PARAMETRIC INTERACTION OF WHISTLER WAVES AND KINETIC ALFVÉN WAVES IN THE SPACE PLASMAS

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Satellite observations show a close relationship between the whistlers and lower-hybrid waves in space plasmas. Intense whistler waves generated by lighting discharges [17] or by quasi-monochromatic VLF transmitters can be unstable to three-wave parametric interaction. Work [7] demonstrates experimental results about excited lower hybrid waves by VLF whistler mode waves in the topside ionosphere and near magnetosphere. Kinetic Alfvén waves are often observed by satellites [21, 32]. In this paper we consider analytically a possible mechanism of this relationship – parametric interaction of whistler pump waves with lower-hybrid and Alfvén waves in magnetized plasma with small plasma parameter. In the dynamics of Alfvén waves the kinetic effects (finite ion Larmor radius and electron inertia) are taken into account. A nonlinear dispersion equation describing three-wave interaction is obtained in the framework of two-fluid magnetohydrodynamics. The instability growth rates and time of instability development are found. Our theoretical investigation shows that the whistler mode well be an effective source of the lower hybrid and Alfvén waves in the magnetospheric plasma. This nonlinear process can take place in the Earth magnetosphere and in the solar atmosphere. The products of the decay – lower-hybrid and kinetic Alfvén waves – can effectively interact with magnetospheric and solar plasmas.

INTRODUCTION

The interest to the three-wave resonant (TWR) parametric decay closely related with the study of the origin of the waves observed in the laboratory and space plasmas. Also, different wave modes damp with different rates, and a mutual transformation of the wave modes caused by TWR can significantly change the rate of plasma heating, often providing an explanation for enhanced (or reduced) plasma heating. Therefore, the parametric interactions is of great interest [3, 25, 22, 27, 12, 20, 34, 35]. In introduction we shall be interested in processes, which can be united by the same physical phenomenon, the parametric interaction of whistler mode waves with ionospheric and magnetospheric plasmas [23] analyzing the effect of parametric excitation of the electrostatic whistler waves by electron plasma waves. A new excitation mechanism of auroral Langmuir and Alfvén waves in the planetary magnetosphere is presented in [10]. It was shown that a large-amplitude electromagnetic whistler wave propagating along the magnetic field lines can nonlinearly generate Langmuir and Alfvén waves through three wave parametric instability. The question of a generation of the oblique Alfvén waves by the parametric instability of the whistlers in the near Earth Plasma is considered in [28], namely the decay process with participation two whistlers waves spreading under angles to an external magnetic field and oblique Alfvén wave. It is shown that parametric instability whistlers should reduce to fast growth oblique Alfvén waves with perpendicular wave length. There is also very interesting work [13]. In this paper the parametric instability of quasi-monochromatic VLF waves in the upper ionosphere is considered. It is shown that the initial signal scattering into a low-frequency plasma wave and decaying into low frequency and ion-cyclotron waves is the most effective. A parametric interaction between two whistlers and ion-cyclotron wave is considered. The above process also may be responsible for excitation of the low-hybrid resonance in the ionosphere and magnetosphere of the Earth. The processes of whistlers scattering in the fields of big angles to an external magnetic field were analyzed in the paper [29]. In the paper [25] the decay of UHW on the whistler and KAW was studied. In the paper [22] the decay of LHW on the UHW and electromagnetic wave was studied. Later [20] used theoretical results of Murtaza and Shukla for interpretation of the nature of downshifted maximum, and of Stenflo for interpretation of stimulated electromagnetic emission in the ionosphere. In the paper [35] the nonlinear interaction of kinetic Alfvén wave with upper hybrid wave has been investigated, and the results were used for the interpretation of long-period geomagnetic pulsations.

Whistlers are one of the most widespread type of waves in a magnetized plasma. The generation mechanisms are related to flashes, particle beams, anisotropic velocity distributions of particles, and nonlinear processes in space plasmas. In the Earth magnetosphere, whistlers can propagate from one hemisphere to another, and under favorable conditions can be detected by the ground instruments and satellites in "magnetoconjugated" points. The low-hybrid waves were often observed simultaneously with whistlers [4, 8, 11, 26, 6, 7]. Satellite observations shown a close relationship between whistlers and LHWs [30, 5], and suggested that whistlers are the source of the lower hybrid wave.

However, despite of the observational data mentioned above, an exact mechanism of such a close relation between lower hybrid emissions and whistlers is still uncertain. In the present paper the nonlinear interaction of whistlers and lower hybrid waves have been investigated. Lower hybrid wave is excited as a result of parametric decay of whistlers into KAWs and LHW. A high efficiency of the interaction among KAWs and whistlers is caused by the presence of the longitudinal component of KAW electric field.

