MAGNETIC FIELD MODELLING ON THE CP STARS

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The modulus and longitudinal component of the surface magnetic field of a star are constructed applying the model of artificially introduced magnetic charges with their superposed potentials that are spatially distributed in the stellar interior. In the dipole approximation the expression for the mean longitudinal magnetic field given by the proposed model is of the same form as obtained previously by another authors. Such straight-forward calculations of the magnetic field allow one to reconstruct most of possible magnetic field configurations on the stellar surface.

INTRODUCTION

A comprehensive study of chemically peculiar (CP) stars spectra and elements abundance analysis at their surfaces should take into account all the possible physical processes, which contribute to the spectral line broadening. For the magnetic CP stars one of these processes is a magnetic broadening due to the Zeeman or Pashen-Back effects. In order to estimate this contribution we have to know the structure of surface magnetic field at the CP stars.

Most of our knowledge about magnetic field structure at these stars is based on the measurements of *a mean longitudinal magnetic field* variation with rotation of the star. Many observational data were obtained recently by Mathys & Hubrig [4] as a result of observation campaign of magnetic CP stars.

The modelling of the magnetic structure of CP stars was initially based on the *oblique rotator model* with dipolar field that allowed to explain the sine character of mean longitudinal field variation with the phase of star rotation $\langle B_z\rangle(\varphi)$. But most of the observational data testify that there are some deviations from the sine form. To describe these observable deviations Hensberge *et al.* [2] propose *the decentered dipole model*. Recently, Landolfi *et al.* [3] developed a (non-axisymmetric) *multi-polar model* for explanation of the detected mean longitudinal field variations. But not all the observable features of $\langle B_z\rangle(\varphi)$ could be explained in the frame of the mentioned models.

From the other point of view Gerth *et al.* [1] proposed *the model of the spatially distributed point-like field sources* with *virtual magnetic charges*. Though the isolated magnetic charges have not been found in the nature, we, however, can artificially introduce such *a system of the point-like field sources* distributed throughout the volume **D** (see Fig. 1) somewhere in the *interior* of a stellar body for the best recovering the magnetic field structure at the star surface. Due to the disruption of magnetic field lines in the vicinity of such sources and the presence of some *whirls* inside the **D**-volume, the results of magnetic field modelling are valid only outside this volume, where $\mathbf{div}\ \vec{\mathbf{B}} = 0$ and $\mathbf{rot}\vec{\mathbf{H}} = 0$. In such a way for the surface magnetic field modelling the **D**-volume should be located in the stellar interior.

The superposition of potentials of artificially introduced virtual magnetic charges results in the scalar magnetic potential for any surface point **M** (see Fig. 1):

$$U = \sum_j \frac{Q_j}{|\vec{\mathbf{r}}_j|}, \tag{1}$$

where Q_j is a virtual magnetic charge of the j-source and $|\vec{\mathbf{r}}_j|$ is a distance from this field source to the surface point [1]. In order to provide a zero magnetic flux trough the star surface ($\oint \vec{\mathbf{B}}\mathbf{d}\vec{\mathbf{S}} = 0$), the sum of virtual

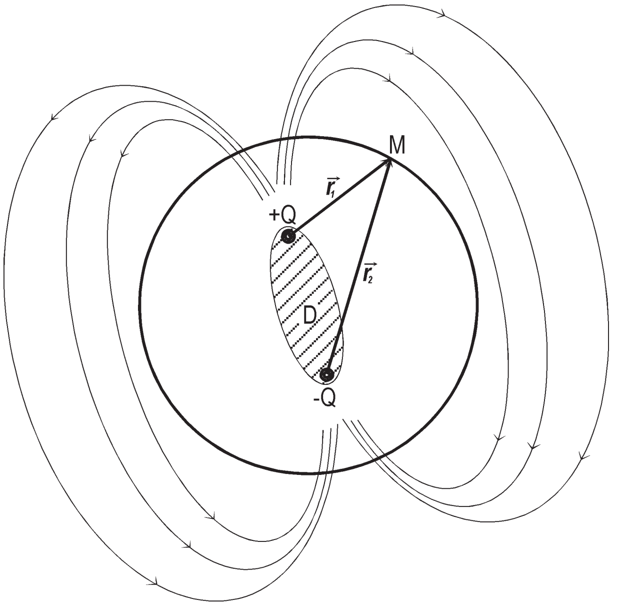

Figure 1: System of the point-like field sources artificially introduced inside the star for modelling the surface magnetic field structure

