AN INSTABILITY OF THE TRIGONOMETRIC SOLUTION FOR THE PERIODICAL COMPONENTS OF THE POLAR MOTION

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Estimation of the real errors of the polar motion approximation with trigonometric functions is made by a model of simulated polar motion with variable amplitudes of the Chandler and seasonal components. It is shown that the real errors of the trigonometric coefficients are much more than the formal estimation errors, obtained by the least-squares method (LSM).

INTRODUCTION

The determination of the Earth's orientation parameters (EOP) is important part of the modern scientific investigations. The first determinations of the EOP only include the pole coordinates from optical astrometry, and since 1956 those of the pole coordinates, the Universal Time and the celestial pole offsets. The accuracy of the pole coordinates determination from the optical astrometry varies within 0.01 and 0.03 arcsec. In the last two decades the optical observations are replaced by various space and ground-based observational techniques, which improves significantly the estimation accuracy. The accuracy of the C04 solution of the IERS for pole coordinates since 1996 is 0.2 mas.

Usually, the analysis of the polar motion consists of separation and investigation of the two main periodical components namely seasonal and Chandler oscillations. The successful separation of the seasonal and Chandler oscillation is possible over six-year or longer time spans of observations, due to almost resonance relation 5 : 6 between the annual and Chandler frequencies. The most used mathematical model for determination of the periodical components of the polar motion, involved by [1] and [2], consists of trigonometric functions of common type

$$x = x_0 + \sum_{i=1}^{n} a_{ai} \sin i\omega_a t + b_{ai} \cos i\omega_a t + \sum_{i=1}^{m} a_{ci} \sin i\omega_c t + b_{ci} \cos i\omega_c t, \tag{1}$$

$$y = y_0 + \sum_{i=1}^{n} c_{ai} \sin i\omega_a t + d_{ai} \cos i\omega_a t + \sum_{i=1}^{m} c_{ci} \sin i\omega_c t + d_{ci} \cos i\omega_c t, \tag{2}$$

where x and y are the pole coordinates, x_0 and y_0 are the mean coordinates in the middle of the six-year time interval, a_{ai}, b_{ai}, c_{ai}, d_{ai}, $i = 1, \ldots, n$; a_{cj}, b_{cj}, c_{cj}, d_{cj}, $j = 1, \ldots, m$ are unknown harmonic coefficients of the components with known seasonal (annual) frequency w_a and Chandler frequency w_c; t is the observation epoch. Usually, the number of harmonics is equal to 1 ($n = m = 1$), for study of the semi-annual and semi-Chandler oscillations the value of harmonics n and m may increase to 2.

SEASONAL AND CHANDLER AMPLITUDE VARIATIONS, DETERMINED FROM THE SOLUTION OA04 FOR THE POLE COORDINATES

The recent Vondrak and Ron solution OA04 for the pole coordinates [3] is obtained from all optical observations for the period 1899.7–1992.0 using the combined star catalogues, based on Hipparcos catalogues and ground observations (Fig. 1). The variations of the amplitudes of the seasonal and Chandler oscillations of the polar motion are determined from the solution OA04 by the model (1, 2). The estimates are obtained by the LSM with running 6-year interval. The variations of the amplitude of the seasonal oscillations of the polar motion consist of decadal oscillation within the limits 0.05″–0.15″ (Fig. 2). The variations of the amplitude of the Chandler oscillations are from 0.0″ to 0.25″ with periods between 20 and 40 years (Fig. 3).

