EXTERNAL GRAVITATIONAL POTENTIAL OF GIANT PLANETS WITH THE POTENTIAL OF THE SYSTEM OF INHOMOGENEOUS SELF–GRAVITATING DISKS

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We introduce parameters of the systems of inhomogeneous disks, which can describe the external gravitational potentials of giant planets. Dynamical characteristics of giant planets were taken from the astronomical observational data. We assumed that giant planets have figures close to equilibrium ones.

There are various methods of description of gravitational potentials of volume bodies [1]. The description based on the potential of a simple fiber (disk) is quite effective. When using such a model, number of dimensions of a body reduces in fact that allows to simplify the expression for the gravitational potential. If one applies this method to planets of the solar system then it's necessary to divide the planet's figure into some ellipsoidal strata. Then, according to the potential theory, one have to find an equivalent disk for each stratum and collect all the disks into a system.

It's known [6, 7] that the external gravitational potential of a homogenous oblate ellipsoid of revolution can be exactly described like the external potential of a real inhomogeneous circular thin disk having the radius

$$R_x = \left(a^2 - c^2\right)^{1/2} \tag{1}$$

and surface density distribution

$$\sigma(R) = \sigma_0 \left(1 - \frac{R^2}{R_x^2}\right)^{1/2}, \tag{2}$$

where $\sigma_0 = \dfrac{2a^2 c\rho}{a^2 - c^2}$; a and c are the major and minor semi-axes of ellipsoid, ρ is the volume density.

If one divides the planet's figure into some homogenous ellipsoidal strata having their own densities ρ_i and oblatenesses f_i, *i.e.*, as a figure composed of some homogenous ellipsoids of revolution, then it will be possible to describe the external gravitational potential of a planet exactly as a sum of the external potentials of inhomogeneous circular disks obtained by fitting the disks' parameters.

According to the proposed algorithm, let us compose a multi-layer ellipsoidal model of a planet using parameters of the internal structure of giant planets [2, 3, 5]. Let divide the planet's figure into some homogenous strata and satisfy the condition of mass parity

$$M = \sum_{i=1}^{n} M_i, \tag{3}$$

where M is the planet's mass; $M_i = \dfrac{4\pi}{3}\left(\rho_i - \rho_{i+1}\right) R_i^3$, $\rho_{n+1} = 0$ are the masses of the homogenous ellipsoids of revolution composed the planet's figure; $R_i = \sqrt[3]{a_i^2 c_i}$ is the equivalent radius of a sphere; n is the number of strata. Then let us determine geometrical oblateness of each stratum from the solution of the Clairaut equation (4). Equilibrium planets' figures have to satisfy it but the figures of giant planets are only closed to equilibrium ones [8].

$$\frac{d^2 f}{dr^2} + \frac{6}{r}\frac{\rho}{D}\frac{df}{dr} - \frac{6}{r^2}\left(1 - \frac{\rho}{D}\right) f = 0, \tag{4}$$

where $D = D(r) = \frac{3}{r^3}\int\limits_0^r \rho r'^2 dr'$ is the average density of a sphere of radius r.

The initial conditions on the external surface for finding the oblateness f are following:

$$f\,|_{r=R} = f_0, \quad \left.\frac{df}{dr}\right|_{r=R} = \frac{1}{R}\left(\frac{5}{2}m - 2f_0\right),$$

where f_0 is the surface oblateness of a planet; $m = \frac{\omega^2 R^3}{GM}$; ω is the angular velocity of the planet's self-rotation.

The defined model parameters are given in Table 1 (columns from 1 to 4).

