MICROLENSING EFFECTS AND STRUCTURE OF GRAVITATIONAL LENS SYSTEMS

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A study of gravitational microlensing of distant objects is presented. We performed simulations of light curves and trajectories of the image centroid of an extended source in the Chang–Refsdal lens with shear and continual dark matter. Various brightness distributions over the source (Gaussian, power-law, Shakura–Synyaev accretion disc) have been studied. We considered in detail approximate relations and corresponding algorithms used to fit observational data on high amplification events (HAE). The results are applied to interpretation of HAE observed by OGLE and GLITP groups. The source size and caustic crossing moment are estimated from these data, however, the determination of the brightness profile is statistically not reliable.

1. Gravitational microlensing may provide valuable information on masses of microlensing stars and structure of distant sources. To obtain more detailed information about the source from microlensing processes a long-term continuous monitoring is needed. Therefore, a considerable attention is paid to analysis of high amplification events (HAE) that usually may be described by a small number of fitting parameters. This requires a detailed investigation of the reliability of fitting procedures, which use some asymptotic relations for the source crossings of a caustic of the gravitational lens system (GLS). In this connection, we have investigated the accuracy of asymptotic relations used to treat caustic crossing microlensing events and model dependence of source parameters that are obtained using existing observational data.

2. We performed simulations of magnifications and trajectories of the image centroid (IC) of an extended source in the Chang–Refsdal lens with shear and convergence due to continuous dark matter. Various brightness distributions over the source have been studied: Gaussian, power-law (PL)

$$F_R(r) = \frac{(p-1)}{\pi R^2 [1 + (r/R)^2]^p},$$

and Shakura–Synyaev accretion disc (SSAD)

$$F_R(r) = \frac{3R\theta(r-R)}{2\pi r^3}\left(1 - \sqrt{\frac{R}{r}}\right),$$

where R is a characteristic source size. We show that astrometric microlensing effects may be helpful to reveal important details of the GLS local structure, providing that astrometric accuracy of image centroid positioning will reach the microarcsecond level. In particular, in case of a small source the occurrence of rapid jumps of the image centroid in HAE seems to be an indicator of caustic crossing at the fold point. For more detailed presentation see [2, 4].

3. We have studied in detail the approximate relations and corresponding algorithms used to fit observational data on HAE in rather a general situation. The simplest approximation used to treat HAE is an approximation of linear caustic, which is correct only in a very small area near the caustic. This may be used to obtain the amplification coefficient of the extended source intensity only if its size is small as compared with the caustic curvature radius. To modify the relation for this coefficient, we must expand the lens mapping $(\xi, \eta) \rightarrow (x, y)$ in the vicinity of a critical point $\xi = \eta = 0$ up to the fourth order:

$$\begin{aligned} x &= 2\xi + a(\xi^2-\eta^2) + 2b\xi\eta + c(\xi^3 - 3\xi\eta^2) - d(\eta^3 - 3\xi^2\eta) + e(\eta^4 - 6\xi^2\eta^2 + \xi^4) - 4f(\xi\eta^3 - \xi^3\eta), \\ y &= b(\xi^2-\eta^2) - 2a\xi\eta + d(\xi^3 - 3\xi\eta^2) + c(\eta^3 - 3\xi^2\eta) + f(\eta^4 - 6\xi^2\eta^2 + \eta^4) + 4e(\xi\eta^3 - \xi^3\eta). \end{aligned}$$

We restrict ourselves to an harmonicity condition, which means the absence of continuous matter on the line of sight: $\partial x/\partial\xi + \partial y/\partial\eta = 2$. Then, we suppose that the critical point is a fold; this means that $b \neq 0$. In this case we derived the relation for the brightness amplification of the Gaussian source on the caustic as follows

$$K = \frac{1}{4\sqrt{\pi|b|R}}\left\{\Gamma(1/4) + \Gamma(3/4)\frac{R}{|b|}\left[5(a^2+b^2)\frac{3a^2+2b^2}{8b^2} + \frac{15c(c-2a^2)}{8b^2} + \left(\frac{3f}{2b} - \frac{15ad}{4b}\right)\right]\right\}.$$

The expression in the square brackets defines the first non-zeroth correction to the linear caustic approximation. In case of, *e.g.*, Q2237+0305 GLS, the source size is $\sim 0.1R_E$ and applicability of the linear caustic approximation seems to be marginal. Four additional parameters (as compared to linear caustic) must be defined if we want to increase the accuracy and this is difficult because they are related with an effect of roughly the same order as the measurement accuracy.

4. Taking into account simulations of microlensing light curves in a simple lens models we derived following conclusions: (i) only a small area near the caustic ($< 0.2R_E$) can be used to fit correct results; (ii) moment of caustic crossing is defined rather well and it is almost independent on the brightness distribution over the source. If the simulated model is known, then the fitting yields the source size with an accuracy of about 10–15%, in case of unknown model, the accuracy is about 15–30%.