BASIC EQUATIONS

Parametric instability, where the whistlers wave decay to the LHW and kinetic Alfvén wave W = LHW + KAW is considered.

Whistler pump wave

$$\vec{E}_0 = (E_{0x}\vec{e}_x + E_{0y}\vec{e}_y)\exp i\psi_0 + c.c., \tag{1}$$

where $\psi_0 = -\omega_0 t + k_{0x}x + k_{0z}z$, $\omega_0 = \left(k_0^2 c^2/\omega_{pe}^2\right) \mid cos\theta\omega_{Be} \mid$, propagates in the homogeneous magnetized plasma ($\overrightarrow{B_0} = B_0\overrightarrow{e_z}$). It is assumed that the wave synchronism condition is satisfied

$$\omega_0 = \omega + \omega_1, \quad \vec{k}_0 = \vec{k} + \vec{k}_1, \tag{2}$$

where $\omega_0, \vec{k}_0$ – the frequency and wave vector of whistler pump wave, $\omega, \vec{k}$ – the frequency and wave vector of kinetic Alfvén wave, $\omega_1, \vec{k}_1$ – the frequency and wave vector of lower hybrid wave. Let us consider that all wave vectors are situated in OXZ plane.

To study three-waves parametric interaction we use two-fluid magnetohydrodynamics (MHD):

$$\frac{\partial \vec{v}_\alpha}{\partial t} = \frac{1}{m_\alpha}(e_\alpha \vec{E} + \vec{F}_\alpha) + (\vec{v}_\alpha \times \vec{\omega}_{B\alpha}) - \frac{T_\alpha}{m_\alpha n_\alpha}\nabla n_\alpha, \tag{3}$$

$$\frac{\partial n_\alpha}{\partial t} = -\nabla(n_\alpha \vec{v}_\alpha), \tag{4}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}, \tag{5}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{6}$$

$$\nabla \cdot \vec{E} = 4\pi\rho, \tag{7}$$

where

$$\vec{j} = e(n_i\vec{v}_i - n_e\vec{v}_e),$$
$$\rho = e(n_i - n_e),$$
$$\vec{F}_\alpha = \frac{e_\alpha}{c}(\vec{v}_\alpha \times \vec{B}) - m_\alpha(\vec{v}_\alpha\nabla)\vec{v}_\alpha.$$

Index $\alpha = i, e$ corresponds to the ion and electron components of plasma, respectively. Electron density and velocity, electric and magnetic field are presented in the form:

$$\begin{aligned} n_e &= n_0 + \tilde{n}_A + \tilde{n}_1, \\ \vec{v}_e &= \vec{v}_0 + \vec{v}_A + \vec{v}_1, \\ \vec{E} &= \vec{E}_0 + \vec{E}_A + \vec{E}_1, \\ \vec{B} &= B_0\vec{e}_z + \vec{B}_A, \end{aligned} \tag{8}$$

where n_0 is the mean value of plasma density, index 0 in expression for $\vec{v}_e$ and $\vec{E}$ denotes the variable related to the pump wave, and indexes A and 1 denote the variables related to KAW and UHW, respectively.

DISPERSION EQUATION OF ALFVÉN WAVES

To obtain dispersion equation we use plasma approximation

$$\tilde{n}_e = \tilde{n}_i, \tag{9}$$

as Alfvén waves are slow. Here $\tilde{n}_e$ and $\tilde{n}_i$ are the perturbations of electron and ion density, respectively. From the motion Equation (3) and continuity Equation (4) we find the expression for $\tilde{n}_e$, $\tilde{n}_i$:

$$\frac{\tilde{n}_e}{n_0} = \left(1 - \frac{v_{ph}^2}{v_{T_e}^2}\right)^{-1} \frac{e}{T_e}\left[\varphi - A + \frac{k_x}{k_z^2}\frac{\omega}{\omega_{Be}}\frac{1}{e}\left(i\frac{\omega}{\omega_{Be}}F_x + F_y\right) + \frac{F_z}{iek_z}\right], \tag{10}$$

$$\frac{\tilde{n}_i}{n_0} = -\frac{e}{T_e}\frac{\mu_i}{1+\mu_i}\left(\varphi + \frac{\omega_{Bi}^2 k_z^2}{\omega^2 k_x^2}A\right), \tag{11}$$

