NONLINEAR INTERACTION OF MHD WAVES AND SOLAR CORONA HEATING

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The nonlinear parametric interaction of Alfvén waves with magnetosonic and ion-acoustic waves is considered on the basis of two-fluid magnetohydrodynamics. A nonlinear dispersion relation describing three-wave interaction, instability growth rate have been calculated and estimated. The analysis of theoretical results shows that kinetic effects in the Alfvén waves (the finite ion Larmour radius) are essential for the parametric interactions of waves.
Nonlinear parametric processes studied in the paper could take place in the solar coronal loops, where plasma parameter is small. The products of the decay – magnetosonic and ion-acoustic waves, can effectively heat the coronal plasma owing to rapid dissipation.

INTRODUCTION

Magnetohydrodynamic waves play an important role in the dynamics of the Earth magnetosphere, solar wind and atmosphere of the Sun. The long-period geomagnetic pulsations observed in the Earth's magnetosphere have been considered as MHD Alfvén waves (Nishida, 1980; Volokitin and Dubinin, 1989). The satellite observations show that the most part of turbulent pulsations are the Alfvén and magnetosonic waves in the solar wind (Belcher, 1971). They are thought to play an essential role in the solar wind heating. The MHD waves can provide a source of energy for corona heating. Alfvén waves have enough energy for heating of coronal loops (see, *e.g.*, Goossens, 1994 and references therein). But they are slowdamped. This is the reason why Alfvén wave can't heat coronal loops directly, and the problem arise: how the energy can be transferred from waves to plasma particles. The ion-acoustic and magnetosonic waves are rapid damped waves. Therefore, if the propagating Alfvén wave decays into ion-acoustic and magnetosonic waves, they can transfer rapidly their energy to particles due to dissipation. In the paper (Uchida, 1974; Wentzel, 1974) it was shown that the processes of formation of acoustic waves from Alfvén waves are effective in the region, where $V_A/V_s - 1/30 - 30$. The dissipation of wave energy arises on distances which are comparable with the length of coronal loops. Nonlinear interaction between Alfvén and ion-acoustic waves was considered for the case of linear dispersion , *i.e.*, without accounting thermal effect (finite Larmor radius of proton). But in the magnetosheric and coronal plasma thermal effects play an important role. Taking into account the thermal effects leads to nonlinear dependence of frequency from wavenumber, which is essential for nonlinear wave interaction.

In the present work we consider nonlinear parametric interaction of Alfvén wave with the ion-acoustic and magnetosonic waves.

BASIC EQUATION

It is assumed that Alfvén wave with finite amplitude propagates in homogeneous magnetized plasma ($\vec{B}_0 = B_0\vec{e}_z$):

$$\vec{E}_0 = (E_{0x}\vec{e}_x + E_{0z}\vec{e}_z)\exp i(-\omega_0 t + \vec{k}_0\vec{r}) + c.c., \tag{1}$$

where the frequency and wave vector are related by equality:

$$\omega_0^2 = k_{0z}^2 V_A^2(1 + t\mu_i). \tag{2}$$

Here $\mu_i = k_{0x}^2 \rho_i^2, \rho_i = V_{Ti}/\omega_{Bi}$ – proton Larmor radius, $\omega_{Bi} = eB/m_i c$ – ion cyclotron frequency, $t = T_e/T_i$. Our governing equations are a set of two-fluid MHD equation:

$$\frac{\partial \vec{v}_\alpha}{\partial t} = \frac{1}{m_\alpha}(e_\alpha \vec{E} + \vec{F}_\alpha) + (\vec{v}_\alpha \times \vec{\omega}_{B\alpha}) - \frac{T_\alpha}{m_\alpha n_\alpha} \nabla n_\alpha, \tag{3}$$

$$\frac{\partial n_\alpha}{\partial t} = -\nabla(n_\alpha \vec{v}_\alpha), \tag{4}$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}, \tag{5}$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}, \tag{6}$$

$$\nabla \cdot \vec{E} = 4\pi\rho, \tag{7}$$

where

$$\vec{j} = e(n_i \vec{v}_i - n_e \vec{v}_e),$$
$$\rho = e(n_i - n_e),$$
$$\vec{F}_\alpha = \frac{e_\alpha}{c}(\vec{v}_\alpha \times \vec{B}) - m_\alpha (\vec{v}_\alpha \nabla)\vec{v}_\alpha.$$

Index $\alpha = i, e$ corresponds to the ion and electron components of plasma, respectively. As $F_{pi} = \frac{m_e}{m_i} F_{pe}$, influence of F_{pi} force is small, and can be neglected. In Ampere-Maxwell's equation we also neglect the displacement current.

