INFORMATIONAL EFFICIENCY OF A MULTY–BEAM INTERFEROMETER IN THE OBSERVATION OF AN OBJECT THROUGH THE TERRESTRIAL ATMOSPHERE

Yu. V. Kornienko and V. Pugach

Institute of Radiophysics and Electronics, NAS of Ukraine
12 Ak. Proskury, 61085 Kharkiv, Ukraine
e-mail: pvv@ire.kharkov.ua

A consideration of the influence of the registration noise on the accuracy of the image reconstruction of a remote sensing object being observed with a multibeam interferometer (the evolution of the stellar Michelson interferometer concept) through a thin layer of the medium having random refraction index inhomogeneities is given. This accuracy appears to decrease due to phase distortion but this effect is rather small. It is concluded that the interferometric imaging method has a high degree of resistance to phase distortion in the atmosphere.

The aim of this study is to estimate quantitatively the information efficiency of a multi-beam interferometer [1] and its immunity to phase distortions.

The set of equations for undisturbed phases of Fourier components of an image consists [2] of equations of the kind

$$\varphi_\nu + \delta_i - \delta_k = \psi_{ik}, \tag{1}$$

where φ_ν is the required actual value of phase of the ν-th Fourier component; δ_i and δ_k are the phase distortions at the i-th and k-th subapertures of the interferometer; and ψ_{ik} is the phase of the Fourier component. The parameters of the solution can be chosen arbitrarily, *i.e.*, like a constant additions to the phase distortions φ and the total phase front inclination (two components). In order to avoid this uncertainty, we complement the set of Equations (1) by equating these parameters to zero:

$$\lambda \sum \delta_i = 0, \quad \mu \sum x_i \delta_i = 0, \quad \mu \sum y_i \delta_i = 0, \tag{2}$$

where x_i and y_i are the coordinates of the i-th aperture.

Let us present the derived set in the form

$$A\,|X\rangle = \,|Y\rangle, \tag{3}$$

where X is the vector whose components are the required undisturbed phases φ and atmospheric phase distortions δ; Y is the vector consisted of the measured values of ψ; A is the matrix of the coefficient that is completely determined by the interferometer geometry, *i.e.*, by the spatial arrangement of its subapertures. (All its components are equal to 1, –1 or 0). The right-hand side of this set, being a result of the experiment, contains random errors which are assumed to be normally and independently distributed with the same variance. This problem can be optimally solved using the least squares method. First, we multiply both sides of (3) by the A^+ – matrix conjugated with A and derive the system

$$M|X\rangle = |Z\rangle \tag{4}$$

with the most probable value of the vector X being its solution. Here $M = A^+A$, and $|Z\rangle = A^+|Y\rangle$.

If the interferometer aperture consists of n subapertures and transmits l spatial frequencies, then the vector X consists of $l+n$ components (l phases of the image Fourier-components and n phase distortions), the vector Y consists of $n(n-1)/2+3$ components, the matrix A of $n(n-1)/2+3$ lines and $l+n$ columns, the vector Z of $l+n$ components and the matrix M of $l+n$ rows and the same number of columns.

Let the right-hand side of (3) are to be known with the error $|\eta\rangle$. It results in an error $|\xi\rangle$ in the solution of system (4) that, due to its linearity, satisfies the equation