Table 1: Parameters of models of giant planets

Stratum number i	Average radius R_i, km	ρ_i, g/cm^3	F_i	Disk number m	R_{xm}, km	σ_{0m}, 10^{11} g/cm^2
			Jupiter			
1	4893	12.60	0.00429	1	453.17	4.678
2	9086	8.50	0.00976	2	1270.72	5.900
3	53818	2.15	0.03431	3	14139.60	2.105
4	63604	0.80	0.05102	4	20409.00	0.531
5	69894	0.37	0.06487	5	25462.36	0.390
			Saturn			
1	5247	9.08	0.00874	1	694.25	2.361
2	18073	5.14	0.02132	2	3738.31	2.855
3	28267	1.76	0.04829	3	8919.32	0.287
4	39644	1.27	0.07168	4	15108.85	0.595
5	58300	0.18	0.09500	5	25639.85	0.108
			Uranus			
1	8122	4.37	0.00902	1	1091.85	1.681
2	19543	2.50	0.01350	2	3214.77	3.438
3	25380	0.12	0.02070	3	5192.48	0.145
			Neptune			
1	12311	4.99	0.01197	1	1907.08	2.422
2	19205	2.63	0.01500	2	3330.47	3.130
3	24622	0.18	0.01710	3	4560.24	0.258

Using obtained results, one can compute parameters of inhomogeneous disks by formulas:

$$R_{xm} = \sqrt{a_m^2 - c_m^2}, \quad \sigma_{0m} = \frac{2a_m^2 c_m\left(\rho_m - \rho_{m+1}\right)}{a_m^2 - c_m^2}, \quad \rho_{n+1} = 0, \quad m = 1, \ldots, n. \tag{5}$$

Then models of systems of inhomogeneous self-gravitating disks can be composed to describe the external gravitational potentials of planets (see Table 1, columns from 5 to 7).

A criteria of the estimated accuracy of description of the external gravitational potentials of planets is a correspondence of the dynamical characteristics (6)–(7) of the models [4] and the data of astronomical observations (see Table 2).

$$V = \frac{GM}{r}\left\{1 - \sum_{k=1}^{\infty} J_{2k}\cdot P_{2k}\left(\cos\theta\right)\cdot\left(\frac{a_e}{r}\right)^{2k}\right\}, \tag{6}$$

$$J_{2k} = (-1)^{k+1}\,\frac{3}{(2k+1)\,(2k+3)}\,\frac{\sum\limits_{i=1}^{n} M_i\left(a_i^2 - c_i^2\right)^k}{Ma^{2k}}, \tag{7}$$

where $P_{2k}(\cos\theta)$ are the odd Legendre's polynomials; J_{2k} are the odd gravitational moments of the models; (r, φ, θ) are the spherical coordinates of the point where such a potential is found.

Table 2: Dynamical characteristics of giant planets (J_{2n} are the observational values; J^*_{2n} are the values computed from composed models)

Parameter	Jupiter	Saturn	Uranus	Neptune
$M, 10^{26}$ kg	18.97	5.68	0.8687	1.0243
a_e, km	71492	60268	25559	24764
m	0.083	0.139	0.0285	0.0259
$J_2 \cdot 10^2$	1.4697±0.0001	1.6331±0.0018	0.351323±0.000032	0.3539±0.0010
$J^*_2 \cdot 10^2$	1.47	1.63	0.351	0.353
$J_4 \cdot 10^4$	–5.84±0.05	–9.14±0.61	–0.319±0.005	$-0.28\ ^{+0,12}_{-0,20}$
$J^*_4 \cdot 10^4$	–5.85	–9.12	–0.322	–0.313
$J_6 \cdot 10^5$	3.1±2.0	10.8±5.0	0.021÷0.067	
$J^*_6 \cdot 10^5$	3.475	8.03	0.04784	

It follows from the fulfilled computations that the description of the external gravitational potential as the potential of a system of inhomogeneous self-gravitating disks is an effective method to be applied for problems of celestial mechanics and astronomical geodesy. The appeared discrepancies between the values of gravitational moments are explained by heterogeneity of internal structure and by absence of information about geometrical oblateness of internal strata of giant planets. The mean relative error of determination of the first gravitational moments is as follows $\delta(J_2) = 0.02\%$, $\delta(J_4) = 0.17\%$ for Jupiter, $\delta(J_2) = 0.19\%$, $\delta(J_4) = 0.22\%$ for Saturn, $\delta(J_2) = 0.09\%$, $\delta(J_4) = 0.94\%$ for Uranus, $\delta(J_2) = 0.25\%$ for Neptune. They were calculated by comparison with the astronomical observational data. It is impossible to check the calculated higher gravitational moments because of the lack of reliable astronomical data.

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