FRICTION EFFECT IN A SPHEROIDAL ENVELOPE ON THE MODIFICATION OF ORBITAL ELEMENTS OF PARTICLES

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Effect of a spheroidal spinning envelope on the modification of orbital elements of a particle moving in a central gravitational field is considered. The resistance force is considered as a small perturbation; the envelope's self-gravitating is negligible.

The self-gravitating systems during their evolution may be separated into a central body and a peripherals in the form of rings, disks or spheroidal envelopes as a result of a global instability. Such phenomena are observed in galaxies, globular stellar clusters, and interstellar medium clouds [2, 3, 5]. The Oort's cloud is likely a formation of such a kind [1, 6]. If a particle moves around a central body and crosses a spheroidal envelope, which surrounds this body, then a friction caused by the envelope matter have to act on the particle. In this connection we consider the problem of influence of the friction on the orbital elements of a particle moving in the central gravitational field.

As a simplest model let's take a central body as a point gravitating center, mass M of which is much greater than the envelope mass, which is negligible. Let's image the envelope as a spheroidal stratum having the internal and external radii r_1 and r_2, respectively (see Fig. 1). Such on equilibrium form can be supported by self-rotation of the envelope, gravitational forces, intrinsic pressure, and magnetic fields. Let us suppose the friction force to be depended on the relative velocity $\vec{v}$ of a particle and envelope's matter as well as on the distance r between the particle and the gravitating center: $\vec{R} = -c\dfrac{v^{\alpha}}{r^{\beta}}\dfrac{\vec{v}}{v}$, where α, β are constants, c is a coefficient that is small enough to consider the friction force as a small perturbation in comparison with the gravitational force.

Let's consider firstly a stationary (*i.e.*, non-rotating) envelope. Since the resistance force is tangent directional to the particle's trajectory then the Lagrange equation for the major semi-axis a and the eccentricity e can be converted to the form [4]:

$$\frac{da}{dr} = -\frac{2Ra^2\sqrt{r\ (2a-r)}}{GM\cdot\xi}, \quad \frac{de}{dr} = -\frac{2Ra\left(1-e^2\right)\sqrt{r}}{GM\cdot e\xi\sqrt{2a-r}}, \tag{1}$$

where $\xi = \sqrt{(ae)^2 - (a-r)^2}$; R is the value of the resistance force, M is the mass of the central body.

Since friction is considered as a small perturbation then values of a and e vary weakly inside the envelope and can be regarded as constants while $r_1 < r < r_2$. Substituting the values of a and e that describe the orbit before the particle enters the envelope and integrating the right parts of (1) one can obtain the first approximation of modifications of a and e after passing the envelope by the particle:

$$\Delta a \approx \int\limits_{r_1}^{r_2} -\frac{2Ra^2\sqrt{r\ (2a-r)}}{GM\cdot\xi}dr, \quad \Delta e \approx \int\limits_{r_1}^{r_2} -\frac{2Ra\left(1-e^2\right)\sqrt{r}}{GM\cdot e\xi\sqrt{2a-r}}dr. \tag{2}$$

Thus, Δa and Δe depend on the resisting force, size of the envelope and initial values of a and e. Antiderivatives of definite integrals (2) are represented in Table 1.

If the particle passes consequently through the envelope then it lose its energy and major semi-axis of the orbit decreases, but the intersection point of the orbit and the envelope moves to the orbit's apocenter. Thus, any stage of decreasing of eccentricity down to some critical value have to become the stage of its continuous increasing.

Table 1: Modification of the major semi-axis and the eccentricity in a case of non-rotating envelope

