STATISTICAL ANALYSIS OF GRAVITATING SUBSTANCE DISTRIBUTION IN THE SOLAR SYSTEM

P. M. Fedij[1], **Yu. K. Gulak**[2], **V. I. Yeremin**[3]

[1] *Pedagogical University, Poltava, Ukraine*

[2] *Gravimetrical Observatory, NAS of Ukraine, Poltava, Ukraine*

[3] *Institute of Magnetism, NAS of Ukraine, Kyiv, Ukraine*

The article presents a statistic estimation of the correlation between the existing radial distribution of gravitating substance in the Solar System and the series of the $a_n = na_0/2$ values ($n = 2, 3, 4, ...$). This correlation has been predicted theoretically by Gulak (1980). The average orbital semi-major axes by Bretagnon were used for eight planets (except Pluto) while for 11 statistically significant asteroidal complexes (AC), which we have found out throughout the whole asteroidal belt by three independent methods of restoring the probability density function, maxima of this function were taken. These methods are: 1) the Parsen-Rozenblatt's algorithm with exponential kern (Milen'kiy, 1983); 2) the method of separation of distributions in a mixture (Ayvasian, 1974); 3) the method of non-parametrical estimation of unknown probability density function (Vapnik, 1984). There is a good agreement of all the received results. Fig. 1 represents results of the first method used. For the final analysis we prefer the values of maxima derived by the second method, which are as more adequate, in our opinion, to the classification problem.

The optimal value $a_0 = 0.2726$ AU of the commensurable element was found by minimizing two different norms of deviation. A comparison of the empirical values a_* of the orbital semi-major axes with those of "theoretical" a as well as the corresponding quantum integers n with the empirical fractional values $n_* = 2a_{*n}/a_0$ for the planets and asteroidal complexes of the Solar System are shown in the table.

Deviations Δn of the fractional empirical values n_* from the expected quantum integers n are normally distributed ($P(\tilde{\chi}^2 > \tilde{\chi}_0^2) > 70\%$, $\tilde{\chi}_0^2 = 0.08$) with the average value $\overline{\Delta n} = 0.056$ and standard deviation of $\sigma = 0.19$. The statistical hypothesis about the zero-value of the average (expected value) was proved. The standard deviation can be considered sufficiently small in comparison with 0.29 (for the uniform distribution in the case when any regularities are absent) and also with a maximal possible deviation of 0.5. It is important to note that six consequent quantum rates correspond to 6 AC of the main belt, which contains a sufficient number of individual objects.

Object	a_*	a	Δa	n_*	n	Δn
Mercury	0.39	0.41	–0.02	2.84	3	–0.16
Venus	0.72	0.68	0.04	5.31	5	0.31
Earth	1.00	0.95	0.05	7.34	7	0.34
Mars	1.52	1.50	0.02	11.18	11	0.18
AC	1.89	1.91	–0.02	13.86	14	–0.14
AC	2.23	2.18	0.05	16.37	16	0.37
AC	2.39	2.45	–0.06	17.57	18	–0.43
AC	2.59	2.59	–0.01	18.98	19	–0.02
AC	2.71	2.73	0.01	19.91	20	–0.09
AC	2.87	2.86	0.01	21.05	21	0.05
AC	3.01	3.00	0.01	22.11	22	0.11
AC	3.15	3.14	0.01	23.08	23	0.08
AC	3.42	3.41	0.01	25.06	25	0.06
AC	3.96	3.95	0.01	29.05	29	0.05
Jupiter	5.2026	5.18	0.02	38.17	38	0.17
AC	5.2032	5.18	0.02	38.17	38	0.17
Saturn	9.55	9.54	0.01	70.10	70	0.10
Uranus	19.22	19.22	0.00	141.00	141	0.00
Neptune	30.11	30.13	–0.01	220.91	221	–0.09

