ATMOSPHERIC LIMITATIONS TO ASTROMETRIC DETECTION OF EXTRA-SOLAR PLANETS WITH VERY LARGE TELESCOPES

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The report describes a new effective technique of atmospheric image motion suppression for observations with large ground-based one-aperture telescopes. The method is based on the use of an enhanced symmetrization of star reference fields. Another (optional) element of the technique is a special apodization of the telescope entrance pupil that is especially effective for extremely large ($D > 30$ m) apertures. Numerical simulations made for a 10 m telescope show that both atmospheric image motion and photon noise of the star images with a 10 min exposure can be reduced to less than 10 microarcsec. Evaluations refer to the C_n^2 vertical profile, which is typical for the Chilean astronomical sites, a moderate $FWHM = 0.4''$ and star densities at galactic coordinates $l = 0°$, $b = 20°$. For a 100 m telescope, the precision is equal to 0.2 microarcsec/10 min for regions of high star density and drops to 1 microarcsec/10 min at the Galactic Pole. For the 10 m telescope, which measures astrometric reflex motion of stars with an accuracy of 10 microarcsec, a detection limit for Saturn- and Jupiter-sized planets is about 0.5–1 kpc. Application of very high precision astrometry is especially useful for searching extrasolar planets around the Pre-Main-Sequence, early Main-Sequence, and low-mass stars whose investigations with the aid of the radial velocity technique is difficult.

INTRODUCTION

The list of detected planets orbiting solar type stars contains now more than a hundred of objects, and the number increases very rapidly. Since the discovery of these planets was performed by measuring fluctuations of star's radial velocity, masses obtained indicate only the lower limit which depends on the inclination of the orbit that is unknown. A shortcoming of the present technique based on reflex velocity measurements is that it can not be applied to stars with a deficit of absorption lines necessary to determine radial velocities. It is very sensitive to close massive planet systems (independent upon the distance) but does not provide good data for systems with a larger star–planet separation.

Astrometric method, on the contrary, is very useful in order to obtain complementary characteristics of the known planet systems; it may be applied to study systems with a comparatively large star to planet separation. The precision of the modern ground-based optical astrometry, however, is insufficient and limited to about $\simeq 1$ mas/1 hr. This error set is caused by various effects, the most important of which is the atmospheric image motion. This limitation is not fundamental and caused by the present technique of observations. I suggest a new unconventional technique of differential astrometric observations that offers a substantial improvement in the accuracy (a few decades effect in respect to the variance) by means of an enhanced filtration of atmospheric image motion spectrum. A new approach to the narrow-field astrometry makes it to be quite a powerful tool for investigations in different fields of astronomy, in particular, for studies of exoplanet systems. A concept of the method that allows one to reach microarcsecond accuracies by observations with ground-based very large telescopes, is described in brief below.

THE CONCEPT

An enhanced attenuation of an image motion spectrum is achieved by introducing two principal modifications to the classic method of differential measurements. The first is some special apodizing mask (light transmission modulation) applied to the telescope entrance pupil

$$P(r) = (1 - 4r^2/D^2)^{(\nu-3)/2}, \tag{1}$$

where D is diameter of the telescope and $\nu \geq 3$ is an odd integer. In the frequency domain, such a change causes strong suppression of high-frequency fluctuations of the random phase distortions and corresponds to a filter $Y(q)$ whose response at high spatial frequencies q goes as $q^{-\nu}$. Note that normally ν is equal to 3 (no apodization).

Suppression of image motion spectrum at low-frequencies is realized with the second modification, or introduction of a new measured quantity

$$W = N^{-1} \sum_{i=1}^{N} a_i(\bar{x}_0 - \bar{x}_i) \tag{2}$$

formed as a weighted sum of measured differences of the target $\bar{x}_0$ and i-th reference star cartesian (standard) coordinate $\bar{x}_i$; N is the number of stars and a_i are weights that satisfy a normalizing condition

$$\sum_{i=1}^{N} a_i = N. \tag{3}$$

Expression for the y-axis is in the similar form.

Weights a_i are found from a system of equations

$$\begin{aligned} &\sum a_i = N, \\ &\sum a_i(x_i - x_0)^\alpha (y_i - y_0)^\beta = 0, \quad \alpha + \beta = 1 \ldots \tfrac{k}{2} - 1, \end{aligned} \tag{4}$$

allowing to eliminate some first low modes of random phase fluctuations. Here $k \leq k(k+2)/8$ is an optional even integer, α and β are positive integers, and weights a_i can be negative.

It can be shown that introduction of the quantity W instead of the normally non-weighted differential positions is equivalent to a filter $Q(q)$ with a frequency response $\sim q^k$ at short q. For conventional differential technique k is equal to 2.

