THE EVALUATION OF THE EFFECTS OF VISCO-ELASTIC MANTLE ON LUNI-SOLAR NUTATIONS

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Based on the variational finite element method was carried out evaluation of the effects of visco-elastic mantle on luni-solar nutations. The finite element approach permits to take into account inhomogeneities as of the geometrical, so rheological characters in the Earth mantle. It makes possible to evaluate influence of the mantle visco-elasticity on the forced nutations by more accurate models of the shear quality factor of mantle distribution. One of such models is modificated model "M_1" of Dorofeev and Zharkov (1978). The resulting corrections of mantle visco-elasticity on the forced nutations were obtained for such models of shear quality factor of mantle distribution as: "QMU" of Sailor and Dziewonski (1978); "B" of Sipkin and Jordan (1980); "PREM" of Dziewonski and Anderson (1980); "M_1" of Dorofeev and Zharkov (1978). The comparison of the obtained data shows that results detecting for "QMU" and "PREM" models are very close, while calculation of absorption bands in the low mantle zone and D$''$-zone based on "M_1" model leads to increasing the corrections of mantle visco-elasticity about 10 percents for in-phase nutation components and about 40 percents for out-of phase components in comparison with "QMU" model.

INTRODUCTION

The influence of visco-elastic mantle consists in less changing of it's shear module μ and bulk module λ due to out loading frequency ω, that leads to changing of the Earth reaction. The visco-elastic effects are accompanied by energy dissipation of surrounding. The measure of surrounding energy dissipation defined by quality factor Q, which shows relation of the whole energy emitted over oscillation period in the unit of surrounding to the dissipated one for that period.

At first, the idea of, that quality factor of visco-elastic mantle Q can be described by power law from the loading frequency ω, was proposed by Jeffreys in 1958 ($Q \sim \omega^{\alpha}, \alpha = const$). That idea was essentially improved by Anderson and Minister (1979), as they had obtained for visco-elastic mantle relations, connecting complex shear module μ^* with power frequency depending shear quality factor $Q^{\mu}(\omega)$. Smith and Dahlen (1981) based on simple two level average shear quality factor Q^{μ} of mantle distribution models such as "QMU" Sailor, Dziewonski (1978) and "B" of Sipkin, Jordan (1980) had obtained values $\alpha = 0.15$ and $\alpha = 0.09$, respectively. Based on Smith and Dahlen results, Wahr and Bergen (1986) had evaluated the influence of visco-elastic mantle on the Earth rotation parameters from the Wahr theory (1981). They had also showed that calculation of mantle visco-elasticity leads to appearing of out of phase nutation components. Similar investigations were carried out by V. Dehant (1987). Based on Smith approach (1974) generalization for the case of complex mantle shear modulus μ^*, from "QMU" and "B" models she had obtained visco-elastic corrections for some tidal gravimetric factors δ for such values α as 0.09, 0.15, 0.25, respectively.

As so far for definition of mantle visco-elasticity influence on Earth rotation parameters were only used simple two level of mantle rheology models "QMU" and "B" it will be interesting to apply more complicated models of shear quality factor of mantle distribution, for instance, for evaluating nutation corrections. The variational finite element method permits to calculate inhomogeneities as of the geometrical, so rheological characters in the Earth's mantle [2]. It takes possibility for evaluating influence of mantle visco-elasticity on the forced nutations by more accurate models of shear quality factor of distribution in mantle. One of such models is modificated model "M_1" of Dorofeev and Zharkov [1]. This model did not have a lot of changing so far and now it is accurate enough. It permits to calculate rheology of the mantle inhomogeneity layers good enough.

