IMPACT OF THE STELLAR OBLATION EFFECT ON ESTIMATION OF THE MAGNETIC DIPOLE STRENGTH

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The surface magnetic field structure of an ellipsoidal star is modelled in the frame of the magnetic charge description (MCD) approach. It is shown that the stellar oblation effect can lead to the essential overestimation of the magnetic dipole strength value obtained from the mean crossover effect (up to 12%) and quadratic magnetic field (up to 8%) in comparison with its theoretical value obtained for the case of the spherically symmetric star. Taking into account the gravity-darkening phenomenon is argued that this overestimation increases with the growth of the effective gravity difference at the equator and poles of the star. The data of the mean longitudinal magnetic field provide the most correct estimation of the magnetic dipole strength value in the ellipsoidal star.

INTRODUCTION

The upper layer of stellar gas is stressed by the mutual action of radiation and gas pressure, centrifugal and gravitational forces and forms the respective (often symmetrically spherical) shape of stellar surface. For a rapidly rotating star the rotational deformation of the stellar surface could be essential and leads to the observable stellar oblation on the rotational poles. Furthermore, the essentially high stellar rotation leads to the surface brightness distribution, as it was originally shown in [8]. Nowadays, a common model of a differentially rotating star is characterized by the highly anisotropic turbulence which has a strong horizontal component [7]. This horizontal turbulence strongly reduces the horizontal differential rotation, so that rotation varies only radially. This means that the angular velocity is uniform at the surface of isobaric shells and, consequently, the rotation is usually said to be “shellular”. The presence of a magnetic field generated, for example, by the Tayler–Spruit dynamo mechanism does not affect the shape of the equipotentials, but strongly amplifies the horizontal coupling.

The structure of a surface magnetic field for an ellipsoidal star can be sufficiently well described in the frame of the magnetic charge description (MCD) method [1, 2, 4]. The aim of this research is to consider the stellar oblation influence on the results of the surface magnetic field modelling.

MAGNETIC FIELD AT THE SURFACE OF AN ELLIPSOIDAL STARS

The surface of a rapidly rotating star has a shape that can be well described with the help of the model of a rotating Maclaurin ellipsoid in which a small semi-axis $R_{\rm p}$ is directed along the rotational axis, while a large semi-axis $R_{\rm e}$ lies in the equatorial plane. For such a model the distance of an arbitrary surface point M from the stellar centre in the $R_{\rm p}$ units is:

$$\rho = \frac{1}{\sqrt{1-\varepsilon^2\cos^2\delta}}\,, \tag{1}$$

where eccentricity is defined as $\varepsilon = \sqrt{1-R_p^2/R_e^2}$ and δ specifies point’s latitude in the Spherical reference frame related to the Star (SS). According to the main relations obtained in [5] the integrated radiation flux from a visual surface of the ellipsoidal star with a weak oblation ($\varepsilon^2 \ll 1$) can be expressed as

$$I_0 \simeq \frac{\pi}{30}\left[10\,(3-u) + \varepsilon^2(15\,(1+\cos^2 i) - u\,(7-\cos^2 i))\right], \tag{2}$$

where i specifies the inclination of the stellar rotational axis to the line of sight and u is the limb darkening coefficient.

The corresponding equations for the Cartesian components of the magnetic field vector at the point M of ellipsoid's surface can be deduced from [2] and applied for calculation of the surface magnetic field characteristics. Supposing the presence of a centered symmetric magnetic dipole inside the ellipsoidal star, we can obtain the expressions for the magnetic field characteristics according to [5, 6] for the star with the weak oblation effect. The corresponding equations have the following form: for the mean longitudinal magnetic field

$$<B_z>\simeq \frac{\pi B_d}{120 I_0}\left\{2Z_0(15+u)+3\varepsilon^2 X_0\sin i\cos i(35-11u)-\varepsilon^2 Z_0[15(3-7\cos^2 i)-u(5-17\cos^2 i)]\right\}, \tag{3}$$

for the mean crossover effect

$$\begin{aligned}< B_c > \simeq -\frac{\pi B_d v_e Y_0 \sin i}{2240\, I_0}\{\, 56\,(8-3u) - b\,[\,64\,(1+2\cos^2 i) - u\,(29+23\cos^2 i)] \\ -4\varepsilon^2[\,8\,(13-16\cos^2 i) - u\,(34-23\cos^2 i)]\},\end{aligned} \tag{4}$$