$R = cv$	$\Delta a = k\,a^{3/2}\,(a\eta - \xi)\,,$	$\Delta e = k\gamma^2\xi e^{-1}$
$R = cvr^{-1}$	$\Delta a = ka^{3/2}\left(2\zeta\gamma^{-1} - \eta\right),$	$\Delta e = k\sqrt{a}\,(1+e)\,e^{-1}\,(\zeta - \eta\gamma)$
$R = cvr^{-2}$	$\Delta a = k\sqrt{a}\left(1+e^2\right)\,\gamma^{-2}\left(\zeta\gamma^{-1} + 2\xi r^{-1}\right),$	$\Delta e = ka^{-1/2}e^{-1}\left(e^2\gamma^{-1}\zeta - \xi r^{-1}\right)$
$R = cv^2$	$\Delta a = 2ace^{-1}\,(E\,(e,x) - 2F\,(e,x) - \tau)\,,$	$\Delta e = ce^{-2}\gamma^2\,(2E\,(e,x) - 2F\,(e,x) - \tau)$
$R = cv^2r^{-1}$	$\Delta a = ce^{-1}\left[6F\,(e,x) - 8E\,(e,x)\,\gamma^{-2} + \tau + 8\lambda\sqrt{1-ex}\,\chi\,\gamma^{-2}e^{-1}\right]$ $\Delta e = -ce^{-2}a^{-1}\left[4E\,(e,x) - 4F\,(e,x)\,\gamma^2 - \gamma^2\tau - 4\lambda\sqrt{1-ex}\,\chi e\right]$	
$R = cv^2r^{-2}$	$\Delta a = \frac{2}{3}ca^{-1}\gamma^{-4}\left[\left(3e^2+5\right)\gamma^2e^{-1}F(e,x) - 8\left(1+e^2\right)e^{-1}E(e,x) - 4\chi\sqrt{1+ex}\left(2ex\gamma^2 + e^2 + 3\right)\lambda^{-3}\right]$ $\Delta e = -\frac{2}{3}c\,(ea\gamma)^{-2}\,\left[E\,(e,x)\,\left(1+7e^2\right) - F\,(e,x)\,\left(3e^2+1\right) + e\chi\,(1-ex)^{-1}\,\lambda^{-1}\,\left(7e^3x - 5e^2 + ex - 3\right)\right]$	

Used designations: $\xi = \sqrt{(ae)^2 - (a-r)^2}$; $\zeta = \arcsin\left(\frac{r + a\,\left(1-e^2\right)}{re}\right)$; $\eta = \arcsin\left(\frac{a-r}{ae}\right)$; $k = -\frac{2c}{\sqrt{GM}}$; $\gamma = \sqrt{1-e^2}$; $x = (1 - r/a)/e$; $\chi = \sqrt{1-x^2}$; $\lambda = \sqrt{1-(ex)^2}$; $\tau = \ln 2 - \ln\left(2ex^2 - e - e^{-1} + 2\chi\lambda\right)$; $F\,(...)$ and $E\,(...)$ are elliptical integrals of the first and second kinds.

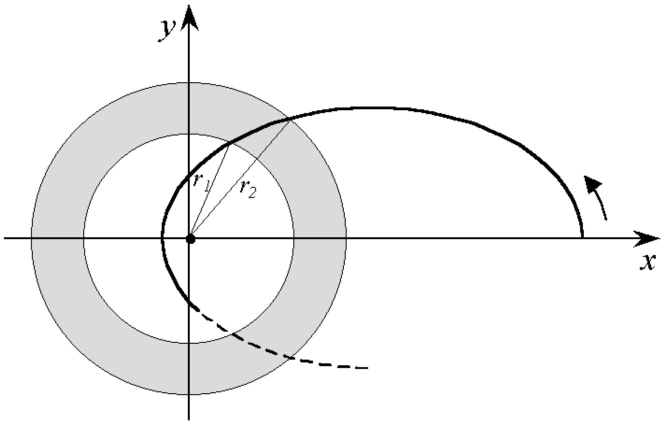

Figure 1: Particle passes trough the envelope

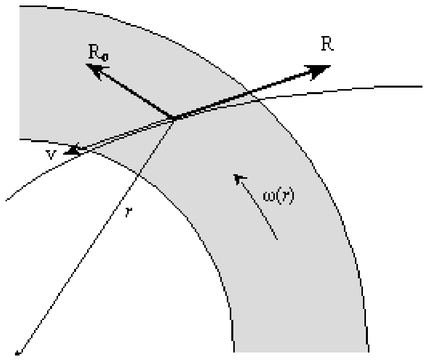

Figure 2: Resistance force components

Now we consider passing of a particle through a self-rotating envelope. Such an envelope acquires its spheroidal form due to self-rotation, but if one limits oneself with orbits lying in the equatorial plane then nonsphericity is negligible. A new transversal force appears due to the axial rotation (see Fig. 2). It produces the additional increments of the major semi-axis and eccentricity that can be found from the Lagrange's equations:

$$\Delta a^* \approx -\int_{r_1}^{r_2} \frac{2a^3R'_\varphi\sqrt{1-e^2}}{GM\cdot\xi}\,dr, \quad \Delta e^* \approx \int_{r_1}^{r_2} \frac{R'_\varphi\sqrt{1-e^2}\left(r^2 - a^2\left(1+e^2\right)\right)}{GM\cdot\xi}dr. \tag{3}$$

Thus, now the full modifications of a and e are equal to $\Delta a + \Delta a^*$ and $\Delta e + \Delta e^*$. The force component