FORMULATION OF THE PROBLEM

As at that case we are considering only influence of mantle inelasticity on the forced nutation it is not necessary to calculate inner core dynamics, because their mutual effect will be slightly less. As working model we take dynamical model of Moritz [3], which based on Molodensky's theory (1961). The comparison of main nutational terms, obtained based on that model by variational finite element method [2] has showed a good agreement with respective results of Wahr model (1981). Let us consider that Earth is two axes ellipsoid of revolution, has absolutely rigid inner core, stratification liquid core in an ellipsoidal shell of revolution form and also visco-elastic isotropic mantle and crust in the form, which distinguishes small from spherical shell. Liquid core can rotate relatively the mantle. The Earth is self-gravitating, hydrostatically prestressed body. It rotates around own axis, taking under influence from the luni-solar attraction. The influence of an ocean and atmospheric loads not takes into account. We shall neglect the flattening and the rotational effects in the visco-elastic shell, as they are vanishing small [3], and shall take into account their on the elliptical liquid core. Then, using operation for eliminating static meanings of the stress tensor in the visco-elastic shell and static pressure in the liquid core, obtained from hydrostatically equilibrium conditions of the Earth [3], come to the equilibrium equation of the visco-elastic shell and to the motion equation of the liquid core relatively tidal actions, presented in the Tisserand reference system (X, Y, Z) [3]:

$$0 = \operatorname{grad}\left(V_e + V_1 + u_R\, g(R)\right) - \operatorname{div}\vec{u}\operatorname{grad}W_g + \frac{1}{\rho}\operatorname{div}\hat{P}_1; \tag{1}$$

$$\ddot{\vec{u}} + 2\vec{\omega}\times\dot{\vec{u}} = \operatorname{grad}\left[V_e + V_1 + u_R\, g(R) + \left(1 + \frac{\sigma}{\Omega}\phi\right)\right] - 2\frac{\sigma}{\Omega}\frac{\partial\phi}{\partial z}\vec{e}_z - \frac{1}{\rho}\operatorname{grad}p_1. \tag{2}$$

Here $V_e = K(xz\cos\sigma t + yz\sin\sigma t)$ is the tesseral part of tidal wave potential; $\phi = -\Omega^2\epsilon\,(xz\cos\sigma t + yz\sin\sigma t)$ is the changing of centrifugal potential due to nutation; $V_1 = kV_e$ is the changing of tidal potential due to Earth deformations; k is the Love number; W_g is the self-gravitating potential; ρ is a density; ϵ is the polar motion radius of the tidal wave; σ is the frequency of the tidal wave; R is the radius of the Earth point; u_R is the radial displacement component; $g(R)$ is the gravity acceleration; $\hat{P}_1$ is the changing of shell stress tensor due to tidal deformations; p_1 is the tidal pressure changing in the liquid core.

Let us assume that vibrations into liquid core are results from the forced tidal wave frequency σ, taking into account rigidity of the inner core and absence of the Earth surface loads. Make up functionals of Lagrange, which are present whole energy for the visco-elastic shell and for the liquid core, respectively, in the cylindrical coordinate system (z, r, φ), where axis r coincides with the Tisserand axis Z:

$$\begin{aligned}
E_1 &= \pi\iint_{F_s}\left[c_1\left(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2\right) + 4c_2\varepsilon_{zr}^2 + 2c_3\left(\varepsilon_{zz}\varepsilon_{rr} + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi}\right)\right] rdzdr \\
&\quad -\pi\iint_{F_s}\Bigg[2\,(1+k)\,Krw + w\left(\frac{\partial w}{\partial z}\cos\alpha + 2\frac{\partial u}{\partial z}\sin\alpha\right)g(R) + 2w\,(w\cos\alpha + 2u\sin\alpha) \\
&\quad \times g_R'\cos\alpha - 2w\left(\frac{\partial w}{\partial z} + 2\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right)\right)g(R)\cos\alpha + 2\,(1+k)\,Kzu + u\left(2\frac{\partial w}{\partial r}\cos\alpha + \frac{\partial u}{\partial r}\sin\alpha\right) \\
&\quad \times g(R) + 2u\,(2w\cos\alpha + u\sin\alpha)\,g_R'\sin\alpha - 2u\left(2\frac{\partial w}{\partial z} + \frac{\partial u}{\partial r} + \frac{u}{r}\right)g(R)\sin\alpha\Bigg]\rho rdzdr;
\end{aligned} \tag{3}$$