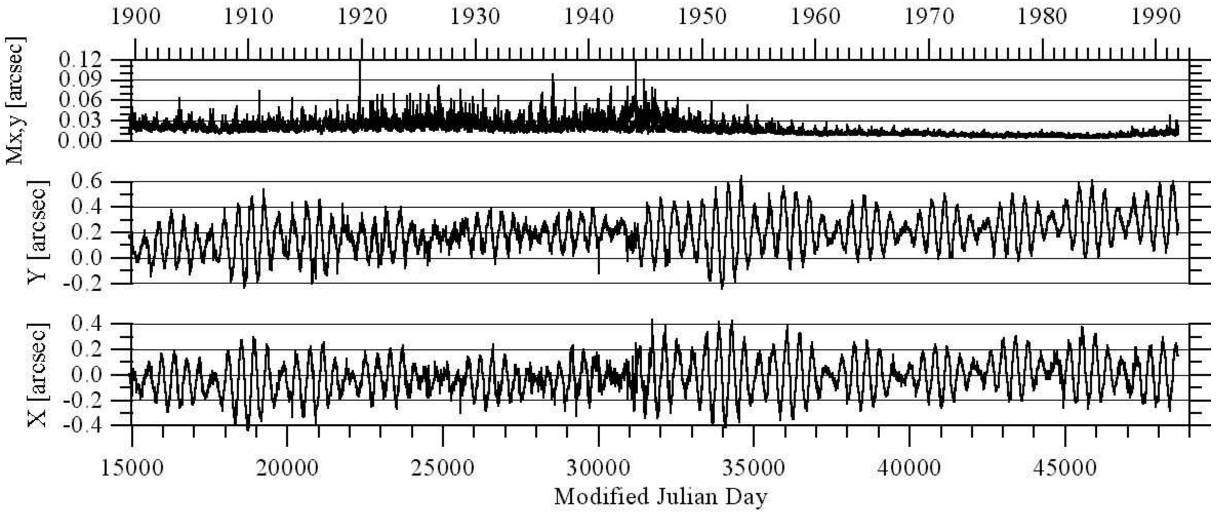

Figure 1. The last solution OA04 of Vondrak and Ron [3] for the pole coordinates X, Y and their errors M_x, M_y

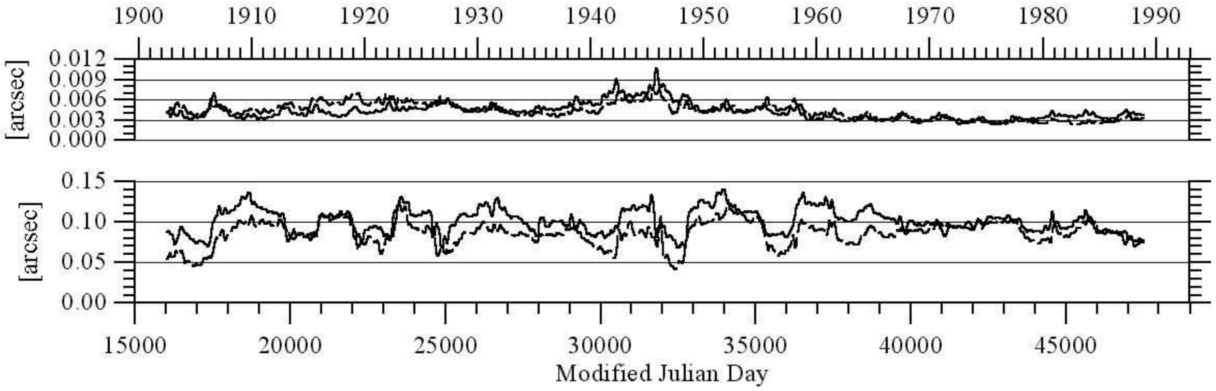

Figure 2. Variations of the amplitude of the seasonal oscillations of the polar motion (the graph below, solid line for x-coordinate, dashed line for y-coordinate), determined by the LSM estimation, according to the model (1, 2). The upper graph represents the formal estimation errors

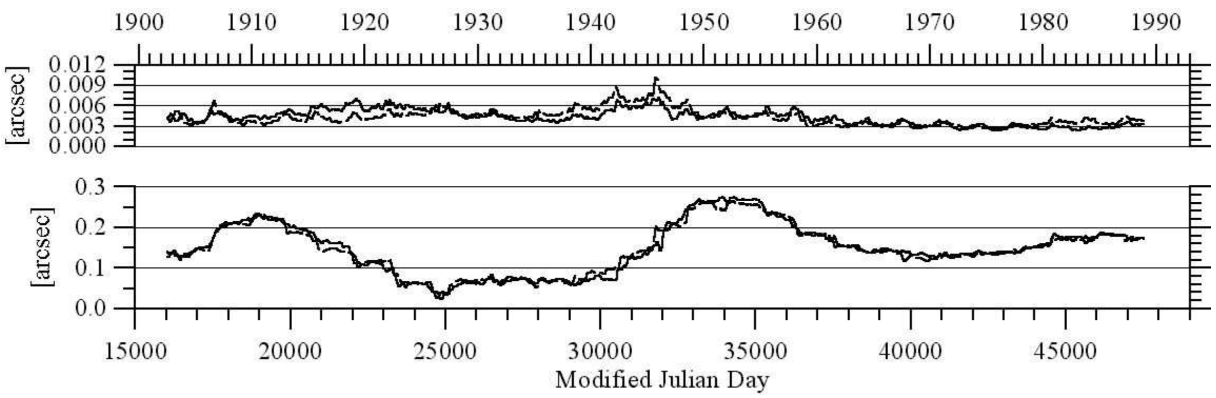

Figure 3. Variations of the amplitude of the Chandler oscillations of the polar motion (the graph below, solid line for x-coordinate, dashed line for y-coordinate), determined by the LSM estimation, according to the model (1, 2). The upper graph represents the formal estimation errors

