DARK ENERGY AND LARGE-SCALE STRUCTURE OF THE UNIVERSE

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The evolution of matter density perturbations in the two-component model of the Universe filled with dark energy (DE) and dust-like matter (M) is considered. We have analysed it for two kinds of DE with $\omega \neq -1$: a) an unperturbed energy density and b) a perturbed one (uncoupled with the matter). For these cases, the linear equations for the evolution of the gauge-invariant amplitudes of matter density perturbations are presented. It is shown that, in the case of the unperturbed energy density of DE, the amplitude of matter density perturbations increases slightly faster than in the second case.

INTRODUCTION

The measurements of luminosity distances d_L to the SN Ia stars as a function of the redshift have revealed the accelerated expansion of the Universe [10, 11]. Until recently, for the interpretation of the gathered data on the large-scale structure of the Universe, models with the positive cosmological constant Λ were preferred by cosmologists. For such models, one can calculate the evolution of matter density perturbations up to the formation of gravitationally bound systems of galaxies and clusters of galaxies (see [6] and references therein). Search for a plausible physical interpretation of Λ-constant has introduced new terms in astrophysics: dark energy (DE) and a quintessence for the notation of energy of unknown nature that repulses and involves the self-attracting matter into an accelerated expansion. The classical Λ-constant is the simplest kind of such energy. Now, the more general models of this component are under considerations (see for review [9] and references therein). In some papers, the assumption of absence of coupling between DE and matter is used. But in this case, the matter density perturbations lead to perturbations of the dark energy density [2, 3, 7]. Another kind of DE is based on the assumption of an homogeneous and isotropic distribution of this component. Such models predict an energy flow from one component to another or, in other words, DE and matter are coupled in perturbed regions. In this paper, we will analyse the evolution of matter density perturbations for both kinds of DE using the constant equation of state and $\omega^{(DE)} = P^{(DE)}/\varepsilon^{(DE)} \neq -1$, where $P^{(DE)}$ and $\varepsilon^{(DE)}$ are pressure and energy density of DE, respectively.

DARK ENERGY

The influence of DE on the dynamics of the Universe and evolution of matter density perturbations can be studied using analysis of Einstein's equations and its presentation as an ideal fluid with the equation of state $P = \omega\varepsilon$, where ω is negative. In the case of $\omega = -1$, we have Λ-constant or the Lorentz-invariant dark energy which can also be presented by the density of Lagrangian function $\mathcal{L} \equiv \mathcal{L}(\{g_{ik}\}, \{g_{ik}, l\})$ that satisfies the equation

$$\frac{1}{2}\sqrt{-g}\Lambda g_{ik} = \frac{\partial(\sqrt{-g}\,\mathcal{L})}{\partial g^{ik}} - \frac{\partial}{\partial x^l}\frac{\partial(\sqrt{-g}\,\mathcal{L})}{\partial g^{ik};l},$$

where $\Lambda \equiv \Lambda(\{g_{ik}\}, \{g_{ik}, l\})$ is some arbitrary function. Einstein's Λ-constant has the physical interpretations of zero-point vacuum fluctuations, vacuum polarization or follows from some versions of the supersymmetry theories. But all these interpretations converge to the fine tuning problem: at the Planckian epoch, the energy density of matter was by ~ 120 orders larger than the dark energy one. This issue can be essentially relaxed when $\omega \neq -1$ and depends on time, this case is usually referred as dark energy of tracker field kind. For such a type of DE besides of the state equation and coupling of DE with the matter, we should define the vector of 4-velocity ($\vec{u}$, $(\vec{u})^2 = -1$) which indicates the direction of the energy flow. Using this vector we can define the 3d metrics tensor $h_{ik} = u_i u_k - g_{ik}$ in terms of the 4d space-time and thermodynamical parameters such as *energy density* ε, *pressure density* P, and *stress-tensor* σ_{ik}:

$$\varepsilon = T_{ik}u^i u^k, \quad P = \frac{1}{3}T_{jl}h^{jl}, \quad \sigma_{ik} = T_{jl}\left(h_i^j h_k^l - \frac{1}{3}h^{jl}h_{ik}\right).$$

For cosmological applications, we used the constant equation of state $P = \omega\varepsilon$ with $\omega = 0$ for the case of dust-like pressureless matter, $\omega = 1/3$ for electromagnetic field, and $\omega = -1$ for the case of Einstein's Λ-constant. For the case of scalar fields, the general form of $\omega = \omega(\tau)$ may exist. The scalar field has a density of Lagrangian: $\mathcal{L} = \frac{1}{2}g^{ij}\varphi_{,i}\varphi_{,j} - V(\varphi)$ leading to $\varepsilon^{(\varphi)} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)$ and $P^{(\varphi)} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)$ for φ which is homogeneously and isotropically distributed on a 3d hypersurface. Such a kind of DE finds its interpretation in the framework of the gravitation theory generalizations, *e.g.*, the branes theory and theory of gravitation with a more general geometry than the Riemann's one or some unified theories of fundamental physical interactions.