$$\begin{aligned}
E_2 &= \pi\iint_{F_s}\left[c_4\left(\varepsilon_{zz}^2 + \varepsilon_{rr}^2 + \varepsilon_{\varphi\varphi}^2\right) + 2c_4\left(\varepsilon_{zz}\varepsilon_{rr} + \varepsilon_{zz}\varepsilon_{\varphi\varphi} + \varepsilon_{rr}\varepsilon_{\varphi\varphi}\right)\right] rdzdr \\
&\quad -\pi\iint_{F_s}\left[\sigma\,(\sigma + 2\Omega)\,w^2 + \sigma^2u^2\right]\rho rdzdr - 2\pi\iint_{F_s}\left[\Omega\,(\Omega+\sigma)\,\epsilon w + \Omega\,(\Omega-\sigma)\,\epsilon u\right]\rho rdzdr \\
&\quad +\pi\iint_{F_s}\Bigg[2\,(1+k)\,Krw + w\left(\frac{\partial w}{\partial z}\cos\alpha + 2\frac{\partial u}{\partial z}\sin\alpha\right)g(R) + 2w\,(w\cos\alpha + 2u\sin\alpha)\,g_R'\cos\alpha \\
&\quad +2\,(1+k)\,Kzu + u\left(2\frac{\partial w}{\partial r}\cos\alpha + \frac{\partial u}{\partial r}\sin\alpha\right)g(R) + 2u\,(2w\cos\alpha + u\sin\alpha)\,g_R'\sin\alpha\Bigg]\rho rdzdr;
\end{aligned} \tag{4}$$

here $c_1 = (\lambda + 4\mu^*)/3$, $c_2 = \mu^*$, $c_3 = (\lambda - 2\mu^*)/3$ are complex coefficients; $c_4 = \lambda/3$ is real coefficient; λ is real bulk module; μ^* is complex shear module; ε_{ij} are strain tensor components; w, u are the displacement components along axes z and r, respectively; F_s is meridian cross section area of the Earth; $\cos\alpha = z/R$; $\sin\alpha = r/R$.

THE FINITE ELEMENT METHOD RESOLVING PROBLEM

For resolving the system of equations (1, 2), taking into account rigidity of the inner core and absence of the surface loads, let us apply the finite element method based on the variational Lagrang principal in the displacement form [4], which expresses minimum of the whole energy of system, and tends to resolving system of the variational equations such as:

$$\delta E_{1,2}(u_i) = 0. \tag{5}$$

For resolving system of the equations (5), 8-node isoparametric quadrilateral curve finite element is used [4]. As global coordinate system, so system in which all finite elements of the dividing Earth are connected, the cylindrical coordinate system (z, r, φ) is used. As local coordinate system, so system in which the approximation functions of the element are defined and the numerical integration is carried out, the normal coordinate system (ξ, η) is used. As approximation functions in the 8-node element, functions of such form as [4] are used:

$$\begin{aligned}
&\phi_1 = 1/4\,(1-\xi)\,(1-\eta)\,(-\xi-\eta-1)\,; \qquad \phi_2 = 1/4\,(1+\xi)\,(1-\eta)\,(\xi-\eta-1)\,;\\
&\phi_3 = 1/4\,(1+\xi)\,(1+\eta)\,(\xi+\eta-1)\,; \qquad \phi_4 = 1/4\,(1-\xi)\,(1+\eta)\,(-\xi+\eta-1)\,;\\
&\phi_5 = 1/2\left(1-\xi^2\right)(1-\eta)\,; \qquad \phi_6 = 1/2\left(1-\eta^2\right)(1+\xi)\,;\\
&\phi_6 = 1/2\left(1-\xi^2\right)(1+\eta)\,; \qquad \phi_8 = 1/2\left(1-\eta^2\right)(1-\xi)\,.
\end{aligned} \tag{6}$$

