NONLINEAR THREE–WAVE INTERACTION OF KINETIC ALFVÉN WAVES IN THE CORONAL LOOPS

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The interaction of kinetic Alfvén waves (KAW) with particle is very effective and essentially depend on the transversal length of the waves, which propagate in the coronal loops. Therefore, it is very important to know how the Alfvén wave spectrum is formed. In this work the resonance interaction of three Alfvén waves is shown to be one of the most important mechanism of such a spectrum formation. This interaction is very efficient when kinetic effects have considerable influence on the Alfvén wave dispersion. For some waves (with sufficiently small transverse wave lengths) the rate of transformation of one wave into another may even reach the frequency of initial wave. The results obtained may be used for the interpretation of the low frequency waves spectra observed in the solar atmosphere and explanation of the particle acceleration and coronal loops heating mechanisms.

INTRODUCTION

The basic dynamic processes in the photosphere-coronal plasma occur as wave processes. The photosphere is permanently disturbed being continuous source of waves propagating in the upper atmosphere. However, the sound and magneto-sound waves are known to undergo a strong refraction in the chromosphere, which prevents their emerge in the corona. Thus, to explain processes occurring in the coronal plasma, the Alfvén waves leaving freely the photosphere through the open magnetic force lines are the most suitable. Besides, such waves can be generated in the corona, for example in flaring coronal loops [1]. When propagating in the chromosphere and corona, the Alfvén waves can essentially change the spectrum owing to their nonlinear interaction with particles of plasma [2, 3]. Really, as follows from the two-fluid hydrodynamics [4], the nonlinear interaction of Alfvén waves appears very large and the characteristic time of interaction can be of an order of frequencies of interacting waves. It is necessary to note that the hydrodynamics does not give absolutely exact results, as apparent from comparison of the dispersion of kinetic Alfvén waves obtained in the hydrodynamics and kinetics.

From hydrodynamics for Alfvén waves we have

$$\omega^2 = (k_z v_A)^2 \left[1 + \left(1 + \frac{T_e}{T_i}\right) z_i\right], \tag{1}$$

where $z_i = (k_\perp v_{Ti}/\omega_{Bi})^2$, ω is the frequency of the wave, k_z, $k_\perp$ are the longitudinal and transverse wave vectors, v_A is the Alfvén velocity, v_{Ti} is the thermal velocity of ions, T_e, T_i are the electron and ion temperatures, ω_{Bi} is the ion cyclotron frequency.

From the kinetic equations for Alfvén waves we have [2]

$$\omega^2 = (k_z v_A)^2 \left[1 + \left(\frac{3}{4} + \frac{T_e}{T_i}\right) z_i\right], \quad z_i << 1,$$
$$\omega^2 = (k_z v_A)^2 \left[1 + \frac{T_e}{T_i}\right] z_i, \quad z_i >> 1. \tag{2}$$

It is apparent that dispersions well coincide only for rather large values of cross wave vectors. Besides, the complete dispersion of kinetic Alfvén waves obtained in the kinetic approximation contains the modified Bessel function [2, 3]

$$\omega^2 = (k_z v_A)^2 \left[\frac{z_i}{1 - A_0(z_i)} + \frac{T_e}{T_i}\right], \tag{3}$$

which in the hydrodynamics fails to be obtained. In (3) $A_0(z) = I_0(z)\exp(-z)$, $I_0(z)$ is the zero-order modified Bessel function. The greatest difference in dispersion is observed at a finite ratio of the Larmor radius to the wave length. That is the most interesting part of the spectrum, where the interaction between waves, as well as the interaction of waves with particles intensively occur. The kinetic approach is more exact because all quantities including the tensor of pressure are calculated instead of being chosen from the equation of state or otherwise.

NONLINEAR INTERACTION OF KINETIC ALFVÉN WAVES (KINETIC APPROACH)

Interaction of Alfvén waves in the kinetic approximation was considered in [5]. In this work, two potential approximations were carried out. In present work, such approximations were not made. The complete set of Maxwell and Vlasov kinetic equations was used. The system of three nonlinear equations describing nonlinear three wave interactions of kinetic Alfvén waves and taking into account of polarization of fields and currents was obtained

$$\begin{aligned}
\frac{dB_{\perp k}}{dt} &= \frac{1}{2}\omega_{Bi}\frac{v_A}{v_{Ti}}\int dk_2 dk_1 \delta(k-k_2-k_1)\frac{\mathbf{k}_{\perp 1}\times\mathbf{k}_{\perp 2}}{k_{\perp 1}k_{\perp 2}}\frac{1}{z_i^{1/2}}B_{\perp k_1}\frac{B_{\perp k_2}}{B_0} \\
& [z_i(D_1-D_2)+D(z_{i1}-z_{i2})+DD_1D_2(A_{01}-A_{02})]\,, \\
\frac{dB_{\perp k_1}}{dt} &= -\frac{1}{2}\omega_{Bi}\frac{v_A}{v_{Ti}}\int dk_2 dk \delta(k-k_2-k_1)\frac{\mathbf{k}_{\perp 1}\times\mathbf{k}_{\perp 2}}{k_{\perp 1}k_{\perp 2}}\frac{1}{z_i^{1/2}}B_{\perp k}\frac{B_{\perp k_2}}{B_0} \\
& [z_{i1}(D-D_2)+D_1(z_i-z_{i2})+DD_1D_2(A_0-A_{02})]\,, \\
\frac{dB_{\perp k_2}}{dt} &= -\frac{1}{2}\omega_{Bi}\frac{v_A}{v_{Ti}}\int dk_1 dk \delta(k-k_2-k_1)\frac{\mathbf{k}_{\perp 1}\times\mathbf{k}_{\perp 2}}{k_{\perp 1}k_{\perp 2}}\frac{1}{z_i^{1/2}}B_{\perp k}\frac{B_{\perp k_1}}{B_0} \\
& [z_{i2}(D_1-D)+D_2(z_{i1}-z_i)+DD_1D_2(A_{01}-A_0)]\,,
\end{aligned} \tag{4}$$

where $D_n = \dfrac{k_{zn}v_A}{\omega_n}\dfrac{z_{in}}{1-A_0(z_{in})}$, $z_{in} = (k_{\perp n}v_{Ti}/\omega_{Bi})^2$, $\omega = \omega_1+\omega_2$, $\mathbf{k} = \mathbf{k}_1+\mathbf{k}_2$.

As one can see, contrary to [5], the nonlinear interaction of waves depends on the modified Bessel functions. For the real events and phenomena occurred in the chromosphere-coronal plasma with participation of Alfvén waves, Eq. (4) will give more exact result in the spectral region where both the waves coupling and waves interaction with particles are the most intensive.

As Eq. (4) shows, the nonlinear interaction of Alfvén waves in the solar atmosphere occur very actively and the spectra of waves can significantly vary from the photosphere to the corona. Therefore, in concrete models of heating and acceleration of particles by kinetic Alfvén waves it is necessary to take into account nonlinear interaction of Alfvén waves and to calculate the form of a spectrum obtained as a result of such interaction.

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