CALCULATION OF DAMPING CONSTANT OF FRAUNHOFER LINES IN THE SOLAR ATMOSPHERE

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Reliable data on spectral lines damping are required to define chemical content of stellar atmospheres. We propose explicit expressions for the damping constant, which take into account several mechanisms influencing the extension of spectral lines. Considering inelastic collisions, we calculate the damping constant for some spectral lines under various temperatures. Using the most correct and empirically tested method for investigations of spectral lines we calculate the damping constant for spectral lines of iron group in the solar atmosphere.

The problem of the damping constant emanates from deficiency of reliable theoretical results, which according to opinion of many authors is related to computational complications. A theoretical value of the damping constant does not always coincide with the value determined from the observed spectra profiles. This is the case even for the solar atmosphere. Furthermore, for the conditions close to the solar atmosphere the damping constant is calculated theoretically only for several lines. The corresponding value lays within the interval $(1.1\ldots1.9)\gamma_6$, where γ_6 is the classical (Unsold) approximation for van der Waals interaction. Results of laboratory measurements appear to be even more inconsiderable in number and unreliable since it is practically impossible to reproduce in a laboratory the conditions prevailing in the solar atmosphere. Specifically, it is difficult to carry out an experiment with hydrogen as a basic agent of van der Waals collisions in conditions of the solar atmosphere. However, the results of laboratory investigations yield a correction factor for γ_6 only a bit greater than 1. Many attempts of empirical studies of Fraunhofer's lines damping constant were made in order to verify and improve theoretical and laboratory results.

Recently, modern devices have allowed an improvement of the precision of the observed data; there appeared more accurate laboratory measurements of atomic parameters and interaction constants and more adequate models of the solar atmosphere have been devised. However, no convenient formulae have been introduced so far which provide theoretical estimation of the damping constant by account of all the mechanisms bringing about the extension of spectral lines profiles. A problem of explicit calculation of a whole profile when only a few mechanisms act is apparently not simple, and also calls for separate researches. The misarranged empirical methods and inconsistency of the observed data with theoretical calculations are considered as an inability of theory to provide the quantitative description of profiles of Fraunhofer's lines, or as an unreliability of the damping constant theoretical computations.

Starting from the first principles for expressions for the damping constant, a series of mechanisms is found, which lead to its increase in comparison with Weisskopf-Lindholm damping for van der Waals interatomic interactions [1].

As it is known, in order to obtain trustworthy conclusions about nature and structure of space objects, the analysis of star radiation requires the detailed study of mechanisms of interaction between radiation and atomic systems. Atmospheres of the majority of stars consist preliminary of hydrogen and helium, while the quantity of atoms of other elements is of few orders less. Certain amount of atoms and molecules are on various ionization levels due to interactions with radiation and among themselves. The full description of star atmospheres needs a simultaneous solution of the transfer equation and the equations determining occupations of atoms quantum states. Within the approach of one-photon transitions a central role in the description is played by the light absorption coefficient.

The profile of the so-called absorption coefficient in a line, that is its dependence on light frequency, as well as the profile of atom spectral line are formed by a series of mechanisms. Among the mechanisms which are responsible for the full profile the colliding mechanism is determinative for the spectral line wing and leads to the formation of the Lorentz profile.

The Lorentz profile is characterized mainly by the damping constant γ, which determines the profile width. The value and temperature dependence of the constant are of particular importance for star spectra analysis. To adjust the observed profiles of Fraunhofer's lines with theoretically calculated profiles, the damping constant obtained from the Weisskopf-Lindholm theory is often increased artificially. In some cases the (required) magnification is unnaturally large, which is interpreted as theory inability to give a quantitative description of profiles of Fraunhofer's lines. In our opinion this is an incorrect point; it is necessary to carry out a detailed investigation of quantities obtained from the first principles and characterizing profiles of atomic spectral lines.

The damping constant γ and the frequency shift Δ of radiation of an atom interacting with other particles according to mechanisms responsible for the Lorentz constituent of a spectral line:

$$I(\omega) = \frac{\gamma}{2\pi} \frac{1}{(\omega - \omega_0 - \Delta)^2 + (\gamma/2)^2} \tag{1}$$

are connected with scattering cross-section σ by the following equation:

$$\gamma/2 - i\Delta = \frac{N}{V} < v\sigma >, \tag{2}$$

where

$$\sigma = \int\limits_0^\infty (1 - e^{i\delta}) 2\pi\rho d\rho. \tag{3}$$

In the above formula ρ denotes aiming distance, while complex phase reads

$$\delta = \eta + i\Gamma, \tag{4}$$

where real part η is equal to the difference between shifts of atomic phases, among which the transitions of frequency ω_0 occur. This phase shift is caused by the shift of atomic energy levels as a result of interaction among atoms and perturbing particles. Let us consider for simplicity of records N particles of one sort embedded in volume V. Moreover, this is just the case which occurs in star atmospheres, since basically atoms of hydrogen are the perturbing particles there. The value of 2Γ is equal to the total probability of atom transition from states, between which transition of frequency ω_0 takes place, to all other states. The quantity Γ causes an additional extension of spectral lines, the so-called extension caused by elastic collisions. Natural or radiation width of an atomic spectral line is not taken into account. The quantity v means atom velocity relatively to a perturbing particle; brackets in Eq. (2) stay for averaging over velocities.