We consider equations in Cartezian coordinates (x, y, z), supposing that all of the wave vectors are situated in OXZ plane. It is assumed that the wave synchronism conditions are satisfied:

$$\omega_0 = \omega + \omega_1, \quad \vec{k}_0 = \vec{k} + \vec{k}_1, \tag{8}$$

where $\omega_1, \vec{k}_1, \omega, \vec{k}$ – frequency and wave vector of magnetosonic and ion-acoustic wave, respectively.

DISPERSION EQUATION FOR LOW-FREQUENCY PERTURBATIONS

Since Alfvén and ion-acoustic waves are the ultralow-frequency waves, we can use the plasma approximation for obtaining the dispersion equation:

$$\tilde{n}_e = \tilde{n}_i, \tag{9}$$

where $\tilde{n}_e$ and $\tilde{n}_i$ are perturbations of the ion and electron number densities. From the motion and continuity equations, we can write for $\tilde{n}_e$ and $\tilde{n}_i$:

$$\frac{\tilde{n}_e}{n_0} = \frac{e}{T_e}\left(\psi - \frac{k_x^2}{k_z^2}\frac{\omega^2}{\omega_{Be}^2}\phi - Q_{NL}\right), \tag{10}$$

$$\frac{\tilde{n}_i}{n_0} = \frac{e}{m_e}\left(\frac{k_z^2 \psi}{\omega^2} - \frac{k_x^2 \varphi}{\omega_{Bi}^2}\right). \tag{11}$$

Here

$$Q_{NL} = \frac{k_x \omega}{e k_z^2 \omega_{Be}}\left(i\frac{\omega}{\omega_{Be}} F_x + F_y\right) + \frac{F_z}{iek_z}.$$

To find the relation between φ and ψ, we use z component of Ampere's law in two potential approximation and equation of charge conservation. Comparing (10) with (11) and using relation between φ and ψ, we obtain the dispersion equation for ultralow-frequency perturbations:

$$\left[\frac{k_z^2 V_s^2}{\omega^2}\left(1 - \frac{\omega^2}{k_z^2 V_A^2}\right)\left(1 - \frac{\omega^2}{k_z^2 V_s^2}\right) - \mu_s\right]\varphi = -Q_{NL}. \tag{12}$$

In the absence of pump wave ($Q_{NL} = 0$) we have:

$$\begin{aligned} \omega_A^2 &= k_z^2 V_A^2 (1 + \mu_s), \\ \omega_s^2 &= k_z^2 V_s^2 / (1 + \mu_s). \end{aligned} \tag{13}$$

These expressions describe dispersion laws for the Alfvén and ion-acoustic waves, respectively. The special properties of these waves are their ability to propagate in xz plane, thus the wave energy can be transferred across the external magnetic field as well as along it. And since $\omega_s^2 \ll \omega_A^2$ and $\omega_A^2 \simeq k_z^2 V_A^2$, the dispersion equation for an ion-acoustic wave can be rewritten as:

$$\varepsilon\phi = \eta\psi_0 E_{1y}^*, \tag{14}$$

where

$$\varepsilon = \omega^2 - k_z^2 V_s^2 / (1 + \mu_s),$$

$$\eta = -\frac{\omega^2}{\omega_{Be}} \frac{k_{0x}}{k_z} \frac{V_{Te}^2}{V_A} \frac{e}{T_e},$$

In the calculation of the coupling coefficient η, we have taken into account the pondermotive force created by interaction of the pump wave with the magnetosonic wave.

DISPERSION EQUATION FOR MAGNETOSONIC WAVE

To obtain a dispersion equation for magnetosonic wave propagating across magnetic field, we also use plasma approximation. In this case ions and electrons velocity must coincide. From the electron equation of motion we find:

$$V_{ex} = \frac{e}{m_e \omega_{Be}} \left(E_{1y} + i\frac{\omega_1}{\omega_{Be}} E_{1x} \right) + \frac{1}{m_e \omega_{Be}} \left(i\frac{\omega_1}{\omega_{Be}} F_{1x} + F_{1y} \right), \tag{15}$$

where the term proportional to (ω/ω_{Be}) describes the inertial drift, of the wave that propagates across magnetic field. The component of pondermotive force is determined by interaction of pump wave and ion-acoustic wave. From the ion equation of motion and Maxwellś equation we write expression for V_{ix}:

$$V_{ix} = \frac{k_1^2 V_A^2}{\omega_1^2} \frac{eE_{1y}}{m_e \omega_{Be}}. \tag{16}$$