SIMULATION OF THE POLAR MOTION WITH VARIABLE SEASONAL AND CHANDLER AMPLITUDES

To estimate the real errors of the approximation of the polar motion with trigonometric functions of the type (1, 2) it is very useful to use simulation of the polar motion with known variations of the amplitudes of the seasonal and Chandler oscillation, which are close enough to its real changes in the time. A simple model of the polar motion is presented by the equations (3, 4).

$$x = A_{an} \sin 2\pi t + A_{ch} \cos \frac{2\pi t}{T_{ch}}, \quad y = A_{an} \cos 2\pi t - A_{ch} \sin \frac{2\pi t}{T_{ch}}, \tag{3}$$

$$A_{an} = 0.1'' + 0.04'' \sin \frac{2\pi t}{10}, \quad A_{ch} = 0.2'' + 0.075'' \sin \frac{2\pi t}{30}, \quad T_{ch} = 1.18a. \tag{4}$$

The model (3, 4) includes constant part $0.1''$ of the seasonal amplitude A_{an} and variations with period 10 years and amplitude $0.04''$. The Chandler amplitude A_{ch} consists of constant part $0.2''$ plus variations with period 30 years and amplitude $0.075''$. The period of the seasonal oscillation is exactly one year, and the period of the Chandler oscillation T_{ch} is 1.18a. The simulated polar motion is similar to the behaviour of the real polar motion for the period 1940–1970 (Fig. 4). The changes of the amplitudes of the seasonal and Chandler oscillations, according to the model (3, 4) is shown in Fig. 5 by the solid lines: the upper line represents the Chandler amplitude variations, and the lower line is the seasonal amplitude variations.

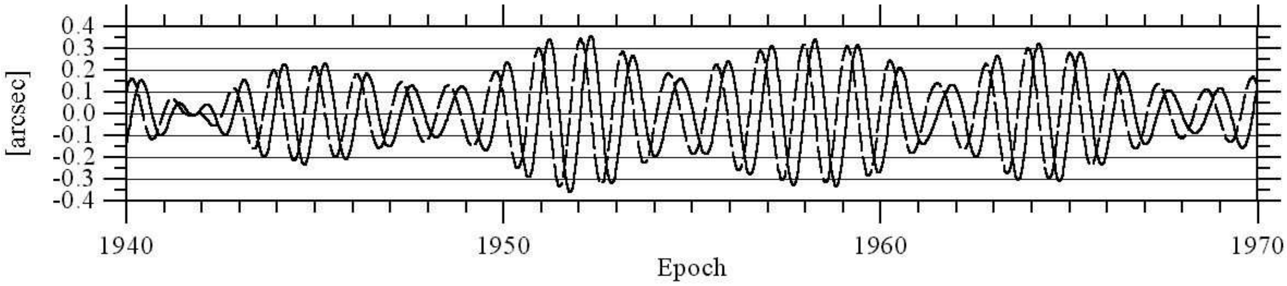

Figure 4. Simulation of the polar motion with variable amplitudes of the Chandler and seasonal oscillation (solid line for x-coordinate, dashed line for y-coordinate)

