THE DISTRIBUTION OF TEMPERATURE IN AN ACTIVE REGION FOR A COMET WITH KNOWN PARAMETERS OF ROTATION AND ORBIT

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We calculated the temperature regime of an active region considered as a conical hole in dust layer. All the calculations are carried out for the case when the season effects are distinctly expressed. The temperature is defined from the geometrical parameters of the active region separately for ice bottom and dust walls. It is shown that dust walls are cooled very slowly for the small vertex angles of the crater structure. The temperature of the ice bottom practically does not change over the whole period of rotation, because it is retained at an approximately constant level by concentration of reradiated energy from the dust walls heated by the Sun.

INTRODUCTION

Since VEGA–GIOTTO missions [5] cometary nuclei are surely considered as non-spherical bodies with inhomogeneous surfaces. It would be unreasonable to persist on the similarity of every cometary nucleus to that of P/Halley, but it is impossible to ignore the fact that outgassing of volatiles may be concentrated in a small number of active zones [5]. It is necessary to replace classic idealized models of a cometary nucleus by more realistic ones. The subject of the present paper is to describe heat transfer and sublimation in an active region [1, 2]. There are different ways of craterization: non-catastrophic evolutionary processes and some explosion mechanisms. The different reasons of the catastrophic craterization are considered: a meteoroid impact, fast sublimation of a species more volatile than water ice, fast crystallization of amorphous ice, recombination of hydrated ions [6]. A simple model of an active region that looks like a conic hole (crater) in the dust mantle of a nucleus is proposed here. The crater has bare ice at the bottom and dust on its side walls (Fig. 1).

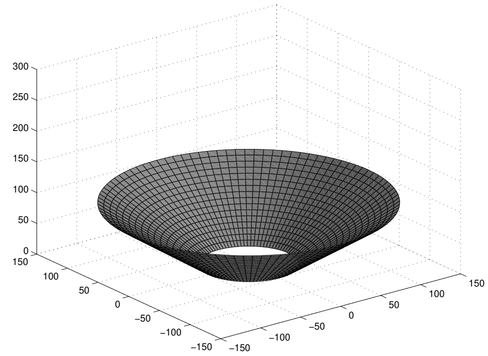

Figure 1. A discrete grid for the numerical solution of the system of integral equation (heat balance on the ice bottom and dust walls of crater)

THE MODEL DESCRIPTION

The temperature regime of an active region on a cometary nucleus are calculated. All the calculations are carried out for the case when the season effects (for different angles between the equator of the nucleus and its orbital plane) are distinctly expressed. The essential assumptions are as follows:

1. The nucleus of the comet is spherical and homogeneous.
2. An active region looks like a conic hole (crater) in the dust mantle on the nucleus. The crater has bare ice on the bottom and dust on its side walls.
3. The temperature distribution is calculated taking into account two ways of heating: directly from the Sun and by re-emission in infrared from the walls of the crater. The temperature distribution over the crater is obtained for different geometrical parameters sets and orbital parameters.
4. The study of temperature regime are carried out for the case when the season effects are distinctly expressed.The different angles between the equator of the nucleus and its orbital plane have value: $\psi = 0,\ 30,\ 60,\ 90°$.
5. A discrete grid (see Fig. 1) was constructed for numerical solving of the system of integral equation (heat balance on the ice bottom and dust walls of the crater). The bottom and lateral surface of the cone were divided into small elements of equal areas.

THE MODEL EQUATIONS

The heat balance of an active region is defined by the local zenith distance of the Sun and the nucleographic latitude of the active region. The local zenith distance z is given by the formula:

$$\left\{ \begin{array}{lcl} \cos z_{bi} & = & \vec{n}_{sun} \cdot \vec{n}_{bi}, \\ \cos z_{si} & = & \vec{n}_{sun} \cdot \vec{n}_{si}, \end{array} \right. \tag{1}$$

where n_{sun} is the unit vector of the direction of the solar rays that is given by the relationship:

$$n_{sun} = [-\cos\phi \sin\psi + \sin\phi \cos\psi \cos(t), -\cos\phi\cos(t), \sin\phi\sin\psi + \cos\phi\cos\psi\cos(t)], \tag{2}$$

where ϕ is the angle of the incident solar radiation on the cometary nucleus, ψ is the slope angle of the orbital plane of the comet, t is the hour angle.