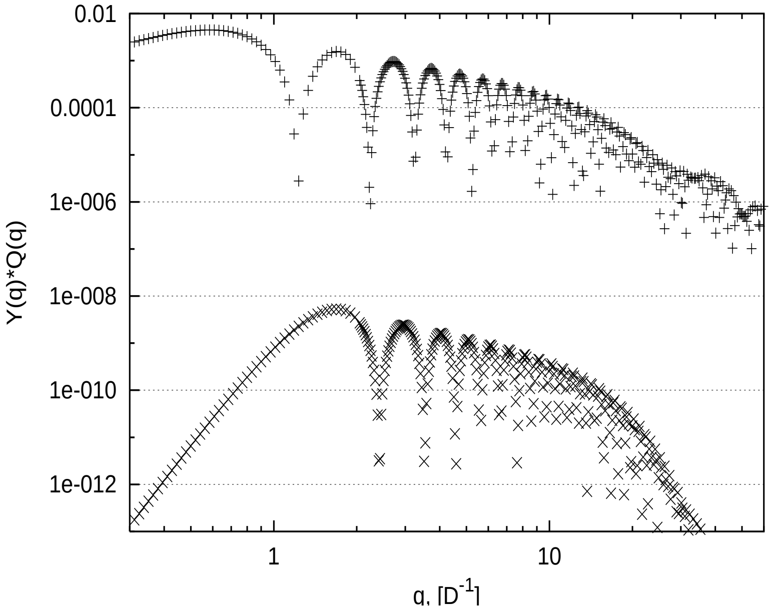

Figure 1. Combined filter $Y(q)Q(q)$ response for the parameters given in the text

The combined filtering effect of the apodization and of introduction quantity W is shown in Fig. 1, where we compare the product of $Y(q)Q(q)$ for the classic ($k = 2$, $\nu = 3$, upper curve) and a new ($\nu = 9$, $k = 8$, lower curve) methods. The plot is given for a 100 m telescope, $1'$ effective angular size of reference field, and a single turbulent layer at 20 km altitude. In this case, a gain in attenuation of image motion spectrum reaches 10^5.

Equations (4) are used for reduction of the variance Δ^2 of atmospheric image motion. However, the total error of the measured quantity W also includes the image-centroiding error component σ_{ph} caused by a Poisson photon noise in star images. It is minimized by a condition

$$\sum a_i^2/n_i = \min \tag{5}$$

that is added to the system of equations (4); n_i is number of photons detected from the i-th star.

ASTROMETRIC PERFORMANCE OF VERY LARGE GROUND-BASED TELESCOPES

Realization of the milliarcsecond accuracy requires a good elimination of various noise sources related to optical aberrations, pixelization effects (especially for small images produced by adaptive telescopes), photon noise in star images, differential chromatic refraction (DCR), *etc.* In particular, there are very intricate problems caused by a DCR that stretches the star images into colored strips. The amplitude of the DCR effect depends on zenith distance, air temperature and pressure, spectral band, and star colours. Thus, two rays with wavelengths of 500 and 600 nm coming from a star at a zenith distance of 30° are imaged with a separation of about 180 mas along the vertical direction. Fortunately, in the proper motion studies, the DCR effect is residual and essentially weak since star motions are to be found from residuals of differential star positions at two epochs.

Evaluating astrometric performance of very large ground-based telescopes, we restricted the error budget by two components: the atmospheric image motion and photon noise with variances Δ^2 and σ^2_{ph}, respectively. A contribution from both effects was estimated as a function of the angular field size R for sky star densities near the Galactic plane and at the Pole. The analysis was restricted to a case of a future extremely large 100 m telescope and a modern 10 m class telescope. A vertical profile of $C_n^2(h)$ was obtained by averaging data from the three Chilean sites: Cerro Tololo, Cerro Paranal, and San Pedro Martir.

Estimates for a 100 m telescope assume $FWHM = 0.1''$ achievable with low-order adaptive optics, and for a 10 m telescope a quite conservative $FWHM = 0.4''$ was adopted. We assumed that observations are obtained in zenith, in R band, with a CCD quantum efficiency of 0.85, transmission of optics 0.8 and of atmosphere 0.9. Total light $\sum n_i$ of the star field was estimated using the Galaxy model from [1]. Stars fainter than $V = 23$ mag were not considered as giving low light signal. To obtain the more robust results, the expected star number in each 1 mag bin was rounded (truncated) to a smaller integer. This procedure trimmed the bright end of stellar magnitudes owing to which very narrow star fields were formed largely by faintest stars. Photon noise in the target star image was always neglected.

Results of simulations are shown in Fig. 2. Each curve starts at small R (star field radius) where only low ($k = 2$ or $k = 4$) orders can be realized with 1–3 reference stars; right ends of curves correspond to $k = 12$ where fields contain $N \sim 100$–1000 stars (except for a case of a telescope with $D = 10$ m operating at the Pole). From Fig. 2 we may conclude that, by a high star density, an enough good accuracy is expected with a field size R of 0.4–1′; the frame is to be at least $R = 2'$ wide at polar regions.