MATTER AND DARK ENERGY

For a two-component model of the Universe consisting of matter and DE, the energy-stress tensor is represented by $T_k^i = T^{(M)}{}_k^i + T^{(DE)}{}_k^i$. Conservation equations are written as $\nabla_i\left(T^{(M)}{}_k^i + T^{(DE)}{}_k^i\right) = 0$, or, in other form, $\nabla_i T^{(M)}{}_k^i = Q_k$ and $\nabla_i T^{(DE)}{}_k^i = -Q_k$, where $\vec{Q} \equiv \{Q_k\}$ is vector of an energy flow [5]. For the general case, one should define the vector of the energy flow between two components.

We assume an unperturbed DE and comoving matter and DE on an homogeneous and isotropic background. This simplification leads to $\overline{\nabla}_i T^{(DE)}{}_k^i = 0$, where $\overline{\nabla}_i$ is a covariant derivative in the isotropic and homogeneous space with a metrics tensor $\overline{g}_{ik}$. The real perturbed space-time presented by the metrics g_{ik}. The motion equations for particles comoving to the unperturbed background and DE (in the case of $\overline{\sigma}_{ik}^{(DE)} = 0$) are as follows:

$$\frac{d\overline{u}^i}{ds} = -\overline{\Gamma}_{jk}^i \overline{u}^j \overline{u}^k = -\Gamma_{jk}^i \overline{u}^j \overline{u}^k + f^i, \tag{1}$$

where $\overline{\Gamma}_{jk}^i$ and Γ_{jk}^i are the Christoffel symbols defined for the metrics $\overline{g}_{ik}$ and g_{ik}, respectively, $f^i = (\Gamma_{jk}^i - \overline{\Gamma}_{jl}^i)\overline{u}^j\overline{u}^l$ is vector of an additional force needed for an homogeneous distribution of DE in the regions of perturbations. The energy flow vector is $Q_k = \overline{\nabla}_i T^{(DE)}{}_k^i - \nabla_i T^{(DE)}{}_k^i$ that gives

$$Q_k = W_{kl}^j T^{(DE)}{}_j^l - W_{jl}^l T^{(DE)}{}_k^j,$$

with a tensor $W_{jk}^i \equiv \Gamma_{jk}^i - \overline{\Gamma}_{jk}^i$. The definition of perturbations is ambiguous and depends on the choice of a gauge.

The metrics in a longitudinal gauge has the form

$$ds^2 = a(\eta)^2[-(1 + 2\Psi(\eta)Y(x^\alpha))d\eta^2 + (1 + 2\Phi(\eta)Y(x^\alpha))\delta_{\beta\gamma}dx^\beta dx^\gamma],$$

where $\Phi(\eta)$ and $\Psi(\eta)$ are Bardeen's potentials [1]. Non-zero components of tensor W_{jk}^i according to this metrics are the following (α, $\beta = 1$, 2, 3):

$$W_{00}^0 = \dot{\Psi}Y, \qquad W_{0\alpha}^0 = W_{\alpha 0}^0 = -k\Psi Y_\alpha, \qquad W_{00}^\alpha = -k\Psi Y^\alpha,$$
$$W_{0\beta}^\beta = W_{\beta 0}^\beta = \dot{\Phi}Y, \qquad W_{\beta\beta}^0 = \left(2\frac{\dot{a}}{a}(\Phi - \Psi) + \dot{\Phi}\right)Y,$$
$$W_{\beta\alpha}^\beta = W_{\alpha\beta}^\beta = -k\Phi Y_\alpha, \quad \text{and} \quad W_{\beta\beta}^\alpha = k\Phi Y^\alpha \quad \text{for } \alpha \neq \beta \text{ (no sum on } \beta).$$

Thus, the components of the energy flow and additional force vectors are

$$Q_0 \;=\; 3\dot{\Phi}(\varepsilon^{(DE)} + P^{(DE)})Y, \quad Q_\alpha = k\Psi(\varepsilon^{(DE)} + P^{(DE)})Y_\alpha \tag{2}$$

and

$$f^0 \;=\; \dot{\Psi}Y, \quad f^\alpha = -k\Psi Y^\alpha, \tag{3}$$

respectively.