and for the mean quadratic magnetic field

$$\begin{aligned}\langle B_{mq}\rangle^2 \;\simeq\; \frac{\pi B_d^2}{4I_0}\Bigg\{\frac{525-193u}{210} + Z_0^2\frac{105+13u}{105} + \varepsilon^2\Bigg(\frac{105-247u}{140} + Z_0X_0\sin i\cos i\,(21-5u) \\ +\sin^2 i\left[X_0^2\frac{245-117u}{40} - \frac{2205-157u}{240}\right] + Z_0^2\left[\frac{1365-181u}{140} - \sin^2 i\frac{651-11u}{48}\right]\Bigg)\Bigg\},\end{aligned} \tag{5}$$

where

$$\begin{cases} X_0 = & \rho_1[\cos\beta\sin i - (1-\varepsilon^2)\sin\beta\cos i\cos(\lambda_1+\varphi)], \\ Y_0 = & \rho_1[1-\varepsilon^2]\sin\beta\sin(\lambda_1+\varphi), \\ Z_0 = & \rho_1[\cos\beta\cos i + (1-\varepsilon^2)\sin\beta\sin i\cos(\lambda_1+\varphi)] \end{cases} \tag{6}$$

and

$$\rho_1 = \frac{1}{\sqrt{(1-\varepsilon^2)^2\sin^2\beta + \cos^2\beta}}\,. \tag{7}$$

Here variable β defines the angle between the magnetic dipole axis and the axis of axial stellar rotation while variables λ_1 and φ specify the longitude of the probe magnetic charge in the SS reference frame and rotational phase of the star with the equatorial velocity v_e, respectively. The magnetic dipole strength is defined as $B_d = 4aQ/R_p^2$ (see, *e.g.*, [3]).

In Eqs. (3)–(5) the influence of the stellar oblation effect on the magnetic field characteristics is taken into account by the terms proportional to ε^2. In order to estimate its contribution the following function is calculated for some magnetic field characteristics

$$\Lambda_m(\varepsilon, u, i, \beta, \varphi) = \left(1 - \frac{< B_m(\varepsilon) >}{< B_m(0) >}\right) \times 100\%, \tag{8}$$

where $< B_m(\varepsilon) >$ denotes the field characteristic estimation (the mean crossover effect or the mean quadratic magnetic field) for the case of non-zero stellar oblation, while $< B_m(0) >$ is the field estimation obtained by neglecting the stellar oblation effect ($\varepsilon = 0$). For the mean longitudinal magnetic field the other expression of the Λ-function is applied as

$$\Lambda_z(\varepsilon, u, i, \beta, \varphi) = \frac{<B_{\rm z}(0)> - <B_{\rm z}(\varepsilon)>}{<B_{\rm z}(0)>_{\rm max}} \times 100\%, \tag{9}$$

in order to avoid the division by zero for the certain sets of the variables i, β and λ_1 (see Eqs. (3), (8)). Dependence of these functions on the variables $\varepsilon, i, \beta, \varphi$ is shown in Fig. 1.

