HAZARD OF COLLISIONS IN GEOSTATIONARY RING

V. U. Klimyk[1], **L. M. Kizyun**[2]

[1] *Space Research Laboratory, Uzhhorod National University*
2a Daleka Str., 88000 Uzhhorod, Ukraine
e-mail: space@univ.uzhgorod.ua

[2] *Main Astronomical Observatory, NAS of Ukraine*
27 Akademika Zabolotnoho Str., 03680 Kyiv, Ukraine
e-mail: kisyun@mao.kiev.ua

A probability of catalogue satellites collision in the geostationary orbit is calculated. The direct method is applied: the dangerous rendezvouses of the satellites are determined, and a probability of collisions under the dangerous rendezvouses is calculated. A density of distribution of the uncontrolled satellites coordinates and their orbital elements is obtained. A density of distribution of the directions and values of the relative velocities under dangerous rendezvouses of the geosynchronous satellites is also calculated.

DISTRIBUTION OF UNCONTROLLED OBJECTS IN GEOSTATIONARY RING

To obtain a distribution of uncontrolled geostationary objects (UGO) we used the catalogue of orbital elements [3]. The orbital elements at 0.01 day interval were obtained using the analytical theory of UGO motion [1], taking into account the Earth's nonsphericity, the Moon and the Sun attraction. Based on the UGO orbital elements the coordinates λ, φ, r_0 (geographic longitude λ, latitude φ (or declination δ_0), and geocentric distance r_0), determined for all UGO positions in the Earth coordinate system were calculated at 0.01 day time interval. In a such way the instantaneous distribution of all UGO was obtained at 1990–2004 time interval. Then the mean values of these coordinates at a long time interval were calculated. The accuracy of the UGO calculating position is 1° at the 15 years time interval.

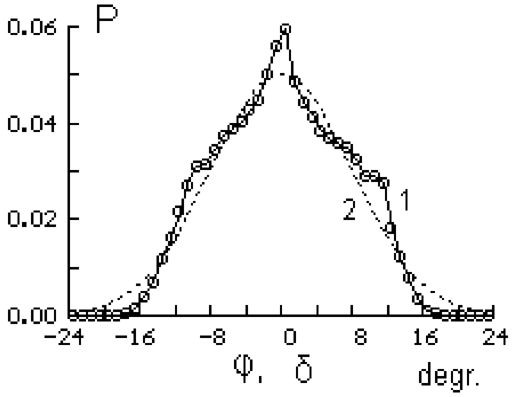

Figure 1. Density function of δ for UGO (1), normal density function (2) and φ density function (isolated points)

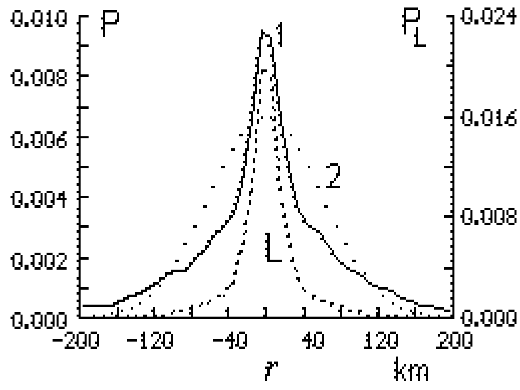

Figure 2. Density function of r for UGO (1) and for geostationary objects in a libration orbit (L) and density of normal distribution (2)

The density function of declination δ calculated using the data obtained by above-mentioned method is shown in Fig. 1 (the curve 1), $\delta = \delta_0 - \delta_s$, where δ_0 is UGO declination, δ_s is its mean value (for Uzhhorod latitude $\delta_s = -6.2°$). The function is calculated for Uzhhorod using the points $(\delta_i \frac{m_i}{n \cdot \Delta\delta})$, where $\Delta\delta$ is the length of splitting interval along δ axis ($\Delta\delta = 1°$, that is about 736 km in orbit), δ_i is the central point of an i-interval,

m_i is the number of δ values falling in i-interval, n is the number of all δ values. The density function of δ is symmetric relatively δ_s, but this distribution is not Gaussian curve (dotted curve 2 in Fig. 1).