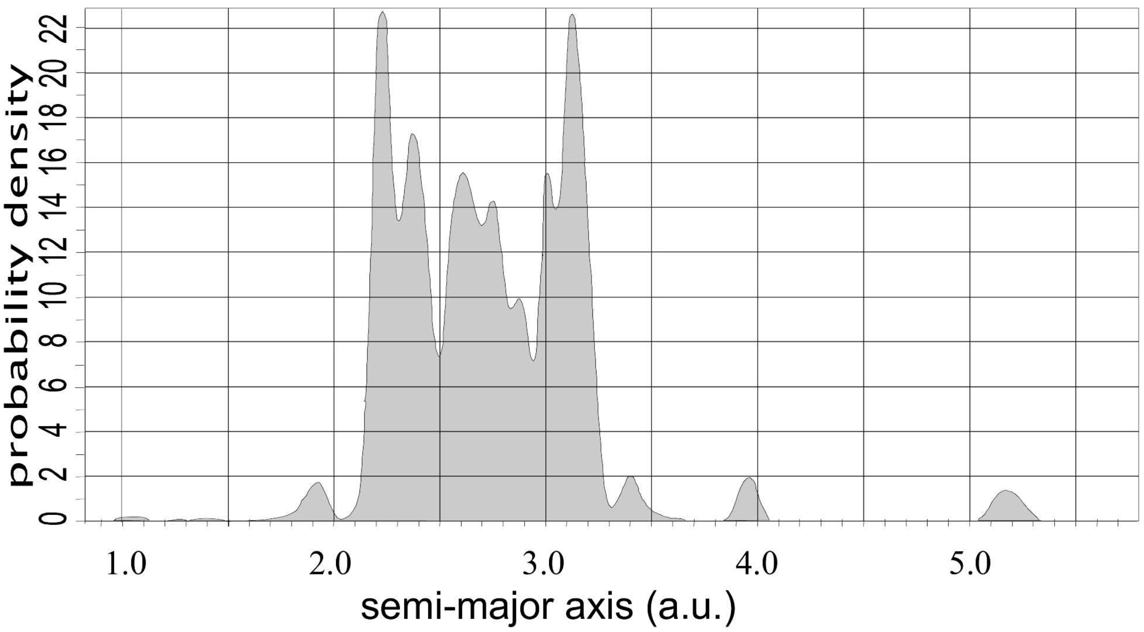

Figure 1: Probability density function of asteroidal semi-major axes distribution by the Parsen-Rozenblatt method

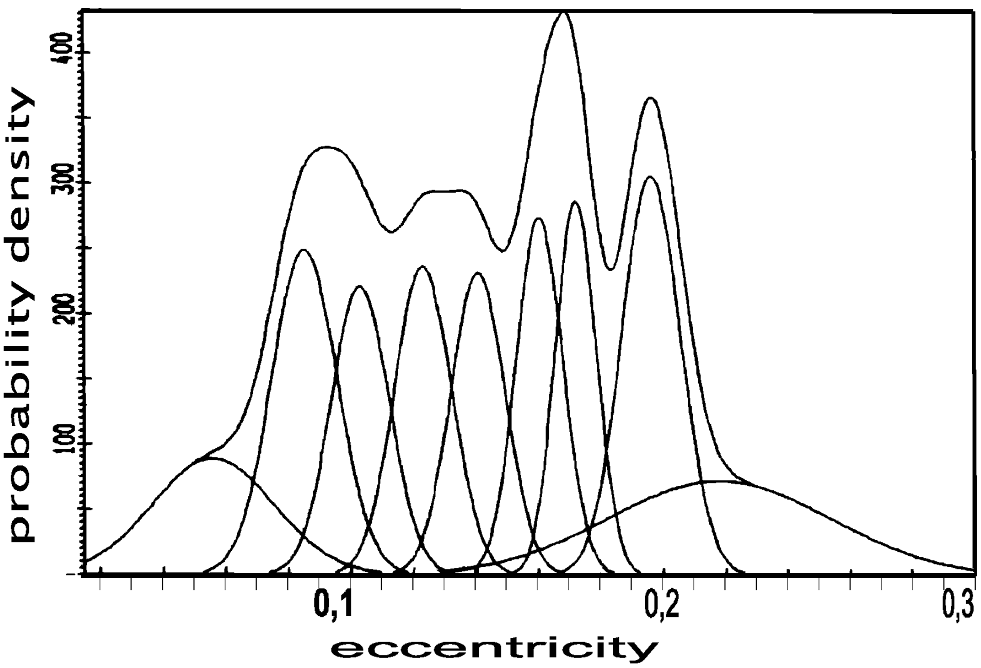

Figure 2: Probability density function of asteroidal orbits eccentricities distribution inside of AC with $2.10 < a < 2.31$ AU obtained by the second method

Thus, we can state that the theory is consistent with the observed distribution of gravitating matter complexes in the Solar System. It is necessary to note that such a distribution means a quantization of the specific total energy of the complexes:

$$E_n = -\frac{GM}{a_n} = -\frac{2GM}{na_0} = -\frac{E_0}{n},$$

where GM is the "gravitational charge" of the Sun.

We have also studied the distribution of orbital eccentricities and inclinations inside of each AC. As a result of this consideration the discrete structure has been found that corresponds qualitatively to the theoretical predictions (for example see Fig. 2).

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