Table 2: Additional modification of the major semi-axis and the eccentricity in case of rotating envelope

$R = cv$	$\omega\ (r) = \sqrt{GM} \cdot r^{-3/2}$	$\Delta a^* = -\frac{1}{3}\, k\, a^{5/2} e^{-1} \sqrt{1-e} \left[-E\left(\rho_1, \sigma_1\right)\ (3e+1) + F\left(\rho_1, \sigma_1\right)\ (1-e) - \left(E\left(i\rho_1, i\sigma_2\right)\ (3e+1)\ (e-1) + F\ \left(i\rho_1, i\sigma_2\right)\ (1+e)\ (5e+1)\right)\ \gamma^{-2} \right]$ $\Delta e^* = -\frac{1}{3}\, k\, \gamma\, a^{3/2}\, e^{-1} \left[3\ (1+e)^{3/2}\, E\left(\rho_1, \sigma_1\right) - E\left(i\rho_2, i\sigma_1\right) \sqrt{1-e}\, (1-3e) + 3F\left(\rho_1, \sigma_1\right) \sqrt{1+e}\, (e-1) + F\left(i\rho_1, i\sigma_2\right) \sqrt{1-e}\, (1+e) + \sqrt{r}\ \xi \right]$
	$\omega\ (r) = \omega_0 \cdot r^{-1}$	$\Delta a^* = -2c\mu^{-1}\omega_0 a^3 \gamma\eta,$ $\Delta e^* = \frac{1}{2}\, c\ (\mu e)^{-1}\ \gamma\, \omega_0 \left[(r+3a)\ \ \xi + 3a^2 e^2 \eta \right]$
	$\omega\ (r) = \omega_0 = const$	$\Delta a^* = -2c\mu^{-1}\omega_0 a^3\ (\xi + a\eta)\,,$ $\Delta e^* = -\frac{1}{6}\, c\, \omega_0\, (\mu e)^{-1}\ \gamma\ \left[15a^3 e^2 \eta + \xi\ \left(r\ (2r+5a) + 5a^2\ \left(1+2e^2\right) \right) \right]$
$R = cv^2$	$\omega\ (r) = \sqrt{GM} \cdot r^{-3/2}$	$\Delta a^* = 2ca^2\zeta, \quad \Delta e^* = -ce^{-1}\gamma\ (a\eta - a\zeta\gamma - \xi)$
	$\omega\ (r) = \omega_0 \cdot r^{-1}$	$\Delta a^* = 2c\mu^{-1} a^3 \omega_0^2 \gamma\eta,$ $\Delta e^* = -\frac{1}{2}\, c\, \omega_0^2\, \gamma\ (\mu\, e)^{-1}\ \left[(r+3a)\ \xi + 3a^2 e^2 \eta \right]$
	$\omega\ (r) = \omega_0 = const$	$\Delta a^* = -c\mu^{-1}\gamma\, a^3 \omega_0^2\ \left[a^2\ \left(e^2+2\right)\ \eta + \xi\ (r+3a) \right]$ $\Delta e^* = \frac{1}{8}\, c\, (\mu e)^{-1}\ \omega_0^2 \gamma \left[7a^4 e^2\ \left(4+e^2\right)\ \eta + \frac{1}{3}\, \xi\ \left(6r^3 + 14a\ \left(r^2 + ra + a^2\right)\right) + 7a^2 e^2\ (13a+3r) \right]$
Used designations: $\rho_1 = \sqrt{\frac{2e}{1+e}}$; $\rho_2 = \sqrt{\frac{2e}{1-e}}$; $\sigma_1 = \sqrt{\frac{a\,(1+e)-r}{2ae}}$; $\sigma_2 = \sqrt{\frac{r-a\,(1-e)}{2ae}}$.		

R'_φ depends on the additional relative velocity of the particle and envelope's matter introduced by rotation: $R'_\varphi = c\ (\omega\,(r)\, r)^\alpha \cdot r^\beta$, where $\omega\,(r)$ is the distribution of the angular velocity of envelope's matter in the equatorial plane. Table 2 gives antiderivatives for the integrals in (3).

If the envelope's matter rotates by the Keplerian law $\omega\,(r) \sim r^{-3/2}$ then the orbit changes and the particle can be rotating by on a circular orbit inside the envelope. In the case of rotation by the law $\omega\,(r) \sim r^n\ (-3/2 < r < 0)$ the orbit turns into a quasi-stationary state when modifications of the major semi-axis and the eccentricity become small, but the orbit become elliptical and its apocenter lies inside the envelope.

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