A distribution of UGO geographical latitude φ doesn't almost differ from δ distribution. In Fig. 1 the value of φ density function for UGO is submitted by isolated points.

The density function of r is shown in Fig. 2 (the curve 1), $r = r_0 - r_s$, where $r_s = 42164$ km. The curve is obtained using the points from 10 km splitting interval. The density function of r is not Gaussian curve too. The objects in a libration orbit change the form of the distribution curve. Almost all satellites of this type are situated at $r_s \pm$ 30 km interval. A density function of r for such UGO is shown in Fig. 2 (the curve L). P_L axis for this curve is on the right.

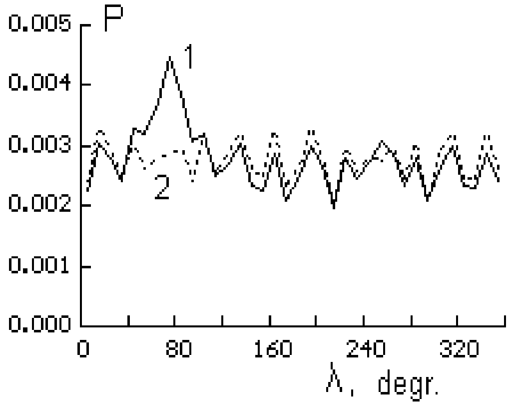

Figure 3. Density function for UGO subsatellite longitude λ (1) and for geostationary objects in a drift orbit (2)

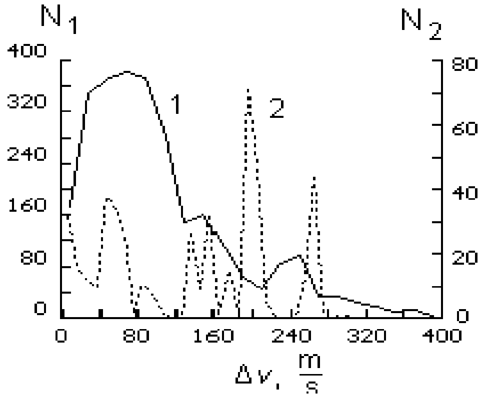

Figure 4. Distribution of relative velocities of two UGO (1) and relative velocities of UGO and CGS (2) at dangerous rendezvouses

The density function of UGO subsatellite longitude λ is shown in Fig. 3 (the curve 1). A splitting interval is 10°. 55 000 points were used to determine this distribution. The curve maximum is founded near the libration point with a longitude $\lambda = 75°$. It depends greatly on many objects of ℓ_1 type in a libration orbit among this distribution. By dashed line in Fig. 3 (the curve 2) is shown a density function of subsatellite longitude only for geostationary objects (GO) in a drift orbit. 60 000 points are used.

Let us consider a spatial (in the coordinate plane $\varphi O r$) frequency function of distribution $P(\varphi,\, r)$ for UGO taken from the catalogue. This function may be presented as an implicit function by ellipse equation:

$$\frac{(736 \cdot \varphi)^2}{[-29.83 \cdot P(\varphi,r) \cdot k + 10953]^2} + \frac{(r - 42164)^2}{[0.001659 \cdot P(\varphi,r)^2 \cdot k^2 - 0.8621 \cdot P(\varphi,r) \cdot k + 121.1]^2} = 1, \tag{1}$$

where the value $P(\varphi,r)$ in denominator is the density function obtained as: $P(\varphi,r) = \dfrac{m_{\varphi r}}{\Delta\varphi \cdot \Delta r \cdot n}$. Here $m_{\varphi r}$ is the number of object positions along all zone of UGO in $\Delta\varphi \cdot \Delta r$ splitting field ($\Delta\varphi \cdot \Delta r = 5000$ km^2), n is the total number of objects positions, using for calculations ($n = 92207$). In (1) $k = \Delta\varphi \cdot \Delta r \cdot n \approx 46 \cdot 10^7$. If $k = 1$, the value $P(\varphi,r)$ transforms to the number of objects positions along all GO zone in $\Delta\varphi \cdot \Delta r$ area.