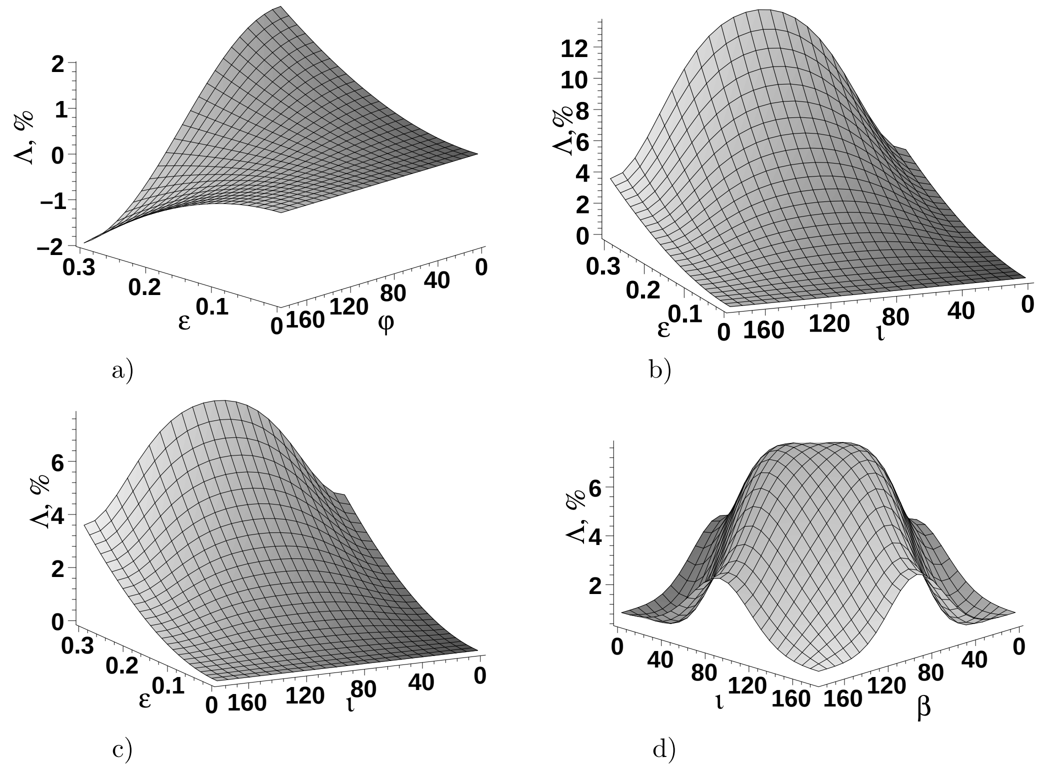

Figure 1. The shapes of the Λ_m-functions: (a) for the mean longitudinal magnetic field with the values of $u = 0.6$, $i = 90°$, and $\beta = 90°$; (b) for the mean crossover effect with $b = 0.2$ (solar differential rotation); (c) for the mean quadratic magnetic field with $u = 0.6$, $\beta = 90°$, and $\varphi = 0$; (d) for the mean quadratic magnetic field with $\varepsilon = 0.3$, $u = 0.6$, and $\varphi = 0$

DISCUSSION

It is widely accepted to consider a spherically symmetric star in the existing models of the magnetic field structure at the stellar surface. An estimate of the strength of the magnetic dipole is based on the main surface magnetic field characteristics such as the mean longitudinal magnetic field, crossover effect and quadratic field is a function of the geometrical characteristics of the stellar surface. In this situation the main question is: *How strong should the rotational deformation be in order to essentially distort our estimation of the magnetic dipole strength?*

From the analysis of Eqs. (3)–(5) it follows that the aforementioned magnetic field characteristics are not equally sensitive to the stellar oblation effect. For the mean longitudinal magnetic field we can ignore the contribution of the oblation effect because it does not exceed 2% (see Fig. 1a). Nevertheless, for the mean crossover effect and the mean quadratic field the effect of stellar oblation increases their theoretical value (under $\varepsilon = 0.3$) up to 12% (see Fig. 1b) and up to 8% (see Figs. 1c and 1d), respectively. *That could lead to the essential overestimation of the magnetic dipole strength value obtained from the mean crossover effect and quadratic magnetic field observations if we neglected the effect of stellar oblation.*

For an ellipsoidal star, its polar zones distorted by rapid rotation are hotter than its equatorial zones, because $T_{\rm eff}^4 \sim (g_{\rm eff})^{\beta_1}$ [8] and the effective gravity is higher at the poles of the star. This phenomenon is known as gravity-darkening. In order to account for the this phenomenon, we construct a simplified approach supposing that the effective gravity at the stellar surface is inversely proportional to the square of its distance from the center of the star. The ordinary limb darkening law is combined with the brightness distribution over the stellar surface $\rho^{-2\gamma}$ (see Eq. (1)). The new expressions for the integrated radiation flux as well as for the mean longitudinal magnetic field, the crossover effect and the quadratic field are obtained and the respective Λ-functions are recalculated. Choosing the values of the rest of the parameters in Eqs. (8), (9) in the way that they provide the maximum of Λ-function (see Figs. 1a, 1b, 1c), we analyse its dependence on the new parameter γ. It is obtained for all the surface magnetic field characteristics that their Λ-functions grow with the increasing of γ parameter. By increasing the value of parameter γ we can change the brightness distribution over the surface of the star and, consequently, the difference of T_{eff} at the stellar pole and equator. The Λ-function obtained for the mean longitudinal magnetic field is almost insensitive to the increase of γ, while for the mean crossover effect the Λ-function grows up to 2%, when we increase the parameter γ from 0 to 1. Therefore, the given in Fig. 1 values of the magnetic dipole strength overestimation due to the effect of stellar oblation should be considered as the bottom limit of the real error.

For the star with an essential axial rotation the local flux depends on the gravity according to the von Zeipel's theorem and a resulting polarized line profile can differ from the line profile that is usual for the spherically symmetric star. This means that correction of the magnetic field measurements for the stellar oblation effect and the corresponding gravity-darkening should be performed on the stage of line profile analysis or later, during the modelling of stellar magnetic field structure.

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