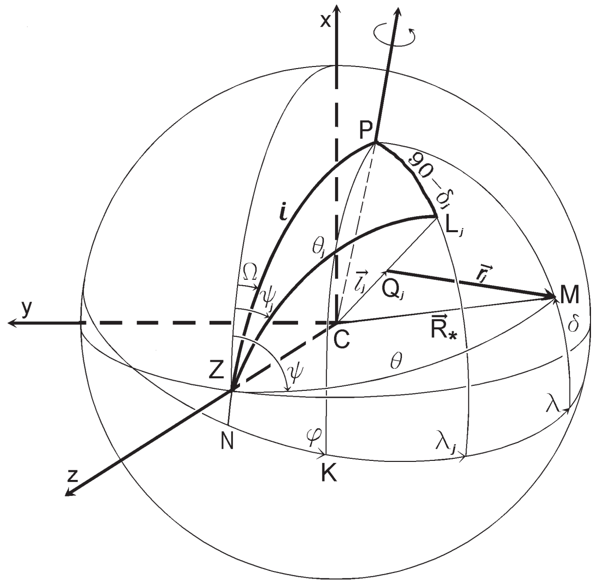

Figure 2: Geometry of the oblique rotator model with one probe point-like magnetic field source

magnetic charges should be kept

$$\sum_j Q_j = 0. \tag{2}$$

Then, the surface magnetic magnetic field vector is determined by the *gradient of potential*:

$$\vec{\mathbf{B}} = -\vec{\nabla} U = \sum_j \frac{Q_j}{|\vec{\mathbf{r}}_j|^3} \vec{\mathbf{r}}_j. \tag{3}$$

METHOD

Let us consider the geometry of oblique rotator model with one probe point-like field source Q_j with the help of three different coordinate frames (see Fig. 2). The first one is a *right-handed Cartesian frame* (x, y, z) with the origin in the centre of the star **C**, the z-axis is directed along the line of sight (pointing to the observer) which crosses star surface in the *subsolar point* **Z**, and the x-axis is in the direction of the *North celestial pole*. The second one is a spherical frame relative to the observer with coordinates (R, θ, ψ) connected with the Cartesian frame coordinates (x, y, z) by the following expressions:

$$\left\{ \begin{array}{ll} x = & R \sin\theta \cos\psi, \\ y = - & R \sin\theta \sin\psi, \\ z = & R\cos\theta. \end{array} \right. \tag{4}$$

In this frame the *positive rotation pole* **P** (the star rotates, by definition, in the counterclockwise direction as shown by the arrow) is specified by the inclination angle i and the azimuth angle Ω, measured from the central meridian in the clockwise direction (see Fig. 2).

The third coordinate frame is the spherical one relative to the star with the origin in its centre **C**. In this frame any surface $(R = R_\star)$ point **M** could be specified by latitude δ and longitude λ, measured from the *zero meridian* in the counterclockwise direction (see Fig. 2). As it follows from spherical triangle $\mathbf{ZPL}_1$ and taking into account the angles i and Ω the relation between these two spherical coordinate frames is specified by the following expressions:

$$\left\{ \begin{array}{l} \cos\theta = \sin\delta\cos i + \cos\delta \sin i \cos(\varphi + \lambda), \\ \sin\theta = \sqrt{1 - \cos^2\theta}; \end{array} \right. \tag{5}$$

$$
\begin{cases}
\cos(\psi - \Omega) = \dfrac{\sin\delta - \cos i \cos\theta}{\sin\theta \sin i}, \\[2ex]
\sin(\psi - \Omega) = \cos\delta \, \dfrac{\sin(\varphi + \lambda)}{\sin\theta},
\end{cases}
\tag{6}
$$

where φ is the angle between zero meridian and **PZ** meridian (see Fig. 2).