The dependence of ellipse semi-major of axis and $P(\varphi,r)$ in (1), is linear with a high correlation coefficient $\rho = -0.995$. The dependence of ellipse semi-minor of axis and $P(\varphi,r)$ was estimated by parabola with a range of semi-minor of axis from 10 km up to 120 km. An implicit function (1) is convenient to use as an inverse function: to set a probability $P(\varphi,r)$ or one of ellipse semi-axes; to receive an ellipse of identical probability $P(\varphi,r)$, that is the coordinates φ, r with a probability of collision $P(\varphi,r)$.

DISTRIBUTION OF UGO RELATIVE MOTION PARAMETERS AT DANGEROUS RENDEZVOUSES

A research problem of GO collisions hazard is in fact the task of the calculation of the probability of their collision at some time interval. The probability of collision and appropriate cumulative distribution function of the Earth's satellites at low heights (up to 3000 km) depending on different conditions and parameters (altitude above the Earth, the satellite size, and the object geographical latitude and longitude) is rather deeply investigated [2]. We used this direct method for investigation of the probability of UGO collisions with a some simplification. For this aim all UGO positions at the certain moments of time with a rather small step Δt at two-year interval were calculated. It is enough to select $\Delta t = 0.01$ of day as the UGO position in the Earth coordinate system slowly changes.

For each instantaneous UGO distribution we fix the moments of dangerous rendezvouses every two objects. Under the dangerous rendezvous we understand the rendezvous of two objects at a distance does not exceeding 100 km. The calculated rendezvouses parameters are transferred by appropriate program in a special archives. These parameters are: the moment of minimum objects rendezvous, the distance between the objects in this moment, the international number and type of two approaching objects, their coordinates (geographical longitude and latitude, geocentric distance) and the rates of their change. The distances between the objects at the moment of rendezvous can be used only as statistical estimations, because the errors of orbits determination exceed the UGO distances at dangerous rendezvouses.

To determine the parameters of UGO relative motion at the dangerous rendezvouses we used the moving rectangular coordinate system and called it a local system. The origin of this coordinate system coincides with O position one of the two approaching objects, for example with a first object. The axis z is directed from the Earth's center along a geocentric radius vector $\vec{r}$ of the first UGO, the x axis is perpendicular to z and directed towards the west, the y axis is perpendicular to the coordinate plane xOz and directed to North Pole of the Earth.

Such coordinate system is connected with geographical coordinates λ, φ, and r by approximate formulas: $x = (\lambda - \lambda_1) \cdot r$, $y = (\varphi - \varphi_1) \cdot r$, $z = r - r_1$, where φ_1, λ_1, r_1 are geographical coordinates and absolute value of radius vector of the first object. Vector of the relative speed $\Delta\vec{\nu} = \vec{\nu_2} - \vec{\nu_1}$ is a main motion parameter at dangerous rendezvous.

Let us consider two scalar parameters: a value $\Delta\nu$ of relative velocity and a slope angle ψ between the x axis and the velocity direction $\Delta\vec{r}$. The value of a slope angle is determined as:

$$\psi = \left| \arctan \frac{\sqrt{(\Delta y)^2 + (\Delta z)^2}}{\Delta x} \right|, \tag{2}$$

where Δx, Δy, Δz are the coordinates of the second object in a local coordinate system. It is obviously that $0^\circ \le \psi \le 90^\circ$.