The algebraic equations were used to calculate the distribution of the temperature over the local active area. For the bottom of the crater:

$$T_{bi} = \frac{B}{A - \ln\left\{ \dfrac{\sqrt{2\pi mkT_{bi}}}{L} \left[\dfrac{q(1-A_{ice})\cos(z_{bi})}{r_{com}^2} - (1-A_{ice})\sigma T_{bi}^4 + \sum\limits_{j=1}^{N_s} \dfrac{\sigma(1-A_{dust})T_{sj}^4}{\pi} K_{ij} ds \right] \right\}}, \tag{3}$$

where q is the solar constant, r_{com} is the heliocentric distance of the comet, A_{ice}, A_{dust} are the albedo of the surface of the nucleus, T_{bi} and T_{sj} are the temperatures of water ice and dust, respectively, σ is the Stefan–Boltzmann constant, $m_{H_2O} = 2.988 \cdot 10^{-26}$ kg denotes the molecular mass of water, $k = 1.38 \cdot 10^{-23} JK^{-1}$ is the Boltzmann constant, L is the energy of sublimation per one molecule. For the side of the crater structure one can write:

$$T_{si} = \left\{ \frac{1}{\sigma} \left[\frac{q(1-A_{dust})\cos(z_{si})}{r_{com}^2} - 2\alpha k(T_{si} - T_{gas}) + \sum_{j \neq i}^{N_s} \frac{\sigma}{\pi}(1-A_{dust})T_{sj}^4 K_{ij} ds \right] \right\}^{1/4}. \tag{4}$$

The gas inside the cone has the temperature T_{gas}. It is assumed that the gas receives the energy from the wall, proportionally to the temperature difference between the gas and the wall. The efficiency of heat exchange is determined by the coefficient α.

The effectiveness of heat exchange in Eqs. (2) and (3) is determined by the matrix K_{ij} given by the relationship:

$$K_{ij} = \frac{\cos\beta_i \cos\beta_j}{l_{ij}^2} \Delta S, \tag{5}$$

where ΔS is the area of an element of the surface. In turn, $\cos(\beta)$ is determined by a dot product of two unit vectors relationship [2]:

$$\left\{ \begin{array}{lcl} \cos\beta_i & = & \vec{n}_i \cdot \vec{l}_{ij}^{\,0}, \\ \cos\beta_j & = & \vec{n}_j \cdot \vec{l}_{ij}^{\,0}, \end{array} \right. \tag{6}$$

here n is a unit vector perpendicular to the centre of the i-element of the dusty wall:

$$n = [n_{xi}, n_{yi}, n_{zi}] \tag{7}$$

and

$$\vec{l}_{ij}^{\,0} = \frac{\{x_{bi} - x_{sj}, y_{bi} - y_{sj}, z_{bi} - z_{sj}\}}{l_{ij}}. \tag{8}$$

The length of the l-vector is

$$\vec{l}_{ij} = \sqrt{(x_{bi} - x_{sj})^2 + (y_{bi} - y_{sj})^2 + (z_{bi} - z_{sj})^2}\,, \tag{9}$$

where x, y, z are the coordinates of two points: one on the side wall (subscripted by s_j) and the second at the bottom (subscripted by b_i) of the crater.

The algebraic system for calculation of the temperature can be solved by an iteration method using the discrete grid shown in Fig. 1. It occurred that this iteration process converged quickly. When calculating the heat balance in an arbitrary point of the crater on a rotating nucleus one has to check if this point is illuminated or shadowed [6].

The distribution of the temperature on conical craters was calculated for the set of parameters that is given in Table 1.

Table 1. Geometrical parameters for models of active area on the surface of a cometary nucleus

r, cm	h, cm	$\varkappa$, degree	ϕ, degree	ψ, degree	t, degree
50	100...300	45	60	0, 30, 60, 90	0...90
50...110	100	45	60	0, 30, 60, 90	0...90
50	100	30...60	60	0, 30, 60, 90	0...90

We applied our model results to Comet Halley. It is obtained that our model explains the absence of a drastic temperature and sublimation variation on the cometary nucleus.

EFFECTS OF CRATERS

It is interesting that the dust walls are cooled very slowly for the small vertex angles of the crater structure. The temperature of the ice bottom retains practically unchanged over the whole period of rotation because of income of reradiated energy from the dust walls heated by the Sun. Sometimes, ice may sublimate even from the night hemisphere due to infrared radiation from dusty side of the crater. Figures 2, 3, and 4 show the results of calculation of temperature of local active area on the cometary nucleus for different angles between the equator of the nucleus and its orbital plane.

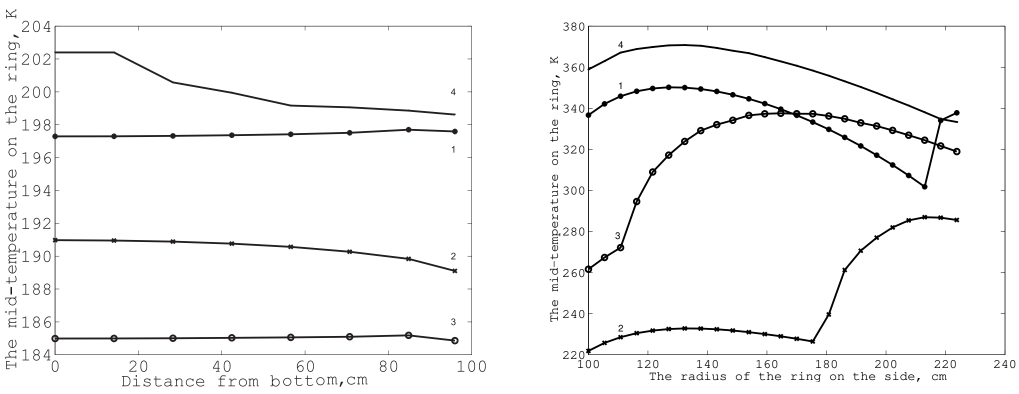

Figure 2. The temperature regime of the crater structure for the geometrical parameters of the cone: $h = 300$ cm, $r = 100$ cm, $\varkappa = \pi/8$. The lines 1, 2, 3, 4 correspond to $\psi = 0,\ 30,\ 60,\ 90^\circ$