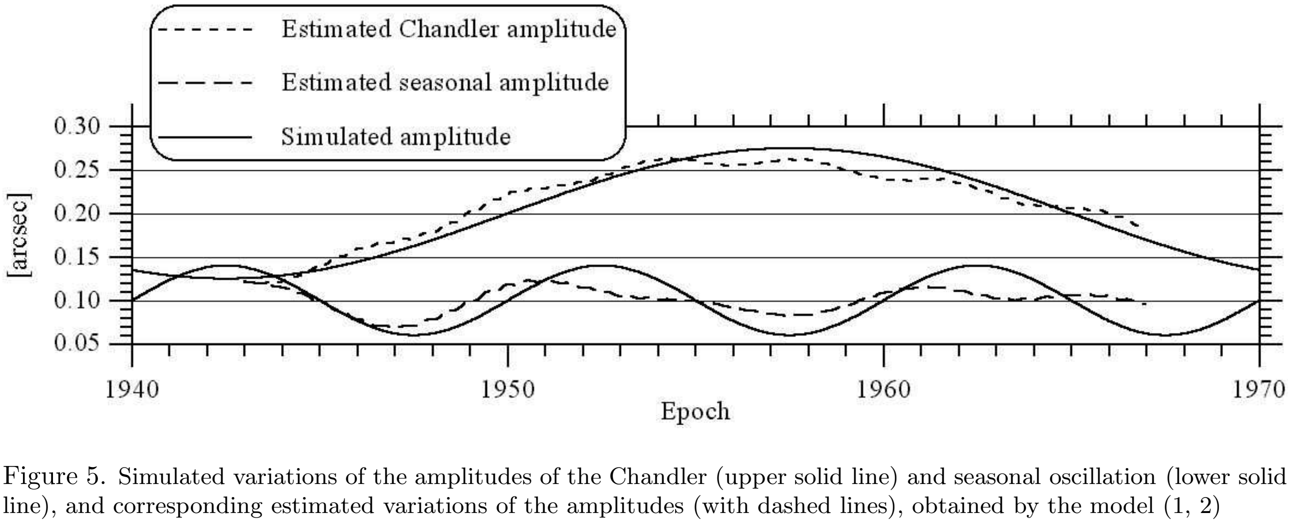

Figure 5. Simulated variations of the amplitudes of the Chandler (upper solid line) and seasonal oscillation (lower solid line), and corresponding estimated variations of the amplitudes (with dashed lines), obtained by the model (1, 2)

The amplitudes of the seasonal and Chandler oscillations, which approximate the simulated polar motion (3, 4) by the model (1, 2), are determined from the estimated trigonometric coefficients by the formulae (5).

$$A_{an,x} = \sqrt{a_{ai}^2 + b_{ai}^2}, \quad A_{an,y} = \sqrt{c_{ai}^2 + d_{ai}^2}, \quad A_{ch,x} = \sqrt{a_{ci}^2 + b_{ci}^2}, \quad A_{ch,y} = \sqrt{c_{ci}^2 + d_{ci}^2}. \tag{5}$$

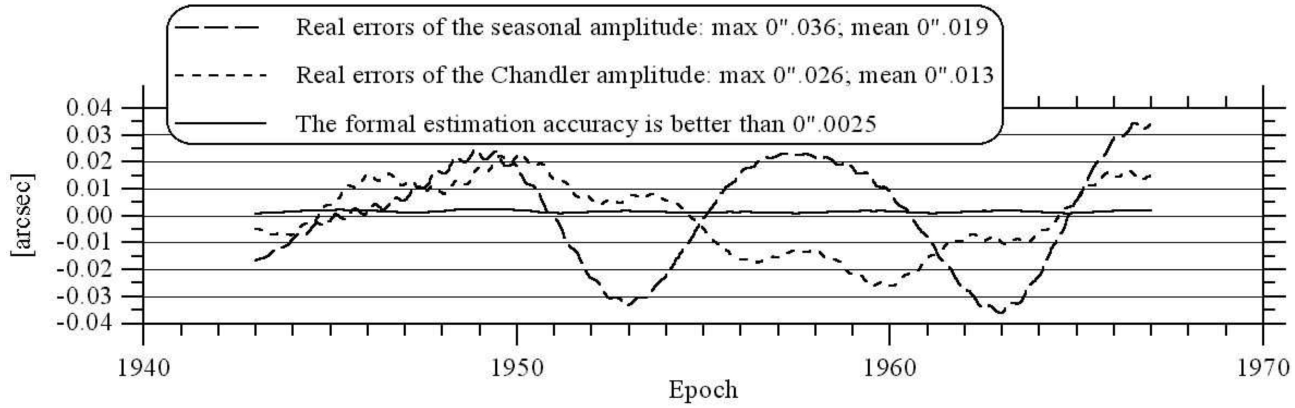

Figure 6. Real errors of the estimated amplitudes of the seasonal and Chandler oscillations of the simulated polar motion. The formal estimation errors are about 10 times lower

The computed seasonal and Chandler amplitudes by equations (5) are shown in Fig. 5 with dashed lines. It is seen significant differences between simulated variations of the amplitudes and corresponding variations, obtained from trigonometric approximation by the model (1, 2). These differences are estimated by the real errors of the polar motion approximations with trigonometric functions. They are shown in Fig. 6 together with the formal estimation errors, determined by the least-squares method. The real errors of the trigonometric approximation of the polar motion by the model (1, 2) are almost 10 times higher than the formal estimation errors. The mean values of the obtained real errors are $0.019''$ for the seasonal amplitude and $0.013''$ for the Chandler amplitude. The maximum values are $0.036''$ and $0.026''$. An additional computation with shifted period of the Chandler oscillation with 1/100 is made, and in this case the maximum errors increase up to $0.048''$ for the annual amplitude and $0.028''$ for the Chandler amplitude.

CONCLUSIONS

1. The trigonometric solution for the periodical components of the polar motion is unstable in the case of time variations of the annual and Chandler amplitudes.
2. This instability of the trigonometric solution increases the maximum value of the real errors of the annual and Chandler amplitudes up to 48 mas and 28 mas when the Chandler period is shifted with 1/100 and up to 36 mas and 26 mas with exact knowledge of the Chandler period value.
3. The real errors of the trigonometric solution for the periodical components of the polar motion exceed about 10 times the formal estimation errors of the annual and Chandler amplitudes.

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