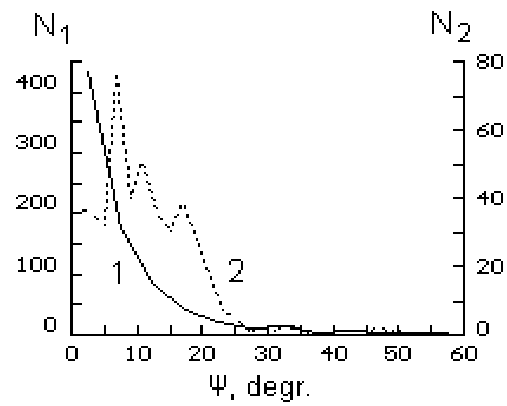

Figure 5. Distribution of ψ angles for two UGO (1), for UGO and CGS (2) at dangerous rendezvouses

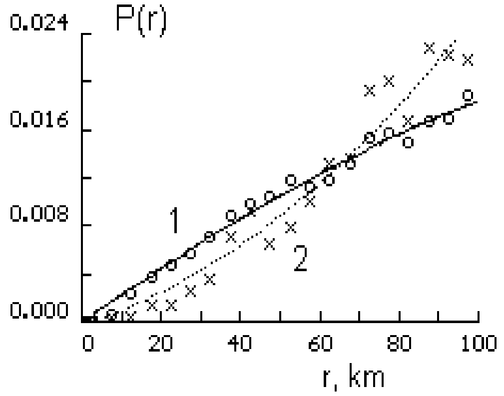

Figure 6. Density function of the distances between two UGO at dangerous rendezvouses (1) and between UGO and CGS (2)

The distribution of the relative velocities $\Delta\nu$ and the angles ψ at the dangerous rendezvouses of two UGO using the archives of dangerous rendezvouses for 1990 is shown in Fig. 4 and Fig. 5 (the curve 1). N_1 in Fig. 4 and Fig. 5 is the number of rendezvouses at 20 m/s velocity interval and at 5°-angle interval, respectively. It is seen that the values of relative velocities at dangerous rendezvouses are situated always in the limits mainly of zero up to 300 m/s. It is more than half of all velocity values (Fig. 4, the curve 1) are taken place in the area from 0 up to 120 m/s with a 20–80 m/s maximum. Some small maximums are near 160 m/s and 240 m/s. The directions of the approaching UGO are almost parallel: the angles ψ between the directions of the velocity vectors of two UGO in the most cases do not exceed 20° (Fig. 5). The distribution curve of ψ angles changes smoothly.

PROBABILITY OF UGO COLLISIONS

The value of a density function $P(r)$ of the minimum distances at dangerous rendezvouses of UGO is calculated (Fig. 6, shown by circles) as $P(r) = \dfrac{m}{n \cdot \Delta r}$. For this aim the data of archives of dangerous rendezvouses are used. Here Δr is the grouping interval, m is the number of dangerous UGO rendezvouses, the minimum distances between which are situated in the given interval Δr, n is the number of all dangerous rendezvouses.

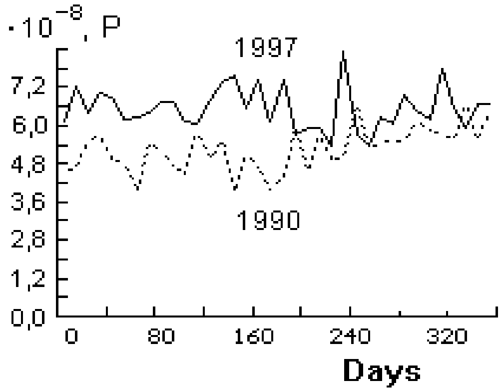

Figure 7. Change of probability of UGO collisions during one day at the one year interval