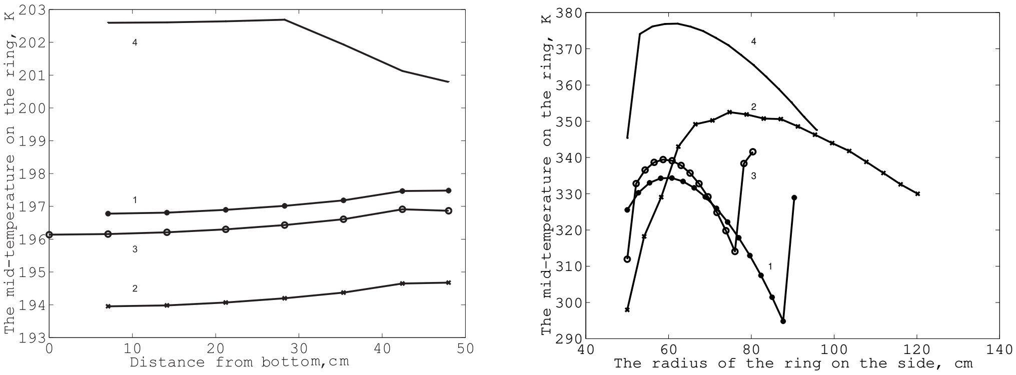

Figure 3. The temperature regime of the crater structure for the geometrical parameters of the cone: $h = 100$ cm, $r = 50$ cm, $\varkappa = \pi/8$. The lines 1, 2, 3, 4 correspond to $\psi = 0,\ 30,\ 60,\ 90^\circ$

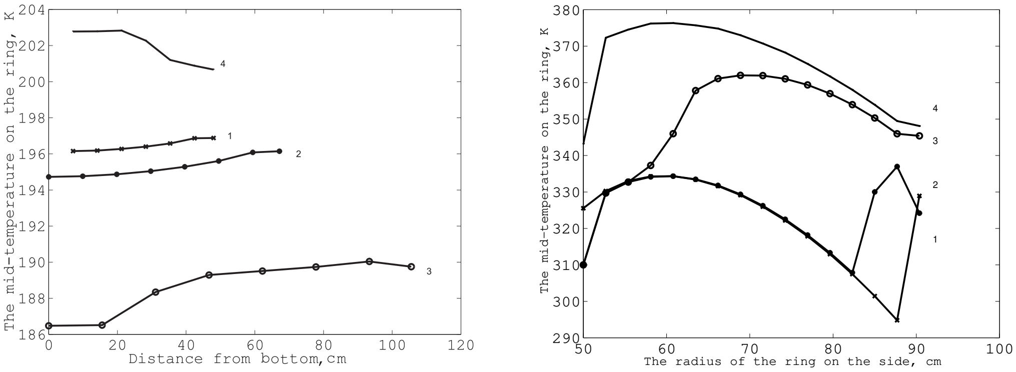

Figure 4. The temperature regime of the crater structure for the geometrical parameters of the cone: $h = 100$ cm, $r = 110$ cm, $\varkappa = \pi/8$. The lines 1, 2, 3, 4 correspond to $\psi = 0,\ 30,\ 60,\ 90^\circ$

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[1] *Ivanova O. V., Shulman L. M., et al.* A model of an active region on the surface of a cometary nucleus // Earth, Moon and Planet.–2002.–**90**.– P. 249–257.

[2] *Ivanova O. V., Shulman L. M.* The effect of amplification of sublimation from an active region on the surface of a rotating cometary nucleus // Kinematics and Physics of Celestial Bodies.–2003.–**19**, N 6.– P. 249–257.

[3] *Mekler Y., Prialnik D., Podolak M.* Evaporation from a porous cometary nucleus // Astrophys. J.–2003.–**356**, N 2.–P. 682–686.

[4] *Prialnic D., Bar-Nun A.* The formation of a permanent dust mantle and its effect on cometary activity // Icarus.–1988.–**74**.– P. 272–283.

[5] *Sagdeev R. Z.* The Comet Halley Dust Environment from the VEGA SP-2 Detectors // Nature.–1986.– **321**, N 6067.–P. 259–261.

[6] *Shulman L. M., Ivanova O. V.* Formation of the crater structure on the surface of a cometary nucleus by meteoroid impact // Kinematics and Physics of Celestial Bodies.–2003.–**19**, N 4.–P. 367–373.