Let one the probe point-like field source be located at the point Q_j at the distance l_j from the centre. Vector $\vec{\mathbf{l}}_j$ crosses star surface in the point L_j, characterized by coordinates (θ_j, ψ_j) or (δ_j, λ_j). Distance between the source location Q_j and surface point $\mathbf{M}$ is $|\vec{\mathbf{r}}_j| = |\vec{\mathbf{R}}_\star - \vec{\mathbf{l}}_j|$. In spherical coordinates relative to the observer it reads (see Eq. (4)):

$$
|\vec{\mathbf{r}}_j| = R_\star \sqrt{1 + a_j^2 - 2a_j(\sin\theta \sin\theta_j \cos(\psi - \psi_j) + \cos\theta \cos\theta_j)},
\tag{7}
$$

where $a_j = l_j / R_\star$.

In such a way, for any surface point $\mathbf{M}$ the components of the magnetic field vector (see Eq. (3)) in the Cartesian frame (x, y, z) are given by the following expressions (see Eq. (4, 7)):

$$
\begin{cases}
B_x = R_\star \sum\limits_j \dfrac{Q_j}{|\vec{\mathbf{r}}_j|^3} (\sin\theta \cos\psi - a_j \sin\theta_j \cos\psi_j), \\[2ex]
B_y = - R_\star \sum\limits_j \dfrac{Q_j}{|\vec{\mathbf{r}}_j|^3} (\sin\theta \sin\psi - a_j \sin\theta_j \sin\psi_j), \\[2ex]
B_z = R_\star \sum\limits_j \dfrac{Q_j}{|\vec{\mathbf{r}}_j|^3} (\cos\theta - a_j \cos\theta_j).
\end{cases}
\tag{8}
$$

Assuming a *limb-darkening law* of the form $1 - u + u\cos\theta$ (where $0 \leq u \leq 1$), we average B_z over the visual stellar disk according to Landolfi *et al.* [3] and obtain expression for the observable mean longitudinal magnetic field:

$$
\langle B_z \rangle = \frac{3}{\pi(3-u)R_\star^2} \sum_j Q_j \int_0^{2\pi} d\psi \int_0^{\pi/2} \frac{\sin\theta \cos\theta (1 - u + u\cos\theta)(\cos\theta - a_j \cos\theta_j) d\theta}{[1 + a_j^2 - 2a_j(\sin\theta \sin\theta_j \cos(\psi - \psi_j) + \cos\theta \cos\theta_j)]^{\frac{3}{2}}}.
\tag{9}
$$

Correspondingly, the *magnetic field modulus* averaged over the visual stellar disk is given by

$$
\langle |\vec{\mathbf{B}}| \rangle = \frac{3}{\pi(3-u)} \int_0^{2\pi} d\psi \int_0^{\pi/2} \sin\theta \cos\theta (1 - u + u\cos\theta) \sqrt{B_x^2 + B_y^2 + B_x^2} \, d\theta.
\tag{10}
$$

The complete expression for $\langle |\vec{\mathbf{B}}| \rangle$ is too cumbersome, so it is not given here.