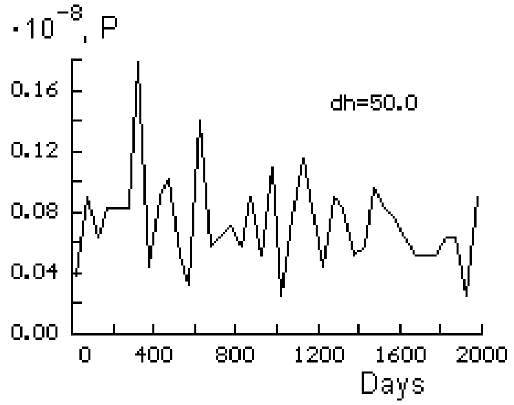

Figure 8. Change of probability of collision CGS with UGO during one day at 5.5 year interval beginning from 1990

There are about 2500–3000 dangerous rendezvouses of two UGO. Using the two years archives data of 5753 dangerous rendezvouses, during one year the approximate frequency function of the distances distribution between UGO at dangerous rendezvouses is calculated (curve 1 in Fig. 6) as:

$$P(r) = 2.30923 \cdot 10^{-4}\, r - 4.099228 \cdot 10^{-7}\, r^2. \tag{3}$$

The mean GO size according to catalogue of 1996 is 8.15 m. Rms residual is about ±8.30 m.

We received in our calculations that the mean size of GO is 8 m. A probability of UGO collisions at dangerous rendezvouses presented in the catalogue is $P_z = \int_0^{0.008} P(r)\, dr = 7.5 \cdot 10^{-9}$, where the frequency function from (3) is used. Under collision we understand such rendezvous of two UGO, that the distance between their centers is less than 8 m.

In Fig. 7 the change of probability of UGO collisions during one day at one year interval is shown. Two curves show the change of probability of collision for 1990 and 1997. The probability was calculated using the mean values of number dangerous rendezvouses during 10 days. The mean probability of collision has increased during one year from $2.0 \cdot 10^{-5}$ in 1990 up to $2.5 \cdot 10^{-5}$ in 1997. The cause of such change is the increasing of UGO number in 1997.

In Table 1 a frequency $P = \dfrac{m}{n}$ of participation of different types UGO in dangerous rendezvouses is submitted. The archives data for 1990 are used. Here m is the number of UGO selected type from the archives of dangerous rendezvouses, n is the number of all UGO from archives. It is seen that UGO in a drift orbit is much more often approach at dangerous distances than UGO in a libration orbit. The reason of this is much more number of UGO in a drift orbit in catalogue.

Let us give an estimation of frequency of UGO rendezvouses, which does not depend on number of selected type objects in the catalogue. Enter the value $Q = \dfrac{P}{P'}$, where $P' = \dfrac{r}{k}$ is the frequency of selected type UGO in catalogue (k is the number of all UGO in catalogue, r is a number of selected type UGO in catalogue). It is seen from Table 1 that for UGO of different types Q is near upon a size, that is the participation of UGO in rendezvouses is a little depends on objects type. Using the archives data one may be seen that the rendezvouses of different types objects are taken place the most frequency (with frequency 0.87). Only 0.13 of all cases are the objects of the same type.

Table 1. Distribution a number of rendezvouses of UGO different types using the data of dangerous rendezvouses archives for 1990

UGO type	ℓ_1	ℓ_2	ℓ_3	d_1	d_2	d_3
Frequency $P = m/n$ in archives	0.134	0.056	0.018	0.407	0.191	0.193
The ratio of frequencies P/P$'$	0.90	0.96	0.73	1.17	0.72	1.25

HAZARD OF COLLISION OF UNCONTROLLED OBJECTS WITH CONTROLLED SATELLITES

On finishing its resource a controlled geostationary satellite (CGS) is transferred outside CGS zone to geosynchronous orbit and goes as UGO. Therefore, the hazard of UGO collision with CGS is considerably less than in the case of two UGO rendezvouses. About 90 dangerous rendezvouses UGO with CGS is taken place during one year; it is equal only about 3 per cent of all dangerous UGO rendezvouses.