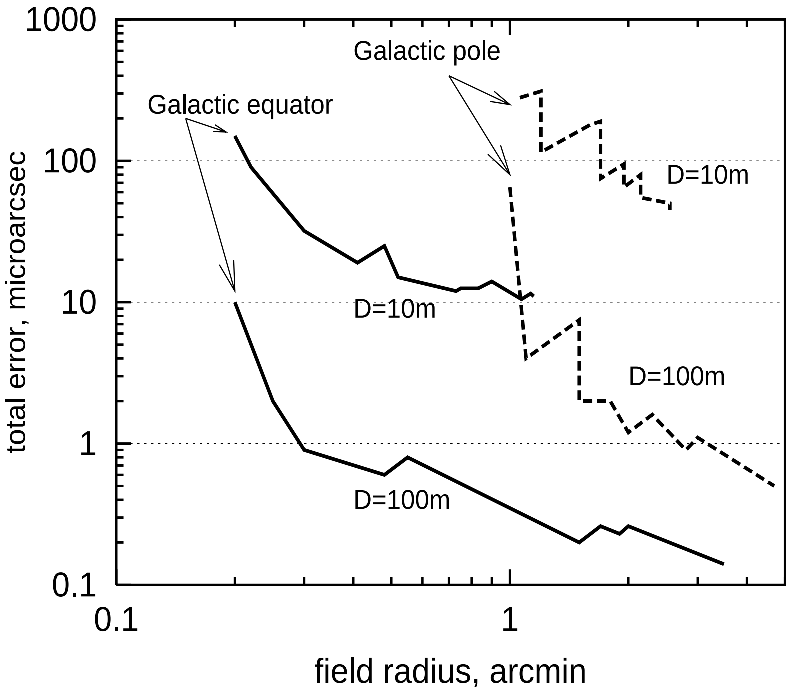

Figure 2. Total error of differential observations as a function of stellar field radius R for the 10 and 100 m telescopes by 10 min exposure; solid lines – at the Galactic plane ($l = 0°$, $b = 20°$); dashed – at the Galactic Pole. Apodization: $\nu = k + 1$ for a 100 m telescope; no apodization ($\nu = 3$) for a 10 m telescope

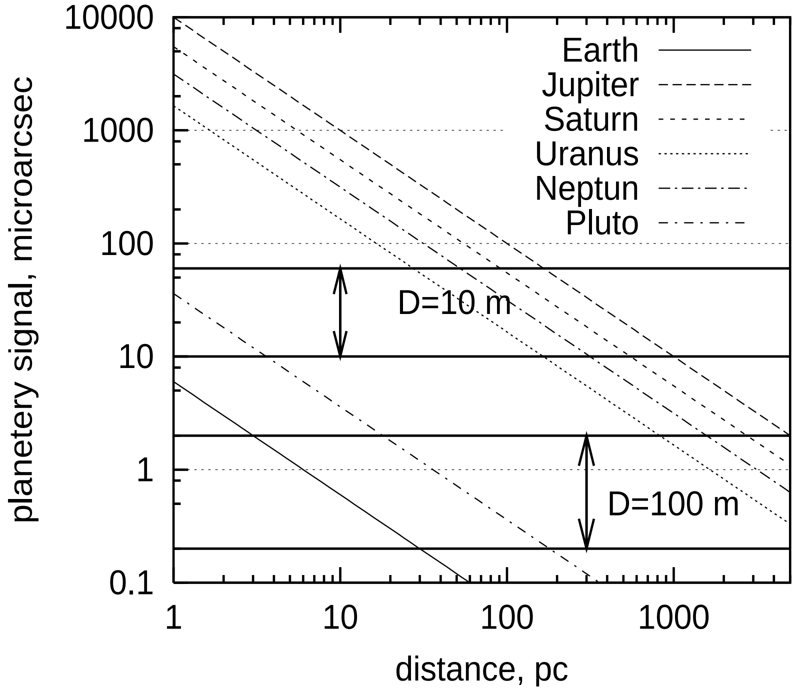

Figure 3. Astrometric planetery signal and astrometric accuracy in a 10 min exposure of the telescopes with diameters of 10 and 100 m at the Galactic plane (lower limits) and at the Pole (upper limit)

With the above assumptions on *FWHM*, and an optimal field size, the expected error of ground-based observations with a 10 m telescope (no apodization) varies from 10 to 60 μas in a 10 min exposure depending on sky star density. For a 100 m telescope, this estimate is 0.2 to 2 μas.

EXTRASOLAR PLANETS' DETECTION LIMITS

Astrometric method of extrasolar planets detection requires measuring a star reflex motion around the planetary system mass center. The planetary signal produced in the solar-type star system is shown in Fig. 3 as a function of distance. The astrometric accuracies for the cases discussed in this report are also shown. The plot shows that 10 m class telescopes are sufficiently precise to perform a search for extrasolar planets. Depending on the sky star density, the application of a new method of differential measurements will allow one to detect Jupiter-type planets at 200 to 1000 pc even with a moderate 10 min exposure.

[1] *Bahcall J., Soneira R.* Models of the Galaxy and the predicted star counts // Astron. and Astrophys. Suppl. Ser.–1980.–**44**, N 2.–P. 73–110.