EVOLUTION OF MATTER DENSITY PERTURBATIONS

Formation of the large-scale structure of the Universe is described by the linear theory of scalar perturbations. We use here the gauge-invariant approach presented in [1, 4, 5, 8]. We have considered a case of a two-components universe with small perturbations in the dust-like matter component and no perturbations in the DE component. The non-zero components of energy-stress tensors for every component are

$$T^{(M)}{}^0_0 = -\overline{\varepsilon}^{(M)}(1+\delta^{(M)}Y), \quad T^{(M)}{}^0_\alpha = (\overline{\varepsilon}^{(M)}+\overline{P}^{(M)})vY_\alpha, \quad T^{(M)}{}^\alpha_0 = (\overline{\varepsilon}^{(M)}+\overline{P}^{(M)})vY^\alpha,$$
$$T^{(M)}{}^\alpha_\beta = \overline{P}^{(M)}[(1+\pi^{(M)}Y)\delta^\alpha_\beta + \Pi^{(M)}Y^\alpha_\beta], \quad T^{(DE)}{}^0_0 = -\overline{\varepsilon}^{(DE)}, \qquad T^{(DE)}{}^\alpha_\beta = \overline{P}^{(DE)}\delta^\alpha_\beta,$$

where δ and v are perturbations of the energy density and velocity, respectively, $\pi^{(M)}$ and $\Pi^{(M)}$ are isotropic and anisotropic components of pressure perturbations (over-lines denote the background magnitudes). From Einstein's equations $\delta G^0_0 = 4\pi G\delta T^0_0$, $\delta G^0_\alpha = 4\pi G\delta T^0_\alpha$, and $\delta G^\beta_\alpha = 4\pi G\delta T^\beta_\alpha$, we obtain the following connection between perturbations of metrics and perturbations of the matter density and velocity:

$$4\pi Ga^2\overline{\varepsilon}^{(M)}D^{(M)} = (k^2-3K)\Phi, \tag{4}$$
$$4\pi Ga^2(\overline{\varepsilon}^{(M)}+\overline{P}^{(M)})V^{(M)} = k\left(\left(\frac{\dot a}{a}\right)\Psi - \dot\Phi\right), \tag{5}$$
$$4\pi Ga^2\overline{P}^{(M)}\Pi^{(M)} = -k^2(\Phi+\Psi). \tag{6}$$

The conservation equations $\delta T^{(M)}{}^i_{0|i} = Q_0$ and $\delta T^{(M)}{}^i_{\alpha|i} = Q_\alpha$ lead to the following equations for the matter density and velocity perturbations:

$$\dot D_g^{(M)} + 3(c_s^2-\omega^{(M)})\frac{\dot a}{a}D_g^{(M)} + (1+\omega^{(M)})kV^{(M)} + 3\omega^{(M)}\frac{\dot a}{a}\Gamma^{(M)} = -3\dot\Phi\frac{\varepsilon^{(DE)}}{\varepsilon^{(M)}}(1+\omega^{(DE)})$$
$$\dot V^{(M)} + \frac{\dot a}{a}(1-3c_s^2)V^{(M)} - k(\Psi-3c_s^2\Phi) - \frac{c_s^2k}{1+\omega^{(M)}}D_g^{(M)} - \frac{\omega^{(M)}k}{1+\omega^{(M)}}\left[\Gamma^{(M)} - \frac{3}{2}\left(1-\frac{3K}{k^2}\right)\Pi^{(M)}\right] = k\Psi\frac{\varepsilon^{(DE)}}{\varepsilon^{(M)}}\frac{1+\omega^{(DE)}}{1+\omega^{(M)}},$$

where $V = v$, $D_g = \delta + 3(1+\omega)\Phi$, $D = \delta + 3(1+\omega)\dfrac{\dot a}{a}\dfrac{V}{k}$, $\Gamma = \pi - \dfrac{c_s^2}{\omega}\delta$ are gauge-invariant amplitudes [1–8] ($c_s^2 = \dot P/\dot\varepsilon$ is a square of the sound speed).