5. We have used our fitting routine to simulate observational light curves of the OGLE (Optical Gravitational Lens Experiment) [5] and GLITP (Gravitational Lens International Time Project) [1] groups of HAEs in the Q2237+0305 "Einstein Cross" GLS. We show that the existing data do not allow one to choose a concrete brightness distribution for a source.

For size estimation, we used the rms radius R_{rms} and half-brightness radius $R_{1/2}$ that are related more directly to the observational quantities than the parameter R. It is supposed that

$$R_{rms} = \left[\int r^2 P(x,y)dxdy \Big/ \int P(x,y)dxdy\right]^{1/2} \quad \text{and} \quad R_{1/2} = R_*/\sqrt{\ln 2},$$

where R_* is defined by the condition

$$\int_0^{R_*} F_R(r)rdr = \frac{1}{2}\int_0^{\infty} F_R(r)rdr.$$

In case of, *e.g.*, the PL distribution $R_{rms} = R/(p-2)^{1/2}$ $(p > 2)$; $R_{1/2} = R\sqrt{\left(2^{1/(p-1)} - 1\right)/\ln 2}$. The Gaussian distribution is the limiting case of the PL one with a fixed R_{rms} and $p \to \infty$; in this case $R = R_{rms} = R_{1/2}$.

We have estimated the characteristic source size R, half-brightness radius $R_{1/2}$ and rms radius for different indexes p for the PL distribution, as well as for Gaussian and SSAD. However, within the least squares method procedure, no preference can be given to any of these distributions. Some of the results are shown in Table 1 in terms of characteristic times $T_{1/2}$ ($T_{1/2} = R_{1/2}/V$, V is the normal velocity of the caustic with respect to the source). The errors have been determined from a statistical simulation.

Figure 1 shows a model dependence for the fitting parameters derived from observational data. The data were fitted using the PL brightness distribution with different p ($T_R = R/V$; $T_{rms} = R_{rms}/V$).

In summary, our numerical simulations show that for a correct determination of source parameters one must use these relations only in a small area near the caustic (less than 0.1–0.2 of typical lens size R_E). The source size and caustic crossing moment can be estimated from these data, however, the determination of the brightness profile appears to be statistically not reliable. Moreover, there is a considerable model dependence of the source size derived from the above data. For more details see [3].

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Table 1. Q2237+030 source crossing times estimated from HAE observed by OGLE and GLITP groups

Initial data, source model	Image (band)	$T_{1/2}$, day	$R_{1/2} \cdot v/300$, 10^{-3} pc km s^{-1}
GLITP, Gaussian	A (R)	38^{+44}_{-17}	$0.34^{+0.4}_{-0.15}$
GLITP, PL, $p = 3/2$	A (R)	80^{+36}_{-22}	$0.71^{+0.34}_{-0.19}$
GLITP, SSAD	A (R)	58^{+6}_{-17}	$0.52^{+0.05}_{-0.16}$
GLITP, Gaussian	A (V)	29^{+25}_{-10}	$0.26^{+0.23}_{-0.09}$
GLITP, PL, $p = 3/2$	A (V)	70^{+17}_{-39}	$0.63^{+0.16}_{-0.34}$
GLITP, SSAD	A (V)	52 ± 12	0.46 ± 0.11
OGLE, Gaussian	A (V)	40^{+17}_{-15}	$0.37^{+0.16}_{-0.14}$
OGLE, PL, $p = 3/2$	A (V)	75 ± 51	0.67 ± 0.39
OGLE, SSAD	A (V)	58^{+12}_{-17}	$0.52^{+0.11}_{-0.16}$
OGLE, Gaussian	C (V)	42^{+21}_{-20}	$0.38^{+0.19}_{-0.18}$
OGLE, PL, $p = 3/2$	C (V)	81^{+34}_{-24}	$0.73^{+0.30}_{-0.22}$
OGLE, SSAD	C (V)	70^{+17}_{-29}	$0.64^{+0.16}_{-0.26}$

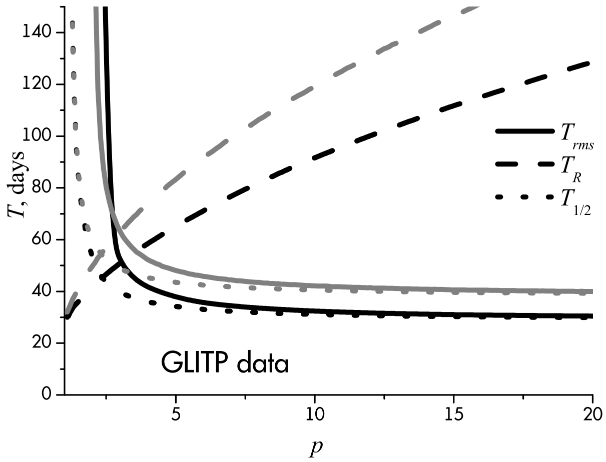

Figure 1. Source sizes (days) derived from GLITP data for different values of the exponent p in the PL brightness distribution. The black lines describe the V band, the grey ones – the R band

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