The computations are performed in assumption that quasi-classical approximation is fulfilled, when a real part of the phase reads:

$$\eta = -\frac{1}{\hbar} \int\limits_{-\infty}^{\infty} \Delta E(t) dt. \tag{5}$$

Here ΔE is the correction to atom energy, caused by its interaction with a perturbing particle, t is the time.

We consider a rare particles system similar to star atmosphere. In this case the contribution to η will be determined mainly by interactions between particles in large distances. Then the potential energy $U(R)$ consists of interactions between their multipole momenta and decreases with the distance R according to a power law:

$$U(R) = -\hbar \sum_{l \geq 0} \frac{C_{n+2l}}{R^{n+2l}}, \tag{6}$$

where n determines the type of interaction, $\hbar$ is Planck constant.

Considering the Maxwell distribution over velocities, the dependence of a damping constant on temperature is obtained within the Weisspopf-Lindholm approximation. The corresponding expression reads:

$$\gamma_0 = \frac{N}{V} 4\sqrt{\pi} \left(\frac{2T}{M}\right)^{\frac{n-3}{2(n-1)}} \Gamma\left(\frac{2n-3}{n-1}\right) \Gamma\left(\frac{n-3}{n-1}\right) \left[\frac{\Gamma(1/2)\Gamma(\frac{n-1}{2})}{\Gamma(n/2)} C_n\right]^{\frac{2}{n-1}} \times$$
$$(1+p^2)^{\frac{1}{n-1}} \cos\left[\frac{2}{n-1} \mathrm{arctg} \frac{1}{p}\right], \tag{7}$$

where M is the reduced atom mass, T is the temperature in atomic units, $\Gamma(x)$ is Euler's gamma-function.

In the case $n = 6$, $p = 0$, the van der Waals damping constant is obtained within the Weisspopf-Lindholm approximation:

$$\gamma_{WL} = 7.901 \frac{N}{V} C_6^{2/5} \left(\frac{8T}{\pi M} \right)^{3/10}, \tag{8}$$

where p is the parameter of inelastic collisions, $0 \leq p \leq 1$.

The value of γ_0 for $p \neq 0$ and $p = 0$, in particular, reads:

$$\gamma_0 / \gamma_{WL} = (1 + p^2)^{\frac{1}{n-1}} \frac{\cos\left(\frac{2}{n-1} \mathrm{arctg} \frac{1}{p}\right)}{\cos \frac{\pi}{n-1}}. \tag{9}$$

The ratio is greater than 1. Therefore, account of inelastic collisions increases the damping constant and the increase is considerable.

In order to estimate the contribution of post van der Waals interactions, let us write the expansion of the Eq. (9) in powers of temperature:

$$\gamma / \gamma_0 = 1 + \sum_{k \geq 1} A_k (T/T_0)^{\frac{k}{n-1}}, \tag{10}$$

where coefficients A_k are determined from constants of post van der Waals interactions.

The repulsion forces are also found to contribute into the damping constant. The contribution is estimated approximately as follows: $\gamma > N \pi a_0^2 < v > /V = N 2 \pi a_0^2 (8T/\pi M)^{1/2} / V$, where a_0 is the scattering length which is of atomic diameter order.

Let us make a quantitative estimate of inelastic collisions contribution to γ. In the case of van der Waals interactions ($n = 6$) for the maximal contribution of inelastic collisions ($p = 1$), elementary calculations yield: $\gamma_0(p = 1)/\gamma_{WL} = 1.3504$. This estimate of inelastic collisions contribution can partially improve the artificial increase mentioned above of the damping constant inherent in analysis of star spectra.

Let us estimate numerically the values of A_k. Consider an example of sodium atom in the solar atmosphere, where hydrogen atoms at the temperature of $T = 1000$ K serve as perturbing particles. Thus, for the Na-H system we find: $A_1 = 0.452, A_2 = 0.295, \gamma/\gamma_0 > 1.2$. Even such not so high temperature in the star atmosphere scale causes significant increase of γ. For $T = 6420$ K, $\gamma/\gamma_0 > 1.32$.

The contribution of repulsion forces for the discussed above Na-H system for $a > 2.6$ Å yields $\Delta A_1 > 0.595$, which results in $\gamma/\gamma_{WL} > 1.92$ for $T = 1000$ K. For $T = 6420$ K, $\gamma/\gamma_{WL} = 2.206$.

By means of the proposed explicit expressions for the damping constant, one can make a numerical estimation of the constant for various temperatures in the solar photosphere attached to various values of inelastic collisions parameter. Computations, performed on the base of different methods of the damping constant investigations [2] are well-consistent with our theoretical computations.

Thus, our new results are consistent with the data of the other authors. They can be used for explanation of experimental studies of Fraunhofer's lines, in particular, for determination of chemical elements content in the solar atmosphere. The theoretical method proposed for the damping constant calculations can be generalized for the case of star atmospheres.

REFERENCES

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