where $v_{ph} = \omega/k_z$, $A = \frac{\omega}{k_z c}A_z$, $\mu = k_x^2\rho_i^2$, $\rho_i = v_{Ti}/\omega_{Bi}$ is the ion Larmor radius, φ and A_z are scalar and vector potentials of KAW electric field, respectively. The first relation between φ and A follows from (10)–(11):

$$\begin{aligned} A \;=\; & \left(1 + \frac{T_e}{T_i}\frac{\mu_i}{1+\mu_i}\left(1 - \frac{V_{ph}^2}{V_{Te}^2}\right)\right)\varphi - \frac{k_x^2}{k_z^2}\frac{\omega^2}{\omega_{Be}^2}\frac{1}{iek_x}\bar{F} + \\ & \frac{1}{iek_z}F_z - \frac{T_e}{e}\frac{V_{ph}^2}{V_{Te}^2}\frac{\vec{k}}{\omega}\cdot\left(\frac{n}{n_0}\vec{V}\right)_e, \end{aligned} \tag{12}$$

where

$$\bar{F} = \left(F_x - i\frac{\omega_{Be}}{\omega}F_y\right).$$

We can find the second relation between φ and A from the perpendicular projection of Ampere low:

$$-k^2 k_z A_z = \frac{4\pi}{c}k_x j_x. \tag{13}$$

The linear part of the transverse current j_x is determined by the plasma ion component, but the nonlinear perpendicular current is generated by the beating of LHW and whistler in electron motions:

$$j_x = en_0 V_{ix}^L + j_{ex}^{NL}. \tag{14}$$

Inserting the expression for ion velocity and nonlinear electron current we get the second relation between KAW potentials:

$$\begin{aligned} A \;=\; & \frac{V_{ph}^2}{V_A^2}\frac{1}{1+\mu_i}\varphi + \left(k_z^2\delta_i^2\right)^{-1}\frac{m_i}{e}\frac{\omega}{k_x}\frac{n_e^L}{n_0}V_{ex}^L + \left(k_z^2\delta_i^2\right)^{-1}\frac{m_i}{e}\frac{\omega^2}{\omega_{Be}^2}\times \\ & \left[\left(\frac{V_{ph}^2}{V_{Te}^2} - 1\right)^{-1}\frac{1}{im_e k_z}F_z + \left(\frac{V_{ph}^2}{V_{Te}^2} - 1\right)^{-1}\frac{\omega}{k_z^2}\vec{k}\cdot\left(\frac{n}{n_0}\vec{V}\right) + \frac{1}{im_e k_x}\bar{F}\right], \end{aligned} \tag{15}$$

where $\delta_i^2 = c^2/\omega_{pi}^2$. Equating (12)–(15) we obtain the nonlinear dispersion equation for KAW:

$$\begin{aligned} & \left[\frac{V_{ph}^2}{V_A^2}(1+\chi_e) - (1+\bar{\mu}_i)\right]\frac{1}{1+\mu_i}\varphi = \\ & \frac{1}{iek_z}F_{ez} - \frac{m_e}{m_i}\frac{V_{ph}^2}{V_A^2}(1+\chi_e)\frac{1}{iek_x}\left(F_{ex} - i\frac{\omega_{Be}}{\omega}F_{ey}\right) - \\ & \frac{m_i}{e}\left(k_z^2\delta_i^2\right)^{-1}\frac{\omega}{k_x}(1+\chi_e)\frac{n_e^L}{n_0}V_{ex}^L, \end{aligned} \tag{16}$$

where $\chi_e = k_x^2\delta_e^2, \bar{\mu}_i = (1+T_e/T_i)\mu_i$. From the motion equation we find electron velocity components in the field of whistler wave:

$$v_{0x} = -i\frac{eE_{ox}}{m_e\left(\omega_0 - \omega_{Be}\right)}, \qquad v_{0x} = \frac{eE_{ox}}{m_e\left(\omega_0 - \omega_{Be}\right)}, \tag{17}$$

and from (6) we find components of magnetic field pump wave:

$$b_{0x} = -i\frac{ck_{oz}}{\omega_0}E_{0x}, \qquad b_{0y} = \frac{ck_{oz}}{\omega_0}E_{0x}, \qquad b_{0z} = i\frac{ck_{ox}}{\omega_0}E_{0x}. \tag{18}$$

Velocity components in the LHW field can be obtained from electron motion equation:

$$v_{1x} = -\frac{ek_{1x}\omega_1}{m_e\left(\omega_1^2 - \omega_{Be}^2\right)}\varphi_1, \qquad v_{1y} = -i\frac{ek_{1x}\omega_{Be}}{m_e\left(\omega_1^2 - \omega_{Be}^2\right)}\varphi_1, \qquad v_{1z} = -\frac{ek_{1z}}{m_e\omega_1}\varphi_1. \tag{19}$$