For DE component, we have $\delta T^{(DE)}{}^i_{0|i} = -Q_0$ and $\delta T^{(DE)}{}^i_{\alpha|i} = -Q_\alpha$ that gives the equations $\dot D^{(DE)} = 0$ and $\dot V^{(DE)} = 0$. If we suppose initial zero perturbations of dark energy $\left(D^{(DE)}_{in} = 0,\ \Gamma^{(DE)}_{in} = 0, \text{ and } V^{(DE)}_{in} = 0\right)$ then $D^{(DE)} = V^{(DE)} = 0$.

For the dust-like matter $\Pi^{(M)} = c_s^{(M)} = \omega^{(M)} = \Gamma^{(M)} = 0$, the conservation equations are simplified:

$$\dot D_g^{(M)} + kV^{(M)} = -3\dot\Phi\frac{\varepsilon^{(DE)}}{\varepsilon^{(M)}}(1+\omega^{(DE)}), \quad \dot V^{(M)} + \frac{\dot a}{a}V^{(M)} - k\Psi = k\Psi\frac{\varepsilon^{(DE)}}{\varepsilon^{(M)}}(1+\omega^{(DE)}). \tag{7}$$

The set of Eqs. (4)–(6) and (7) describes the evolution of scalar perturbations of matter in the Universe with DE which is homogeneously distributed over the whole space. For the case of DE uncoupled with the dust-like matter, the right-hand sides of Eqs. (7) will be equal to zero. In this case, the energy density of DE will trace matter density perturbations by means of metrics perturbations, so, it will be perturbed [2, 3, 7]. The evolution of perturbation amplitudes of the matter density $D_g^{(M)}$ for the two cases of DE (with an unperturbed energy density and a perturbed one) are shown in Fig. 1. For the calculations, the following values of parameters have been used: the constant state equation parameter of DE is $\omega^{(DE)} = -0.8$, the current contents of DE and matter are $\Omega^{(DE)} = 0.7$ and $\Omega^{(M)} = 0.3$, respectively, and the dimensionless Hubble constant is $h = 0.65$. The amplitude $D_g^{(M)}$ is larger for the case of the unperturbed DE that is stipulated by an energy flow from the DE component to the matter one in a perturbed region.

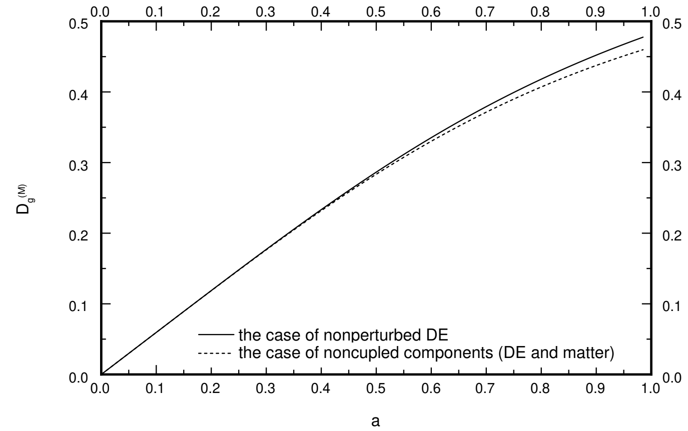

Figure 1. The dependence of $D_g^{(M)}$ on scale factor a for two kinds of DE: the uncoupled with the dust-like matter DE (dashed line) and unperturbed DE (solid line). The state of DE is $\omega^{(DE)} = -0.8$ and other cosmological parameters are $\Omega^{(DE)} = 0.7$, $\Omega^{(M)} = 0.3$, $k = 10^{-1}$ Mpc^{-1}, $h = 0.65$

CONCLUSIONS

We have analysed the evolution of matter density perturbations for two kinds of dark energy: (i) unperturbed homogeneously distributed DE and (ii) uncoupled with dust-like matter DE. The expressions for the energy flow between components (2) and an additional force which keeps a homogeneous distribution of DE (3) as well as Equations for the evolution of matter density perturbations (4)–(6) and (7) are obtained. Their numerical solutions show that the gauge-invariant amplitude of matter density perturbations increases in the case of the homogeneously distributed DE slightly faster than in the case of dark energy uncoupled with the matter. This difference can be explained by existence of an energy flow from DE to the dust-like matter and the additional force smoothing DE.

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