Meridian coordinates of the cross section area z, r; respective displacement components w, u; strain components ε_{zz}, ε_{rr}, $\varepsilon_{\varphi\varphi}$, ε_{zr} within of the finite element, are approximated from functions of the form (6) as follows:

$$\begin{aligned}
&z = \sum^{8} z_i\phi_i; \qquad r = \sum^{8} r_i\phi_i; \qquad w = \sum^{8} w_i\phi_i; \qquad u = \sum^{8} u_i\phi_i; \qquad \varepsilon_{zz} = \sum^{8} \Phi_i w_i;\\
&\varepsilon_{rr} = \sum^{8} \Psi_i u_i; \qquad \varepsilon_{\varphi\varphi} = \sum^{8} \frac{\phi_i}{r} u_i; \qquad \varepsilon_{zr} = \frac{1}{2}\sum^{8} (\Psi_i w_i + \Phi_i u_i);\\
&\Phi_i = \frac{1}{|J|}\left(\frac{\partial\phi_i}{\partial\xi}\frac{\partial r}{\partial\eta} - \frac{\partial\phi_i}{\partial\eta}\frac{\partial r}{\partial\xi}\right); \qquad \Psi_i = \frac{1}{|J|}\left(\frac{\partial\phi_i}{\partial\eta}\frac{\partial z}{\partial\xi} - \frac{\partial\phi_i}{\partial\xi}\frac{\partial z}{\partial\eta}\right); \qquad J = \frac{\partial z}{\partial\xi}\frac{\partial r}{\partial\eta} - \frac{\partial z}{\partial\eta}\frac{\partial r}{\partial\xi}.
\end{aligned} \tag{7}$$

From expressions (3) – (7) we are come to the systems of complex algebraic equations, writing for every finite element in the normal coordinate system (ξ, η). Then, summarizing systems of the complex equations on all finite elements, we are forming a global system of the complex equations in the coordinate system (z, r, φ) such view as:

$$\sum^{N} \frac{\partial E_{1,2}}{\partial w_m} = 0; \qquad \sum^{N} \frac{\partial E_{1,2}}{\partial u_m} = 0; \tag{8}$$

here N is the number of the all finite elements of the dividing Earth; index m changes from 1 to 8.

The resolving of the global system of linear complex algebraic equations (8) is realized based on the Gauss method without main matrix element electing [4], so displacement components in the all node points of the finite element network are defining. From that detecting node displacement meanings, the displacement components and other interesting values are determining in an arbitrary points of the finite element, so at any point of the Earth.

THE INFLUENCE OF MANTLE VISCO-ELASTICITY ON THE FORCED NUTATION

As dispersion changing of the complex shear module μ^* is used the power relation from frequency, which was proposed by Smith and Dahlen (1981):

$$\delta\mu_s{}^*/\mu_s = \left[\cot\left(\alpha\pi/2\right)\left(1-\left(\omega_m/\omega\right)^\alpha\right) + i\left(\omega_m/\omega\right)^\alpha\right]/Q_s{}^\mu\left(\omega_m\right). \tag{9}$$

Here μ_s is a value of elastic shear module in the s-radial layer of mantle; $\delta\mu_s{}^*$ is the dispersion amplitude of complex shear module in the s-layer; $Q_s{}^\mu(\omega_m)$ is a value of shear quality factor of mantle in the s-layer, defining at fulcrum frequency ω_m; ω is the considering dispersion frequency; α is the exponent of the power dispersion relation.