Using expressions (17)–(19) and from (16), we find dispersion equation for the Alfvén wave:

$$\varepsilon_A\varphi = \mu_A\left(E_{0x}\varphi_1^*\right), \tag{20}$$

where μ_A is the coupling coefficient defined by the expression:

$$\mu_A = i\frac{e}{m_e}\frac{k_{1x}}{\omega_{Be}}\frac{k_{0z}}{k_z}\frac{k_z^2V_A^2}{\omega_0}\left(1+\mu_i\right)\left[1 + \frac{m_e}{m_i}\frac{k_z}{k_x}\frac{\omega_{Be}^2}{\omega_1\omega}\frac{k_{1z}}{k_{1x}}\left(1+\bar{\mu_i}\right)\right],$$

$$\varepsilon_A = \omega^2 - k_z^2V_A^2\left(1+\bar{\mu_i}\right).$$

In the calculation of the coupling coefficient μ_A, we have taken into account the pondermotive force created by scattering whistlers pump wave on LHW. And since we consider magnetosphere, where the plasma parameter is as $m_e/m_i << \beta << 1$, and we keep therms with finite-Larmor-radius effects in the dispersion equation for KAW.

DISPERSION EQUATION OF LHW

Dispersion equation of LHW can be find from Puasson equation:

$$\triangle\varphi_1 = -4\pi e\left(\tilde{n}_i - \tilde{n}_e\right). \tag{21}$$

We find expression for $\tilde{n}_i$, $\tilde{n}_e$ from the motion Equation (3) and continuity Equation (4):

$$\tilde{n}_i = \frac{en_0}{m_i}\left(\frac{k_{1x}^2}{\omega_1^2 - \omega_{Bi}^2} + \frac{k_{1z}^2}{\omega_1^2}\right)\varphi_1, \tag{22}$$

$$\widetilde{n_e} = -\frac{en_0}{m_e}\left(\frac{k_{1x}^2}{\omega_1^2 - \omega_{Bi}^2} + \frac{k_{1z}^2}{\omega_1^2}\right)\varphi_1 - \frac{n_0}{m_e}\frac{k_{1x}\omega_{Be}}{\omega_1\left(\omega_1^2 - \omega_{Be}^2\right)} \times \left[i\frac{\omega_1}{\omega_{Be}}F_{1x} + F_{1y} + i\frac{k_{1z}}{k_{1x}}\frac{\left(\omega_1^2 - \omega_{Be}^2\right)}{\omega_1\omega_{Be}}F_{1z}\right] - \frac{\vec{\nabla}\langle\tilde{n_A}\vec{V_0}\rangle}{-i\omega}, \tag{23}$$

where components of pondermotive force are defined by interaction of pump wave and Alfvén wave. From (21)–(23) we find the dispersion equation for LHW:

$$\varepsilon_1 \varphi_1 = \mu_1 \left(E_{0x} \varphi^*\right), \tag{24}$$

where coupling coefficient μ_1 is defined by the expression:

$$\mu_1 = i \frac{e}{m_e} \frac{\mu_s}{1+\mu_i} \frac{\omega}{k_z} \frac{1}{V_{Te}^2} \frac{k_{0z}}{\omega_0} \frac{\omega_1 \omega_{Be}}{k_{1x}} \left[1 + \frac{k_{1z}}{k_{1x}} \frac{k_z}{k_{0z}} \frac{k_z}{k_x} \frac{\omega_0}{\omega_1} \frac{\omega_{Bi}^2}{\omega^2} \frac{V_{Te}^2}{V_s^2} \left(1 + \tilde{\mu_i}\right) + \frac{\omega_0}{\omega} \frac{k_z}{k_{0z}} \right],$$

$$\varepsilon_1 = \omega_1^2 - \frac{\omega_{pi}^2}{1 + \frac{\omega_{pe}^2}{\omega_{Be}^2}} \left(1 + \frac{m_i}{m_e} \frac{k_{1z}^2}{k_1^2} \right).$$