For the system of the two point-like field sources (magnetic dipole) the proposed model has 11 free parameters:

$$
\begin{cases}
Q, |\vec{\mathbf{R}}_\star|; \\
0 \leq a_1, a_2 < 1; \\
0 \leq u \leq 1; \\
0^\circ \leq i < 180^\circ; \\
0^\circ \leq \lambda_1, \lambda_2, \Omega < 360^\circ; \\
-90^\circ \leq \delta_1, \delta_2 \leq 90^\circ,
\end{cases}
\tag{11}
$$

taking into account the Eq. (2). Two (Ω, i) of these parameters specify the orientation of star rotation axis relative to the observer, next two ones $(u, |\vec{\mathbf{R}}_\star|)$ determine the stellar characteristics and the rest seven parameters determine the virtual charges and the location of point-like sources in stellar interior. Varying these free model parameters we try to achieve the best agreement between the observed and simulated characteristics of a surface magnetic field.

SYMMETRIC DIPOLE APPROXIMATION

For the central magnetic dipole in the star with $u = 0.6$, $R_\star = 2R_\odot$ the following assumptions are valid:

$$a_1 = a_2 = a \ll 1,\ Q_1 = -Q_2 = Q,\ \delta_1 = -\delta_2 = \delta,\ (\lambda_1 = \lambda)\ \lambda_2 = 180^\circ + \lambda.$$

As indicated by Landolfi *et al.* [3], in the case of symmetric dipole the value of $< B_z >$ does not depend on the angle Ω. Therefore, without losing in generality, we can keep $\Omega = 0^\circ$ through the rest of calculations. Following to Hensberge *et al.* [2] we use the angle β ($\beta = 90^\circ - \delta$) between the magnetic dipole and rotational axes. The z-component of the magnetic field for the surface point $\mathbf{M}(\theta, \psi)$ is proportional to

$$B_z \sim \left\{ \frac{\cos\theta - aZ}{[1 + a^2 - 2a(A\sin\theta + Z\cos\theta)]^{\frac{3}{2}}} - \frac{\cos\theta + aZ}{[1 + a^2 + 2a(A\sin\theta + Z\cos\theta)]^{\frac{3}{2}}} \right\},$$

where $A = X\cos\psi + Y\sin\psi$ and

$$\left\{ \begin{array}{l} X = \cos\beta\sin i - \sin\beta\cos i\cos(\varphi + \lambda); \\ Y = \sin\beta\sin(\varphi + \lambda); \\ Z = \cos\beta\cos i + \sin\beta\sin i\cos(\varphi + \lambda). \end{array} \right. \qquad (12)$$

Since the value of $a \ll 1$, we decompose expression for B_z into the series for small parameter a and obtain finally

$$B_z \sim 2a[Z(3\cos^2\theta - 1) + 3A\sin\theta\cos\theta],$$

that after averaging over the visible stellar disk (Eq. 9) results in

$$\langle B_z \rangle \sim \frac{15 + u}{20(3 - u)} Z. \qquad (13)$$

The last expression is the same as given in the paper of Hensberge *et al.* [2] for mean longitudinal magnetic field in the case of central symmetric dipole. As it follows from Eq. (12) and Fig. 2, $Z = \cos\alpha$, where α is an angle between the dipole axis and line of sight. In such a way the case of central symmetric dipole is a special case of the model proposed here.

SUMMARY

Basing on the approach developed by Gerth *et al.* [1] we propose the modified method for surface magnetic field modelling. Due to the appropriate choice of orientation of the Cartesian and spherical coordinate frames we obtain the simple expression for the components of surface magnetic field vector $\vec{\mathbf{B}}$ (see Eq. (3)).

The proposed method involves the *point-like field sources* for modelling of the surface magnetic field structure on CP stars. In reality the point-like field sources have not been found. They are introduced into the model artificially as the auxiliary objects only for the best description of the observed characteristics of a surface magnetic field with the least number of model parameters.

In the dipole approximation the expression for $\langle B_z \rangle$ given by the proposed model is of the same form as obtained by Hensberge *et al.* [2].

In principle, any magnetic field configuration can be represented with the help of such artificially introduced *point-like field sources*, while the sum of their magnetic charges is zero. The advantage of this approach consists in arbitrary location of field sources in the interior of a stellar body. Its application is limited only by their number.

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