$$M|\xi\rangle = A^+|\eta\rangle, \tag{5}$$

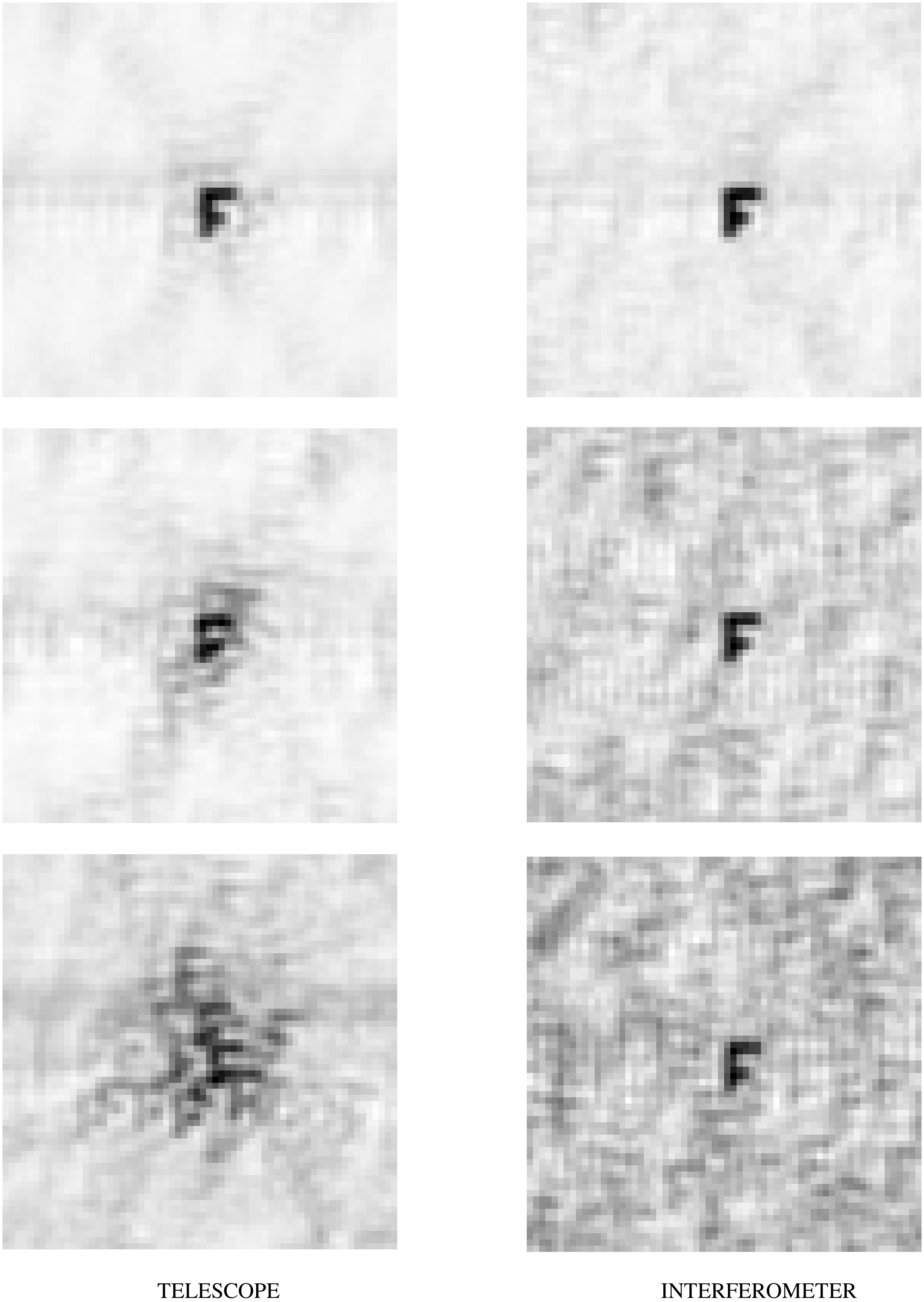

Figure 1: The results of computer simulation of an observing through the Earth atmosphere a stretched object using both a traditional and an interferometric telescope for various wave front disturbances

which gives

$$|\xi\rangle = M^{-1}A^{+}|\eta\rangle. \tag{6}$$

Therefore, the covariation matrix of these errors has the form of

$$C = M^{-1}A^{+}\,\overline{|\eta\rangle\langle\eta|}\,AM^{-1}, \tag{7}$$

where the dash means the averaging over the ensemble of the recording noise realizations. Matrix $\overline{|\eta\rangle\langle\eta|}$ is the auto-covariations matrix of the errors $|\eta\rangle$. By the assumption accepted about the statistical properties of $|\eta\rangle$ this matrix is equal to DE, where E is a unity matrix, and D is the error variance of the vector $|\eta\rangle$ components. For this reason and due to (7),

$$C = DM^{-1}, \tag{8}$$

where D characterizes the intensity of the detection noise, and M^{-1} describes the dependence of the X value error on the interferometer geometry. However, among all the unknowns in system (4), we will restrict our interest to those ones that are related to the object of observation, *i.e.*, φ.

The matrix of their error covariation is

$$C_{\varphi} = DG, \tag{9}$$

where G is a sub-matrix of M^{-1} obtained by crossing out all rows and columns related to the unknowns δ. Since the errors are normally distributed, the desired Fisher matrix F is inverse to C_{φ} and equal to G^{-1}/D.

Matrix $H = G^{-1}$ depends only on the interferometer geometry and characterizes the interferometer efficiency in observation through the atmosphere. We shall call it a matrix of the interferometer information efficiency.

The direct calculation of this matrix for several simplest geometries of the interferometer has shown [3, 4] a surprisingly high immunity of the method suggested in [1] to the effects of atmospheric distortions. However, it remains unclear to what extent it is true for other configurations of the input aperture. To answer this question, a method of analytical calculation of matrix F should be found, which is however associated with certain difficulties, since for rationally designed interferometers [5, 6] containing tens of subapertures, the number of elements of this matrix reaches the values of the order of 10^5 to 10^6. The results of investigation in this area are briefly presented in [7].

Here we show the results of computer simulation of an observing through the Earth atmosphere a stretched object using a traditional and interferometric telescope for various wave front disturbances. The results are demonstrated in Fig. 1.

REFERENCES

[1] *Kornienko Yu. V., Uvarov V. N.* Signal accumulation at observation of an astronomical object through the turbulent atmosphere // Comptes Rendus of Academy of Science of Ukraine.–1987.-Ser. **A**, N 4.–P. 60–63. (in Russian).

[2] *Roddier F.* Redundant versus nunredundunt beam recombination in an aperture synthesis with coherent arrays // J. Opt. Soc. A.–1987.–**A4**, N 8.–P. 1396–1401.

[3] *Kornienko Yu. V.* Interferometric approach to the problem of turbulent atmosphere effects in astronomical observations // Kinematics and Physics of Celestial Bodies.–1994.–**10**, N 2.–P. 98–106. (in Russian).

[4] *Kornienko Yu. V., Leyferov V. A., Pugach V. V.* Informational efficiency of a multibeam interferometer in the observation of an object throughthe inhomogeneous medium // Radiofisika i Electronika.–1997.–**2**, N 1.–P. 132–136. (in Russian).

[5] *Kopilovich L. E.* Multi-beam optical interferometer: the output aperture optimization // Kinematics and Physics of Celestial Bodies.–1998.–**14**, N 2.–P. 138–142.

[6] *Kopilovich L. E.* Non-redundant apertures for optical interferometric systems: maximization of the number of elements // J. Modern Optics.–1998.–**45**, N 11.–P. 2417–2424.

[7] *Kornienko Yu. V.* Construction on Nonredundant Apertures for Interferometric Radiotelescope by Method of Random Search // Physics and Engineering of MSMW: 3rd Intern. Kharkiv Symp., Kharkiv, Ukraine, Sept. 15–17, 1998.–P. 578–580.