NONLINEAR DISPERSION EQUATION FOR THREE-WAVE INTERACTION

From Equations (20) and (24) we find the nonlinear dispersion equation describing three-wave interaction (decay of whistler pump wave into the KAW and LHW):

$$\varepsilon_A \varepsilon_1^* = \mu_A \mu_1^* |\varphi_0|^2. \tag{25}$$

Assuming in (25)

$$\omega = \omega_r + i\gamma_1,$$

$$\omega_1 = \omega_{1r} + i\gamma_1,$$

(where $|\gamma| \ll \omega_r, \omega_{1r}$) and decomposing ε_A and ε_1 to the Taylor series after the small parameter $i\gamma_1$ we obtain the expression for the instability growth rate

$$\gamma_1 = \left. \frac{\mu_A \mu_1^* |E_{0x}|^2}{\frac{\partial \varepsilon_A}{\partial \omega} \frac{\partial \varepsilon_1}{\partial \omega_1}} \right|_{\omega=\omega_r, \omega_1=\omega_{1r}}, \tag{26}$$

where ω_r and ω_{1r} could be found from the equations

$$\varepsilon_A(\omega_r, \vec{k}) = 0, \qquad \varepsilon_1(\omega_1, \vec{k_1}) = 0.$$

Substituting values of derivatives

$$\frac{\partial \varepsilon_A}{\partial \omega} = 2\omega, \qquad \frac{\partial \varepsilon_1}{\partial \omega_1} = 2\omega_1,$$

and coefficients μ_A and μ_1 into (26) we obtain

$$\gamma_1^2 = \frac{W}{4} \frac{\omega_{pe}^2}{\omega_0^2} \mu_s V_A^2 k_{0z}^2 \times \left[1 + \frac{m_e}{m_i} \frac{k_z}{k_x} \frac{\omega_{Be}^2}{\omega_1 \omega} \frac{k_{1z}}{k_{1x}} \left(1 + \bar{\mu_i}\right) \right] \times \tag{27}$$
$$\left[1 + \frac{k_{1z}}{k_{1x}} \frac{k_z}{k_{0z}} \frac{k_z}{k_x} \frac{\omega_0}{\omega_1} \frac{\omega_{Bi}^2}{\omega^2} \frac{V_{Te}^2}{V_s^2} \left(1 + \tilde{\mu_i}\right) + \frac{\omega_0}{\omega} \frac{k_z}{k_{0z}} \right],$$

where $W = \dfrac{|E_{0x}|^2}{4\pi n_0 T_e}$.

DISCUSSION AND APPLICATION

In this paper we consider analytically the nonlinear parametric interaction of whistlers with LHW and Alfvénic wave in a low-β plasma. The kinetic effects (finite ion Larmor radius and electron inertia) are taken into account for dispersion law, when we investigate the dynamics of Alfvén wave. Such Alfvén wave are usually called kinetic Alfvén wave. They have some properties which are different from the MHD Alfvénic wave. They include a presence of nonzero parallel electric field E_z and can propagate in xz-plain. Due to this properties KAW interact effectively with whistler and amplify lower hybrid wave. The nonlinear three-wave parametric interaction we consider can observe in laboratory and space plasma with low-β plasmas. Such processes in space plasmas can take place in the ionosphere and magnetosphere of the Earth, and in the some regions of solar atmosphere where the plasma parameter is small.

In our paper we consider magnetospheric plasma in the capacity of supplement (or application). Typical parameters for magnetospheric plasma at $3-4\ R_E$ are following: $V_{Te} \approx 10^7\,\mathrm{cm\,c^{-1}}$, $\omega_{pe} \approx 10^5\,\mathrm{c^{-1}}$, $\omega_{Be} \approx 10^5\,\mathrm{c^{-1}}$, $\omega_{Bi} \approx 10^2\,\mathrm{c^{-1}}$. Substituting plasma parameters and whistlers intensity $W \simeq 10^{-5}$ into (27), the time of instability development is of $\tau \approx \gamma_{\max}^{-1} = 0.01$ c.

Our theoretical investigation show, that whistler wave, propagating in the magnetosphere, will be a source of lower hybrid and Alfvén waves. Theoretical results can explain satellite observations of the coexistence of whistler, lower hybrid and Alfvén waves.

The nonlinear parametric processes considered in this paper could take place in the solar corona as well, where whistlers are generated by energetic electrons in magnetic traps on the Sun. Whistlers will excite lower hybrid and Alfvén waves while propagating through the solar atmosphere. Lower hybrid wave can effectively accelerate electrons to high energy. Kinetic Alfvén waves interact effectively with plasma particles and participate in the heating and acceleration of space plasma particles owing to presence of the longitudinal component of electric field. Thus, the products of the decay – low-hybrid and kinetic Alfvén waves – can affect the magnetospheric and solar plasmas stronger than the initially excited whistler waves.

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