The density function of distances between UGO and CGS at dangerous rendezvous is shown in Fig. 6 (the curve 2). It is obtained from the least squares adjustment by points $P(r) = \dfrac{m}{n \cdot \Delta r}$ (in Fig. 6 is designed by crosses). The grouping interval is $\Delta r = 5$ m, m is the number of dangerous rendezvouses of UGO with CGS a minimum distance between which is situated in the selected interval Δr, n is the number of all dangerous rendezvouses UGO with CGS. Density function (the curve 2 in Fig. 6) may be presented as polynomial:

$$P'(r) = 1.01003 \cdot 10^{-4}\, r + 15.858612 \cdot 10^{-7}\, r^2. \tag{4}$$

The curves 1 and 2 in Fig. 6 and so the functions (3) and (4) is differ so, that at the same number of dangerous rendezvouses the probability of two UGO collision is much more than probability of collision UGO with CGS.

Really, $P'_z = \int_0^{0.008} P'(r)\, dr = 3.2 \cdot 10^{-9}$, where $P'(r)$ is a function (4). The probability of collision UGO with CGS at dangerous rendezvous P'_z is 2.34 times less than probability P_z in (3). As a result it is obtained that the probability of UGO collision with CGS in geosynchronous orbit is 75 time less a probability of two UGO collision.

The change of probability of UGO collision with CGS during one day at 5.5 year interval is shown in Fig. 8. The probability was calculated as a mean value at 50 day interval. The change of probability is very nonuniform.

The next UGO are the most dangerous: 78113D$\,(d_1)$ (121 rendezvouses with CGS during 1990–1995), 67066G$\,(d_3)$ (45 rendezvouses for six years), 86007A$\,(d_1)$ (27 rendezvouses), 87109A$\,(d_1)$ (27 rendezvouses), 75011F$\,(d_1)$ (25 rendezvouses), 77034C$\,(d_1)$ (23 rendezvouses), 70055A$\,(d_3)$ (19 rendezvouses). Only 121 uncontrolled geostationary objects can approach at dangerous distance to CGS.

The origin of local rectangular coordinate system $x = (\lambda - \lambda_1) \cdot r$, $y = (\varphi - \varphi_1) \cdot r$, $z = r - r_1$ is placed in the center of CGS. The slope angle ψ between the directions of UGO velocities and CGS ones at dangerous rendezvous is calculated as (2).

In Figs. 4 and 5 (curve 2, with axes for them on the right) the distributions of relative velocities of $\Delta\nu$ and angles ψ is shown using the data of archives dangerous rendezvouses of UGO with CGS for 1990–1995. N_2 is the number of rendezvouses at 10 m/s velocity interval in Fig. 4 , N_2 is the number of rendezvouses at 2° angle interval in Fig. 5. It is seen from Fig. 4 that the values of relative velocities at dangerous rendezvouses are situated within the limits of zero up to 300 m/s as well as in case of two UGO rendezvouses. A distribution is very nonuniform. The largest maximum are for velocities: 200 m/s, 260 m/s, 45 m/s.

The ψ angles between velocities directions (Fig. 5) are situated mainly at the same area, as for two UGO rendezvouses and do not exceed 20°. But the distributions are very different. The maximum of curve (2) is in the point of $\psi = 8°$. Sometimes, $\psi = 50°$ and may be higher.

[1] *Bagrov A. V., Erpyliov N. P., Mikisha A. M., et al.* Experimental geodynamic: observations of geostationary satellites // Sci. Inform.–1991.–Issue 69.–P. 35–51.

[2] *Khutorovsky Z. N., Kamensky S. Yu., Bojkob V. F., Smelov V. L.* Risk of collisions of the space objects at low height // Collisions in near Earth space (space debris).–Kosmoinform, IA RAN, 1995.–P. 19–91.

[3] *Sochilina A. S., Kiladze R. I., Grigoriev K. V., Vershkov A. N.* Catalogue of orbits of geostationary satellites.– St.-Petersburg: ITA RAN, 1996.–104 p.