Comparing (15) with (16) we obtain dispersion equation for the magnetosonic wave:

$$\varepsilon_1 E_{1y} = \eta_1 \varphi^* \phi_0, \tag{17}$$

where

$$\varepsilon_1 = \omega_1^2 V_A^2$$

and coupling coefficient is determined by

$$\eta_1 = -2V_A V_S \frac{m_e}{e} \left(\frac{V_{Te}}{V_A} \right)^2 \frac{k_{0x}}{\omega_{Be}} \omega_1^2 \omega_0 \mu_e \tag{18}$$

$$\mu_e = k_x^2 \rho_e^2.$$

NONLINEAR DISPERSION EQUATION AND DECAY GROWTH RATE

From ion-acoustic (14) and magnetosonic (17) dispersion equations we can obtain a nonlinear dispersion relation describing three-wave interaction:

$$\varepsilon\varepsilon_1^* = \eta\eta_1^* \mid \phi_0 \mid^2 . \tag{19}$$

In the absence of the pump wave, $\phi_0 = 0$, two kinds of waves with frequencies $\omega = \omega_k$ and $\omega_1 = \omega_{1k}$ exist in the plasma. In the presence of the pump wave, $\phi_0 \neq 0$, energy transfer from the pump KAW to the other plasma modes occurs, and amplitudes increase with the growth rate. Putting $\omega = \omega_r + i\gamma$, $\omega_1 = \omega_{1r} + i\gamma$ ($\mid \gamma \mid \ll \omega_r, \omega_{1r}$) into (19) and expanding ε and ε_1 in Taylor series, we obtain an expression for the wave growth rate:

$$\gamma^2 = \left. \frac{\eta\eta_1^* |\phi_0|^2}{\frac{\partial\varepsilon}{\partial\omega} \frac{\partial\varepsilon_1}{\partial\omega_1}} \right|_{\omega=\omega_r, \omega_1=\omega_{1r}}, \tag{20}$$

where ω_r and ω_{1r} are defined from equations:

$$\varepsilon(\omega_r, \vec{k}) = 0, \qquad \varepsilon_1(\omega_1, \vec{k_1}) = 0.$$

Substituting the expressions for the coupling coefficients η, η_1 and derivative $\dfrac{\partial\varepsilon}{\partial\omega}$, $\dfrac{\partial\varepsilon_1}{\partial\omega_1}$ we find:

$$\gamma^2 = \frac{W}{2}\frac{V_S}{V_A}\frac{\omega_0^2}{\omega_{Be}k_z V_A}\left(\frac{\omega}{\omega_{Be}}\right)\mu_e\omega_{pe}^2, \tag{21}$$

where

$$W = \frac{\left|E_{0x}^2\right|}{4\pi n_0 T_e}. \tag{22}$$

CONCLUSIONS

Let us estimate the instability growth rate for the coronal plasma. Spectroscopic observations suggest the presence of the Alfvén waves with amplitudes $B/B_0 = 0.01$ in coronal loops (Hara, 1999). To examine existence of the Alfvén waves, these authors used the method first proposed by McClements *et al.* (1991). Typical parameters for such loop are the length $L = (2-5)\times 10^9$ cm, $n = (0.5-1)\times 10^{10}$ cm^{-3}, $B = 100 - 500$ G, $T = 4\times 10^6$ K, $\omega_{pe}/\omega_{Be} \approx 10$, $\mu_e \approx 1$, $\omega_1 \approx 10^{-1}\omega_{Bi}$. Substituting these values of the plasma parameters and the KAW intensity of $W \approx 10^{-5} - 10^{-6}$ into (21), we obtained $\gamma \approx \left(10^2 \div 10^3\right) c^{-1}$ and the time of instability development $\tau = \gamma^{-1} \approx 0.01$ c, respectively.

The nonlinear interaction of the Alfvén waves with the magnetosonic and ion-acoustic waves presented in this paper could occur in the Earth magnetosphere as well as in the solar magnetosphere, where a value of the plasma parameter β is low. In the solar atmosphere the sources of the Alfvén waves can be 5-min. chromospheric oscillation that swing the basis of magnetic loops and lead to propagation of the Alfvén waves into corona. Alfvén waves will transform into the kinetic Alfvén waves during the propagation along magnetic field. The kinetic Alfvén wave can interact more effectively with other kinds of waves because of the presence of a non-zero longitudinal component of the electric field E_z. When KAW reaches the region with low plasma parameter β it will decay into the magnetosonic and ion-acoustic waves. Due to their rapid dissipation, these waves are more effective for heating of the coronal plasma. The excitation of fast magnetosonic waves by phase mixing of an Alfvén wave was considered in Nakarjakov (1997). The relative effective excitation of the slow and fast magnetosonic waves by Alfvén waves needs further investigation.

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