The corrections for mantle visco-elasticity to retrograde and prograde circular components of the forced nutation in phase and out of phase were detected, following to formulas presented by Wahr and Bergen (1986):

$$\begin{aligned}
\delta\eta^+ &= \eta_r{}^+\left(\delta\eta^+/\eta_r{}^+\right) = -\tfrac{1}{2}\left(\varepsilon_r + \Psi_r\sin\varepsilon_0\right)\left(\delta\eta^+/\eta_r{}^+\right) = B_R{}^+ - iB_I{}^+;\\
\delta\eta^- &= \eta_r{}^-\left(\delta\eta^-/\eta_r{}^-\right) = \tfrac{1}{2}\left(\varepsilon_r - \Psi_r\sin\varepsilon_0\right)\left(\delta\eta^-/\eta_r{}^-\right) = B_R{}^- - iB_I{}^-.
\end{aligned} \tag{10}$$

Here ε_0 is the angle of ecliptic inclination; ${B_R}^+$, ${B_R}^-$ and ${B_I}^+$, ${B_I}^-$ are the corrections for mantle visco-elasticity to retrograde and prograde circular components of the forced nutation in phase and out of phase, respectively. The retrograde $\delta\eta^+/{\eta_r}^+$ and prograde $\delta\eta^-/{\eta_r}^-$ are the circular diurnal nutations, which have complex form, were detected based on the above presented variational finite element method, by splitting up the matrix of deformation gradients into symmetric-strain and an antisymmetric-rotation parts. During determination PREM-model of Dziewonski and Anderson (1981) both the standard Earth model, and Bullard (1954) density distribution was also used. The values of Love numbers k and polar motion radiuses ϵ of the diurnal tidal waves were chosen from the Molodensky (1963) model II. The meanings of nutation for rigid Earth in the obliquity ε_r and in the longitude Ψ_r, following Wahr (1981), were taken from the Kinoshita (1977) theory.

The determination was carried out for some shear quality factor ${Q_s}^\mu$ of mantle distribution models based on dispersion relation (9). Two level models such as "QMU" of Sailor, Dziewonski (1978); "B" of Sipkin, Jordan (1980) and much level models such as "PREM" of Dziewonski, Anderson (1980); "M_1" of Dorofeev, Zharkov (1978) were used. Following to conclusions made up in the works of Smith and Dahlen (1981), Wahr and Bergen (1986), V. Dehant (1987), it was choose the $\omega_m = 2\pi/30$ rad/s; $\alpha = 0,09$ for model "B" and $\omega_m = 2\pi/200$ rad/s; $\alpha = 0,15$ for models: "QMU", "PREM" and "M_1", respectively.

The obtained corrections for mantle visco-elasticity to retrograde and prograde circular components of the forced nutation in phase and out of phase, presented for main, annual, semiannual and fortnightly terms in milli-arcseconds are in Table 1.

Table 1. The list of parameters

	"B"-model		"QMU"-model		"PREM"-model		"M_1"-model	
Parameter	in phase	out of phase	in phase	out of phase	in phase	out of phase	in phase	out of phase
18.6-year								
$\delta\eta^+$	0.08220	0.02314	0.04552	0.02014	0.04522	0.01992	0.05017	0.02898
$\delta\eta^-$	−0.62402	−0.17568	−0.33778	−0.14946	−0.33562	−0.14785	−0.37238	−0.21512
Annual								
$\delta\eta^+$	0.32509	0.09152	0.17727	0.07844	0.17614	0.07759	0.19542	0.11289
$\delta\eta^-$	−0.01868	−0.00526	−0.00958	−0.00425	−0.00952	−0.00419	−0.01055	−0.00609
Semi-annual								
$\delta\eta^+$	0.05979	0.01683	0.03354	0.01484	0.03332	0.01468	0.03697	0.02136
$\delta\eta^-$	0.50071	0.14096	0.27070	0.11978	0.26897	0.11849	0.29843	0.17240
Fortnightly								
$\delta\eta^+$	0.00521	0.00146	0.00288	0.00127	0.00286	0.00125	0.00315	0.00182
$\delta\eta^-$	0.11583	0.03261	0.06468	0.02862	0.06427	0.02831	0.07130	0.04119

The comparison of the obtained data shows that results, detecting from "QMU" and "PREM"-models, are very close, while calculation of the absorption bands in the low mantle zones and D$''$-zone from "M_1" model leads to increasing the corrections for mantle inelasticity on the luni-solar nutations about 10 percents for in phase and 40 percents for out of phase components in comparison with "QMU" model.

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