frac{\partial g}{\partial z}. \tag{21}
$$

equations (20) can be expressed via introduced scalar functions:

$$
\begin{aligned}
&\{A, B\} + \varepsilon\eta_{\rm m}\rho\hat{\Lambda}B = 0,\\
&\{A, V\} - \{B, H\} + \varepsilon\eta\rho^2\Delta_1\left(\frac{V}{\rho}\right) = 0,\\
&\left\{A, \frac{H}{\rho^2}\right\} - \left\{B, \frac{V}{\rho^2}\right\} + \varepsilon\eta_{\rm m}\Delta_1\left(\frac{H}{\rho}\right) = 0,\\
&\left\{A, \frac{\hat{\Lambda}A}{\rho^2}\right\} - \left\{B, \frac{\hat{\Lambda}B}{\rho^2}\right\} + \varepsilon\eta\Delta_1\left(\frac{\hat{\Lambda}A}{\rho}\right) = \frac{1}{\rho^3}\frac{\partial}{\partial z}(V^2 - H^2).
\end{aligned} \tag{22}
$$

Here ε is small parameter which in the case of ideal liquid becomes zero.

Due to property of brackets (21) the first of three equations of the system (22) is satisfied if

$$
B = B(A), \qquad V = V(A), \qquad H = B'(A)V(A), \tag{23}
$$

where $B(A)$ and $V(A)$ are some smooth functions of A and $B' = \frac{d}{dA}B(A)$. Substituting B, V, H into the fourth equation of (22) accordingly (23) enables us to rewrite it in the form of commutative relationship

$$
\left\{A, \frac{1}{r^2}[1 - B'^2(A)]\,[\hat{\Lambda}A + V(A)V'(A)] - \frac{1}{r^2}B'(A)B''(A)\,[\mathrm{grad}^2 A + V^2(A)]\right\} = 0.
$$

In its turn it is satisfied if the second term in brackets is some smooth function $S = S(A)$. Taking this into consideration one can obtain the following quasilinear equation

$$
\hat{\Lambda}A + Q(A)\mathrm{grad}^2 A = \rho^2\Psi(A) + \Phi(A), \tag{24}
$$

where

$$
Q(A) = -\frac{B'(A)B''(A)}{1 - B'^2(A)}, \qquad \Psi(A) = \frac{S(A)}{1 - B'^2(A)},
$$

$$
\Phi(A) = -V(A)[V'(A) + V(A)Q(A)].
$$

For sufficiently nonlinear problems it is useful to transform (24) into integro-differential form

$$
A(\xi) = \int_{\rm D} G(\xi, \xi')\{Q[A(\xi')]\mathrm{grad}^2 A(\xi') - \rho^2(\xi')\Psi[A(\xi')] - \Phi[A(\xi')]\}d\xi',
$$

with application the Green function $G(\xi, \xi')$ for D — domain of meridional cross-section and boundary condition of impenetrability $A|_{(D)} = 0$.

If $\alpha^2 \ll 1$ then equation (24) can be linearized and obtain representation

$$
\rho\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial A}{\partial\rho}\right) + \frac{\partial^2 A}{\partial z^2} + (V_1^2 - \Psi_1\rho^2)A = -(V_0V_1 - \Psi_0\rho^2),
$$

where notations

$$B(A) = B_0 + B_1 A \qquad (B_1^2 < 1, \quad Q(A) = 0),$$

$$V(A) = V_0 + V_1 A, \qquad \Psi(A) = \Psi_0 + \Psi_1 A,$$

$$\Phi(A) = \Phi_0 + \Phi_1 A = -V(A)V'(A), \qquad \Phi_0 = -V_0 V_1, \qquad \Phi_1 = -V_1^2$$

are applied. For example, let us determine stationary resolution for tube domain where the Green function takes the form

$$G_k(\rho, \rho') = \rho\rho' \sum_{n=1}^{\infty} \frac{\tilde{Z}_1(\mu_n \rho)\, \tilde{Z}_1(\mu_n \rho')}{\mu_n^2 + \dfrac{(2k-1)^2\pi^2}{b^2} - V_1^2}$$

and finally

$$A \approx A_0(\rho, z) = \frac{4}{\pi} \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{\left[(V_1 - \rho'^2\Psi_0) Z_0(\mu_n \rho')\right]_{\rho'=1-a}^{\rho'=1+a}}{(2k-1)\mu_n \left[\mu_n^2 + (2k-1)^2 \dfrac{\pi^2}{b^2} - V_1^2\right]} \rho Z_1(\mu_n \rho) \sin(2k-1)\frac{\pi z}{b}.$$

Mentioning that if $\varepsilon = 0$ the system (22) is invariant relatively simultaneous interchange $A \Leftrightarrow B$, $V \Leftrightarrow H$ one can write equation for $B = B(\rho, z)$ being equivalent to (24), where

$$Q(B) = -\frac{A'(B)A''(B)}{1 - A'^2(B)}.$$

At last in the case of linear dependence $A = A(B)$ if $A'^2(B) \neq 1$ we obtain

$$Q(B) = 0$$

and equation

$$\hat{\Lambda} B = \rho^2 \Psi(B) + \Phi(B),$$

which is well known in plasma equilibrium theory as the Grad—Shafranov equation.

COMMENTS

Situation mentioned above is concerned only one of possible scenarios which may occur in real planetary evolution. Another case may appear if differential rotation has subcritical level. Then alternative technique should be applied consisting in asymptotic resolution of boundary layer problem. So one should take into account both Coriolis, Lorentz and viscous forces simultaneously. This makes the problem much more difficult than in cases explored above. But some results obtained in this paper will be necessary on subsequent steps.

REFERENCES

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